CRR DISCUSSION PAPER SERIES B

Discussion Paper No. B-6

Short- and Long-term Effects of Economic Growth on Public Debt Dynamics

Atsumasa Kondo

April 2012

Center for Risk Research
Faculty of Economics
SHIGA UNIVERSITY

1-1-1 BANBA, HIKONE,
SHIGA 522-8522, JAPAN

Short- and Long-term Effects of Economic Growth on Public Debt Dynamics[1]

Atsumasa Kondo[2] Shiga University[3]

April 23, 2012

[1]I would like to thank Makoto Yano, Hiroshi Fujiu, Yasunobu Tomoda, Akihiko Yanase and other seminar participants at Mita office of the Institute of Economic Research, Kyoto University for their useful comments and advice.

[2]e-mail: a-kondo@biwako.shiga-u.ac.jp

[3]1-1-1 Banba, Hikone, Shiga, 522-8522, Japan

Abstract

This paper theoretically examines how public debt dynamics are influenced by slight changes in the economic growth rate. If the initial public debt level to budget surplus ratio is sufficiently low, then an improvement in the economic growth rate will reduce the public debt level in every time period. If the ratio exceeds a critical level, economic growth has opposite effects on public debt dynamics in the short-run and in the long-run; while economic growth decreases public debt level from a long-term perspective, it increases the debt level in the short-run.

Key Words: public debt dynamics, economic growth, dynamic general equilibrium

JEL Classification Numbers: H60, O11

1 Introduction

In light of the current worldwide depression, evident in the 2008 subprime mortgage issue and the 2010 Greek debt crisis, economists in many countries are seeking measures for the recovery of economic growth rates. Policy makers, especially those in countries suffering from fiscal government bankruptcy, are apprehensive that the slowdown in economic growth may have damaged their nation's fiscal standing. A decrease in production levels results in decreased tax revenues. However, a slowdown of economic growth leads to lower interest rates because less productivity implies that future goods will become more scarce and valuable relative to current goods. Hence, a slowdown may help a government to pay the interest on accumulated public debt. Does depression always impinge on a government's fiscal health? Under what conditions and in what sense does a slowdown of economic growth aggravate a government's standing? These are important issues that deserve formal theoretical analyses.

The purpose of this paper is to explore these questions using a simple dynamic general equilibrium (DGE) model with persistent economic growth, in which the growth rate is exogenously determined. Focus is placed on public debt dynamics in a DGE path. The way in which the dynamics are influenced by a slight change in the economic growth rate is theoretically examined. In this study, the dynamically optimizing behavior of consumers and their inter-temporal interactions through markets are fully taken into account.

A comparative analysis demonstrates under what conditions public debt decreases (or increases) when the economic growth rate is slightly improved. Three different scenarios are used in this paper to show the decrease or increase, using an initial level of public debt to fiscal balance ratio. (i) If the ratio is low, an improvement in economic growth rate reduces public debt level for every period. In that sense, economic growth benefits the government. (ii) If the ratio exceeds a critical level but is not too high, economic growth has opposite effects on public debt dynamics from short- and long-term viewpoints. Although a slight recovery of the economic growth rate reduces the public debt level from a long-term viewpoint, it increases the debt level temporarily. (iii) If the initial level of government liability to fiscal balance ratio is extremely high, economic growth will have a negative impact on the public debt dynamics for any time period. The critical level of the public debt to fiscal balance ratio is determined in this study—it depends on the discount factor of the future utilities of the consumers.

This study also offers further valuable insights. A recovery of the economic growth rate without fiscal reform may be dangerous given the high level of accumulated public debt. From a government's standpoint, the cost of increased interest payments may exceed the benefits from recovered tax revenue. Public debt levels may instead increase due to increased interest payments. This implies that depression countermeasures aimed at improving the economic growth rate should be accompanied by fiscal reconstruction when there is accumulated public debt.

