CRR DISCUSSION PAPER SERIES B

Discussion Paper No. B-4

A Note on Public Debt Sustainability in an Economy with Declining Fertility
(Revised version)

Atsumasa Kondo

October 2011

Center for Risk Research
Faculty of Economics
SHIGA UNIVERSITY

1-1-1 BANBA, HIKONE,
SHIGA 522-8522, JAPAN

A Note on Public Debt Sustainability in an Economy with Declining Fertility

Atsumasa Kondo*

January 5, 2012

Abstract

This note studies the relationship between population movement and a government's long-term fiscal health. It uses an equilibrium model with dynamic optimization to investigate a situation in which a government repays its fiscal liabilities using tax revenues. The focus is on an upper bound of fiscal liability, which is compatible with a No-Ponzi-game condition. The investigation deals, in particular, with how the upper bound depends on population size, population growth rate and primary fiscal balance per person.

Key Words: public debt sustainability, population growth rate, No-Ponzi-game condition, dynamic general equilibrium

JEL Classification Numbers: E62, H6

1 Introduction

The sustainability of public debt has been a hot topic for a long time. In his seminal work, Domar (1944) focused on a condition for public debt to be sustainable: public debt should not grow faster than GDP. Recently, Greiner (2007, 2008a) used endogenous growth models with infinitely lived consumers. In his 2007 paper, Greiner introduced public capital as a production factor and conducted a simulation to investigate how a public financing investment influences the economy. Greiner's 2008a paper took into account

*I would like to thank Bernd Genser, Makoto Yano, and two anonymous referees for their useful comments and suggestions.

a school for accumulating human capital. A government's expenditures for educational investment were financed by tax revenues and public deficit. Greiner assumed in both papers that the ratio of surplus to gross income linearly depends on a debt income ratio. That assumption, as demonstrated by Bohn (1998), assures that public debt is sustainable. Using a fixed budget deficit to GDP ratio, Bräuninger (2005) explored the relationship between a deficit ratio and growth rates within an endogenous growth framework with overlapping generations. He demonstrated that if the deficit ratio unexpectedly increases, then the growth rate will gradually decline. Building on Bräuninger's work, Yakita (2008) took public capital formation into account. Bräuninger and Yakita showed that a threshold level exists. If public debt is greater than the threshold level at the initial time point, it is no longer sustainable. According to Yakita, the threshold level positively correlates with public capital. In these theoretical studies, however, the relationship between population growth and public debt sustainability is not investigated. Greiner (2007, 2008a) and Yakita (2008) assumed that population size is constant over time. Bräuninger (2005) adopted AK technology, and population size did not play a decisive role with respect to assessing the sustainability in his model.

The main purpose of this note is to reveal how population growth rates affect a government's long-term fiscal health by using a dynamic equilibrium model with long-term optimization. Many developed countries are experiencing population declines. Because decreasing populations may result in lower tax revenues in the future, it is critical for governments to recover population growth rates and population size itself to maintain a healthy financial standing. Although economists and policy makers in such countries have sought measures aimed at raising fertility rates, such countermeasures may have large costs. Given tight budgetary constraints, it is necessary for government officials to carefully assess benefits of recovering fertility rates.

The condition of not being in debt in the future is logically equivalent to a No-Ponzi-game (or pyramid scheme) (NPG) condition. This study derives an upper bound of public debt that is compatible with a dynamic general equilibrium (DGE) and NPG condition. That attempt was conducted by Greiner (2008b). (Please refer to Section 3.2 in his paper (2008b).) Greiner performed a general study and did not rely on a specialized model. Concentrating on a specific model, the present study explicitly reveals how the upper bound depends on population size, population growth rate and primary fiscal deficit per person. Furthermore, the following questions are theoretically examined:

(a) When a population growth rate declines slightly, to what extent does a government need to increase public surplus to sustain a NPG condition? and; (b) To what extent should population size be enlarged to sufficiently compensate for a declining fertility rate? These are important questions for economists and policy makers in many developed countries (e.g., Italy, Germany and Japan) that are suffering from both a declining population and a fiscal crisis.

This note is based on a broad range of studies. Since Solow (1956), many studies on standard growth theory have examined the population growth. Barro and Becker (1988, 1989) treated population growth rate as an endogenous variable with sound micro-foundations. Benhabib and Nishimura (1989) showed that the Barro-Becker models can produce an endogenous cycle of population size. This contrasted with the original Barro-Becker models in which the stationary state is strictly stable and the equilibrium path is immediately trapped in the stationary state.

Yano (1984, 1998) and Kondo (2008) analyzed DGE models with many heterogeneous consumers; however, population growth and public debt sustainability were not considered. The reason was these researchers were primarily concerned with turnpike properties; that is, an equilibrium path will be trapped in a small, steady state area independent of initial conditions. This note assumes, for simplicity, that a decline in the population growth rate is exogenously given and that consumers are homogeneous. It aims to highlight the significant effects of a slight change in population growth rate on the long-term fiscal health of governments.

Neumeyer and Yano (1995) was among the first pair of researchers to make use of a NPG condition to analyze a monetary DGE model and the effects of monetary and fiscal policies beyond a national border. However, this team did not explicitly derive the condition of initial debt compatible with a NPG condition. Kondo (2007) studied the upper bound of a government's fiscal liabilities in a closed economy version of the