This study is in a similar vein to the following research. Greiner and Semmler (2000) studied the effects of various types of policy regimes on economic growth rates using an endogenous growth model with productive public investment developed by Barro (1990) and Futagami et al. (1993). Ghosh and Mourmouras (2004) extended Greiner and Semmler's research to conduct welfare analyses. Groneck (2010) introduced utility enhancing public consumption in addition to productive public capital, and studied the growth and welfare effects of budgetary rules. They especially highlighted the so-called golden rule of public finance: a government is allowed to borrow only if the borrowed funds are intended to be used for productive public investment. Greiner, in his series of papers (2007, 2008, 2011), assumed that the ratio of surplus to gross income linearly depends on a debt income ratio—that assumption assures that public debt is sustainable as demonstrated by Bohn (1998). Futagami et al. (2008) introduced a target level of public debt into an endogenous growth model with productive public capital, and showed the existence of two steady states. In their analyses, government bonds and income tax had clearly different effects on the growth rates in the steady states. However, they did not investigate effects of economic growth on the equilibrium trajectory of public debt.

Many studies concerning the roles of public debt use overlapping generation models. Among others, Bräuninger (2005) studied the role of a government deficit to physical capital ratio within a framework of an endogenous growth model. With a fixed ratio of budget deficit to GDP, he demonstrated that if the deficit ratio unexpectedly increases, the growth rate will decline. Yakita (2008) introduced productive public investments into a Bräuninger-type model. Bräuninger and Yakita commonly showed that a threshold level exists. If public debt is greater than the threshold level at the initial time point, it is no longer sustainable. According to Yakita, the threshold level positively correlates with a level of public capital. In contrast, this paper uses a DGE model with infinitely living consumers, and investigates how public debt dynamics are affected by the GDP growth rate though changes in interest rates and tax revenues.

This paper postulates that an economy grows at a constant rate, which means the absolute level of its economic activity, e.g., GDP, expands to infinity as time elapses. Bewley (1982) and Yano (1984, 1998) studied DGE models without relying on specific assumptions, and proved that if consumers discount their future utilities sufficiently weakly, then the equilibrium path will eventually lie in a small neighborhood of the stationary state that is independent from initial conditions. In their models, however, feasible paths of allocation remained in bounded areas. Jensen (2006) studied a DGE model that allowed for unbounded growth, and showed that the balanced growth path was stable if consumers discounted their future utilities sufficiently weakly. Kondo (2012) exploited a simple DGE model with persistent economic growth, in which sources of growth were improvements in productivity and population growth, and studied the effects of the economic growth rate on the public debt sustainability derived as a no-Ponzi game condition. A theoretical relationship between population growth rate and population size itself on public debt sustainability was highlighted in that paper. Although this paper is strongly influenced by those previous studies, the studies did not consider the relationships between public debt dynamics and economic growth rates. This paper assumes, for simplicity, that the economy is always on a balanced growth path, and largely investigates the effects of the economic growth rate on the public debt level along the equilibrium stream.

The remainder of this paper is structured as follows. The next section offers a model economy. Section 3 derives an equilibrium path. The main analysis is included in Section 4. In Section 5, numerical examples are used in graphs to show the public debt paths. The final section concludes the paper.

2 Model

The economy in the model includes discretely indexed time points $t = 0, 1, 2, \cdots$. The period between time $t-1$ and t is called period t. There are many identical consumers and a government. The government imposes on every consumer a lump-sum style tax T_t that is in a consumption good form, and provides an interest-bearing bond B_t^G. The bond generates an interest rate $1 + r_t$ from the period t to $t + 1$. The consumers and the government transact goods for consumption and bonds in each period.

The representative consumer holds a bond asset B_{t-1} at the beginning of the period t. The consumer receives a consumption good endowment Y_t, consumes it C_t and pays a tax T_t to the government in that period. Budget constraints for the consumer are given by

$$B_t \leq Y_t - T_t - C_t + (1 + r_{t-1})B_{t-1}, \qquad \text{for any } t = 1, 2, \cdots . \tag{1}$$

The consumer obtains utility from the consumption. The utility function is represented by log-form: $u(C) = \log C$. The future utilities are discounted relative to the current consumption with a discount factor $\beta \in (0, 1)$. The behavior of the representative consumer is summarized as the following maximizing problem:

$$\begin{aligned} &\max_{\{C_t, B_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} \log C_t \\ &\text{s.t. Equation (1)} \\ &\text{given } \{r_t\}_{t=0}^{\infty}, \ B_0 \end{aligned} \tag{2}$$

The government sets up a stream of policy variables $\left\{B_t^G, T_t, G_t\right\}$ taking initial debt level B_0^G as given, where G_t is a level of spending during the period t. The government must be subject to flow budget constraints

$$B_t^G = G_t - T_t + (1 + r_{t-1}) B_{t-1}^G, \qquad \text{for any } t = 1, 2, \cdots . \tag{3}$$

Note that the government does not have a clear purpose, i.e. it does not maximize some objective functions. The government's spending is denoted by G_t.

The market clearing conditions are as follows:

$$C_t + G_t = Y_t; \qquad B_t = B_t^G; \qquad \text{for any } t = 1, 2, \cdots . \tag{4}$$

A time stream of price and allocation of resources in the economy is determined so that the conditions (2)-(4) are simultaneously satisfied.

The analyses presented below do not impose inter-temporal budget constraints for the consumers or the government. The parameter constellation is not necessarily compatible with the no-Ponzi game conditions, and outcomes from the bond market diverge to $+\infty$ or $-\infty$.

3 Equilibrium Path

This section explicitly derives the equilibrium path. (See Lemma 1.) Based on the results here, the following section will offer the main analyses.

To solve the equilibrium, the following assumptions are made.

Assumption 1 $Y_{t+1}/Y_t = G_{t+1}/G_t = T_{t+1}/T_t = 1+\theta$ for any $t = 1, 2, \cdots$.

Assumption 2 $\beta < 1+\theta$.

Assumption 3 $0 \leq G_1 < T_1 < Y_1$.

Assumption 4 $B_0^G > 0$.

Because the primary concern of this paper is the effects of economic growth on public debt dynamics $\left\{B_t^G\right\}$, the other aspects of the model economy are kept as simple as possible; government spending, tax levies, and production level are assumed to grow at a constant rate $1+\theta$ (Assumption 1). Assumption 2 guarantees that the net interest rate r_t is positive for any t. (See Lemma 1-(B).) There is no need to explain Assumption 3. Assumption 4 implies the government is a debtor; this paper considers a situation in which the government will repay its fiscal liability with its tax revenue.

All endogenous variables in the equilibrium are determined, with initial conditions $\left(r_0, B_0^G, T_1, G_1, Y_1\right)$, the consumer's subjective discount factor β and the economic growth rate $1+\theta$ as parameters.

Lemma 1 *Endogenous variables along the equilibrium path are as follows:*

$$
\begin{aligned}
(A)\ C_t &= (1+\theta)^t (Y_1 - G_1); \qquad (B)\ 1+r_t = \frac{1+\theta}{\beta}; \\
(C)\ B_t^G &= \left(\frac{1+\theta}{\beta}\right)^{t-1}(1+r_0) B_0^G - (1+\theta)^{t-1}\left[\sum_{s=0}^{t-1}\left(\frac{1}{\beta}\right)^s\right](T_1 - G_1).
\end{aligned}
$$

for any $t = 1, 2, \cdots$.

Proof. (A) On the one hand, the first order conditions for the consumer's optimization problem (2) are as follows:

$$
\frac{C_{t+1}}{C_t} = \beta(1+r_t), \qquad \text{for any } t = 1, 2, \cdots. \tag{5}
$$

On the other hand, (4) and Assumption 1 yield

$$\frac{C_{t+1}}{C_t} = \frac{Y_{t+1} - G_{t+1}}{Y_t - G_t} = 1 + \theta. \tag{6}$$

By (5) and (6), the desired result is obtained.

(B) The result can be obtained from (5) and (6).

(C) Because of the flow budget constraints of the government (3), the public debt dynamics in the equilibrium must be subject to the first-order difference equation,

$$B_t^G = \frac{1+\theta}{\beta} B_{t-1}^G - (1+\theta)^{t-1} (T_1 - G_1), \qquad \text{for any } t = 2, 3, \cdots, \tag{7}$$

with an initial condition of

$$B_1^G = (1 + r_0) B_0^G - (T_1 - G_1). \tag{8}$$

The result is obtained from (7) with (8). ■

Two comments can be made here concerning the equilibrium interest rate. First, as shown in Lemma 1-(B), the interest rate $1 + r_t$ positively depends on the economic growth rate $1 + \theta$, which is exogenous in this paper. The improvement in the economic growth rate implies that the provision of consumption goods in future periods becomes more abundant relative to that in the current period. Thus, the recovery of the economic growth rate increases the interest rate, which may aggravate the government's fiscal health. In spite of the fact that the tax revenue will increase in future periods, the economic growth produces a high level of interest payment burden for the government.[1] Second, because this paper assumes $\beta \in (0, 1)$, it holds that $1 + r_t \,(= (1+\theta)/\beta) > 1 + \theta$, i.e., the Domar condition (1944) is not satisfied. Thus, the sustainability of the government's debt is not an obvious problem. See Kondo (2012) for further details regarding the required conditions for initial levels of public debt B_0^G to be compatible with no-Ponzi game conditions.

[1]In contrast, consumers always benefit from economic growth because the consumption level increases in every period when the economic growth rate is improved.

4 Effects of Economic Growth

This section reveals under what conditions a slight improvement in the economic growth rate positively (or negatively) affects the public debt path in the equilibrium. Consider a situation in which economic productivity is improved during the first period (period 1) and the future expectations for the economic growth rate $1+\theta$ are increased. Consumers revise their expectations and change their current and future behavior within the new environment given the initial conditions $\left(r_0, B_0^G, T_1, G_1, Y_1\right)$. Markets quickly and fully reflect the changes and the equilibrium path is re-determined, captured by Lemma 1. The government's flow budget constraint for each period (3) is also influenced. The government's burden for the interest payment will be greater, while its net tax revenue $T_t - G_t$ is expected to uniformly increase throughout the time horizon.

To compare the positive and negative aspects of economic growth on public debt dynamics, it is most effective to investigate the equation in Lemma 1-(C). The first term of the right hand side of that equation regards the negative (public debt-increasing) effect of economic growth, while the second term concerns its positive (public debt-decreasing) aspect. As seen in the equation, the two effects of economic growth asymmetrically affect public debt dynamics. For an illustration, I especially examine B_3^G. By (7),

$$\begin{aligned} B_3^G &= \frac{1+\theta}{\beta} B_2^G - (1+\theta)^2 (T_1 - G_1) \\ &= \frac{1+\theta}{\beta}\left[\frac{1+\theta}{\beta} B_1^G - \underbrace{(1+\theta)(T_1 - G_1)}_{(a)}\right] - \underbrace{(1+\theta)^2 (T_1 - G_1)}_{(b)}. \quad (9) \end{aligned}$$

The negative effects are obvious; economic growth increases interest rates (the interest rate-effect). In contrast, (a) and (b) in (9) represent the positive effects. The term (b) represents that economic growth increases net tax revenues in the period 3, $T_3 - G_3$. The positive effect on $T_2 - G_2$ is captured by (a), and the values in the square brackets represent the effect of economic growth that reduces the public debt level B_2^G. The first term of (9) reflects the compounding effects of the interest rate-effect and the public debt B_2^G reducing effect.

To completely capture the marginal effects of economic growth on the public debt path, this paper investigates the sign of partial derivatives $\partial B_t^G / \partial \theta$

(for any $t = 1, 2, \cdots$). If $\partial B_t^G / \partial \theta < 0$, then a slight improvement in the economic growth rate will reduce the public debt level at period t. In other words, the positive aspect of economic growth for the government exceeds the negative, which may seem intuitively plausible. However, if $\partial B_t^G / \partial \theta > 0$, then economic growth increases the public debt level at period t. In this sense, economic growth may impinge on a government's fiscal health. Although the latter does not necessarily support our expectation, the analyses below demonstrate that it does occur within particular ranges of the parameter values.

A simple calculation from Lemma 1-(C) yields

$$\begin{aligned} \frac{\partial B_t^G}{\partial \theta} &= (t-1)\left(\frac{1+\theta}{\beta}\right)^{t-2} \frac{1}{\beta}(1+r_0)\, B_0^G \\ &\quad - (t-1)(1+\theta)^{t-2} \left[\sum_{s=0}^{t-1} \left(\frac{1}{\beta}\right)^s\right](T_1 - G_1), \end{aligned} \tag{10}$$

for any $t = 1, 2, \cdots$. Thus, the condition that determines the sign of $\partial B_t^G / \partial \theta$ is derived as follows:

$$\frac{\partial B_t^G}{\partial \theta} \gtreqqless 0 \Longleftrightarrow \frac{(1+r_0)\, B_0^G}{T_1 - G_1} \gtreqqless \frac{1-\beta^t}{1-\beta} \equiv \Psi_t \tag{11}$$

for any $t = 1, 2, \cdots$. As shown in (11), the ratio of the initial level of the public debt to fiscal surplus, $(1+r_0)\, B_0^G / (T_1 - G_1)$, is crucial for determining the sign of $\partial B_t^G / \partial \theta$. The partial derivative $\partial B_t^G / \partial \theta$ changes its sign when the ratio crosses a critical value Ψ_t, which is defined in (11). The critical value Ψ_t has the following properties:

$$\Psi_1 = 1; \tag{12}$$

$$\Psi_t \in \left(1, \frac{1}{1-\beta}\right) \quad \text{for any } t = 2, 3, \cdots; \tag{13}$$

$$\Psi_t \uparrow \frac{1}{1-\beta} \quad \text{as } t \longrightarrow \infty. \tag{14}$$

The expression (14) means that the critical value Ψ_t monotonically converges to its limit value $1/(1-\beta)$ from below as time elapses.

Based on the results obtained above, the effects of economic growth can be analyzed using the three cases with differing ratios of public debt level to fiscal surplus $(1+r_0)\, B_0^G / (T_1 - G_1)$.

Case (i): $0 < (1 + r_0) B_0^G / (T_1 - G_1) < 1$.

First, consider the case in which the ratio of the initial level of public debt to fiscal surplus is relatively low. In this case, it holds by (11)-(13) that $\partial B_t^G / \partial \theta < 0$ for any $t = 1, 2, \cdots$. That is, a slight improvement in the economic growth rate decreases the public debt level in every period. In this sense, economic growth has had a positive impact on the government's fiscal standing both in the short- and long-term viewpoints. The benefits from the increased tax revenue due to the economic growth exceed the costs by the additional interest payment. Conversely, if the economic growth slows, then the public debt uniformly increases from now to the future, which supports our expectation.

Case (ii): $1 < (1 + r_0) B_0^G / (T_1 - G_1) < 1 / (1 - \beta)$.

This case represents a medium level public debt to tax revenue ratio. Because the critical value Ψ_t is monotonically increasing, it crosses ratio $(1 + r_0) B_0^G / (T_1 - G_1)$ from below. Thus, from a short-term viewpoint (for sufficiently small t), Ψ_t is smaller than $(1 + r_0) B_0^G / (T_1 - G_1)$, and, by (11), $\partial B_t^G / \partial \theta > 0$ holds, i.e., the public debt level increases due to economic growth. In the long-run (for sufficiently large t), however, the critical level Ψ_t overtakes $(1 + r_0) B_0^G / (T_1 - G_1)$. By (11), the derivative $\partial B_t^G / \partial \theta$ will show the negative sign. That is, the public debt level decreases thanks to economic growth in the future periods.

In summary, economic growth has opposite effects on the public debt path in short- and long-term standpoints. It has temporary negative impacts for the government, while its positive aspects will surpass its negative aspects after sufficiently long periods.

Case (iii): $1 / (1 - \beta) < (1 + r_0) B_0^G / (T_1 - G_1)$.

The final case describes the situation where the ratio $(1 + r_0) B_0^G / (T_1 - G_1)$ is extremely high. In this case, it holds by (11) that $\partial B_t^G / \partial \theta > 0$ for any $t = 1, 2, \cdots$. A slight improvement in the economic growth rate increases the public debt level in every period. In this sense, the economic growth has a negative impact on the government's fiscal standing. The merits of the increase in tax revenues from the economic growth are not sufficient to compensate the burden from the additional interest payments under a situation in which there is a high level of accumulated public debt.

The results obtained in this section can be summarized as the following theorem.

Theorem 1
(i) If $(1+r_0)\,B_0^G/\,(T_1-G_1) < 1$*, it holds that* $\partial B_t^G/\partial\theta < 0$ *for any* $t = 1, 2, \cdots$*. In this sense, economic growth has a positive impact on the government's fiscal health in every time period.*

(ii) If $1 < (1+r_0)\,B_0^G/\,(T_1-G_1) < 1/\,(1-\beta)$*, there exists a time period* t^* *such that* $\partial B_t^G/\partial\theta > 0$ *(for any* $t = 1, 2, \cdots, t^*-1$*) and* $\partial B_t^G/\partial\theta < 0$ *(for any* $t = t^*+1,\ t^*+2\cdots$*). A slight improvement in the economic growth rate has negative impacts on a government's fiscal standing from a short-term viewpoint, while positive aspects of the economic growth will surpass the negative aspects from a long-term viewpoint.*

(iii) If $1/\,(1-\beta) < (1+r_0)\,B_0^G/\,(T_1-G_1)$*, it holds that* $\partial B_t^G/\partial\theta > 0$ *for any* $t = 1, 2, \cdots$*. In this sense, the economic growth negatively affects the government's fiscal health in any time period.*

As mentioned in the introduction, one of the valuable insights offered by this study is that a recovery of the economic growth rate without fiscal reform may be dangerous if there exists a high level of accumulated public debt. Although case (i) indicates that economic growth can help a government to reduce the public debt level, the presence of condition (i) $(1+r_0)\,B_0^G/\,(T_1-G_1) < 1$ for case (i) is not realistic for many developed countries, for example, Germany, Italy, and Japan. Cases (ii) and (iii) demonstrate that a slight improvement in the economic growth rate increases public debt, at least temporarily. This can be thought of as a warning for economists and policy makers; depression countermeasures aimed at improving the economic growth rate should be accompanied by fiscal reconstruction if there are high levels of accumulated public debt.

5 Numerical Examples

This section provides numerical examples that illustrate the time evolution of public debt paths and the effects of economic growth.

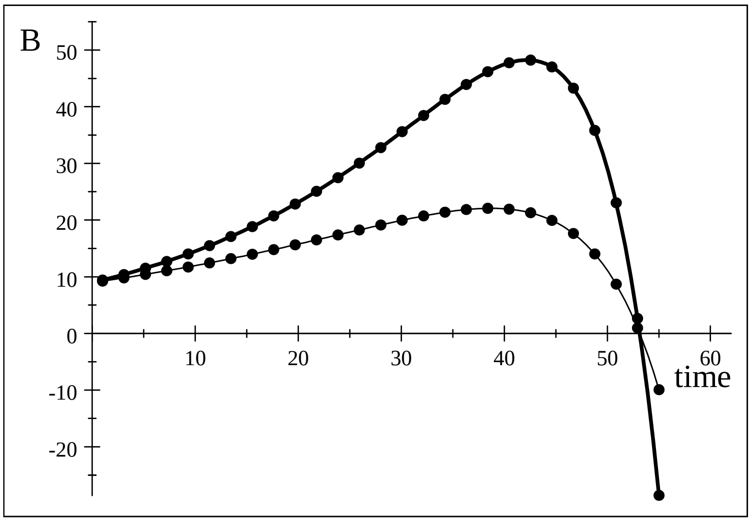
First, the parameter values are set at $\beta = 9/10$, $T_1 - G_1 = 1$ and $(1+r_0)\,B_0^G = 29/3$, as shown in case (ii) discussed in the previous section. With a discount rate of that value, the critical level Ψ_t defined in the previous section is

$$\Psi_t = 10 - 10\left(\frac{9}{10}\right)^t.$$

Further, the economic growth rates are set as $1+\theta_1 = 103/100$ and $1+\theta_2 = 105/100$. Then, the public debt dynamics $\left\{B_t^G\right\}$ correspond to the economic growth rates values, $1+\theta_1$ and $1+\theta_2$, as follows:

$$\begin{aligned} 1+\theta_1 \quad &: \quad B_t^G = 9\left(\frac{103}{100}\right)^t - \frac{1}{30}\left(\frac{103}{90}\right)^t \text{ and} \\ 1+\theta_2 \quad &: \quad B_t^G = 9\left(\frac{21}{20}\right)^t - \frac{1}{30}\left(\frac{7}{6}\right)^t. \end{aligned}$$

The former is drawn with the thin curve in the figure below, and the latter with the bold curve. The two graphs demonstrate that public debt dynamics may temporarily increase if there is a slight improvement of the economic growth rate.



6 Concluding Remarks

This paper used a simple DGE model showing persistent economic growth. It examined the effects of a slight improvement in the economic growth rate on public debt dynamics. I believe that these issues will be of significant interest to both economists and policy makers in many countries, especially those suffering from prolonged periods of depression and a fiscal crisis. However, the model exploited in this research is fairly simple, and some extensions should be conducted in future studies. These are discussed below.

First, this paper analyzed public debt dynamics without regard to physical capital. Although this assumption simplified the analyses, it is desirable to take capital accumulation into account. How are the dynamics of the debt-capital ratio affected by a slight recovery in the economic growth rate? This presents an interesting research topic. Second, the assumption regarding the lump sum-style tax should be changed to take into account a real-world complex tax system. Third, productive or utility enhancing government investment and capital formation should be explicitly included. Finally, welfare analysis should be implemented. In this paper, as pointed out in Footnote 1, consumers always benefited from economic growth. It may be an interesting research topic to consider the possibility where an increased public debt aggravates consumers in some way.

References

[1] Barro, R. J. (1990) "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, Vol. 98, 103–125.

[2] Bewley,T. (1982): "An Integration of Equilibrium Theory and Turnpike Theory," *Journal of Mathematical Economics*, 10, 233-267.

[3] Bohn, H. (1998): "The Behavior of U. S. Public Debt and Deficits," *Quarterly Journal of Economics*, 113, 949-963.

[4] Bräuninger, M. (2005): "The Budget Deficit, Public Debt, and Endogenous Growth," *Journal of Public Economic Theory*, Vol. 7, No.5, 827-840.

[5] Domar, E. D. (1944): "The "Burden of the Debt" and the National Income," *American Economic Review*, Vol. 34, No. 4, 798-827.

[6] Futagami, K., Iwaisako, T. and Ohdoi, R. (2008): "Debt Policy Rule, Productive Government Spending, and Multiple Growth Paths," *Macroeconomic Dynamics*, Vol. 12, No. 4, 445-462.

[7] Futagami, K., Morita, Y. and Shibata, A. (1993): "Dynamic analysis of an endogenous growth model with public capital," *Scandinavian Journal of Economics*, Vol. 95, No. 4, 607-625.

[8] Ghosh, S. and Mourmouras, I. A. (2004): "Endogenous growth, welfare and budgetary regimes," *Journal of Macroeconomics*, Vol. 26, No. 4, p. 623-635.

[9] Greiner, A. and Semmler, W. (2000): "Endogenous growth, government debt and budgetary regimes," *Journal of Macroeconomics*, Vol. 22, Issue 3, 363-384.

[10] Greiner, A. (2007): "An Endogenous Growth Model with Public Capital and Sustainable Government Debt," *Japanese Economic Review*, Vol. 58, No.3, 345-361.

[11] Greiner, A. (2008): "Human capital formation, public debt and economic growth," *Journal of Macroeconomics*, Vol. 30, Issue 1, 415-427.

[12] Greiner, A. (2011): "Economic Growth, Public Debt and Welfare: Comparing Three Budgetary Rules," *German Economic Review*, Vol. 12, Issue 2, 205-222.

[13] Groneck, M. (2010): "A Golden Rule of Public Finance or a Fixed Deficit Regime? Growth and Welfare Effects of Budget Rules," *Economic Modelling*, 27, 523-534.

[14] Jensen, M., (2006): "On Unbounded Growth with heterogenous Consumers," *Journal of Mathematical Economics*, 42, 807-826.

[15] Kondo, A. (2012): "A Note on Public-Debt Sustainability in an Economy with Declining Fertility," forthcomming in *FinanzArchiv/Public Finance Analysis*.

[16] Yakita, A. (2008): "Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting," *Journal of Public Economics* 92, 879-914.

[17] Yano, M., (1984): "Competitive Equilibria on Turnpikes in a McKenzie Economy, I: A Neighborhood Turnpike Theorem," *International Economic Review*, 25, 695-717.

[18] Yano, M. (1998): " On the Dual Stability of a von Neumann Facet and the Inefficacy of Temporary Fiscal Policy," *Econometrica*, 66, 427-451.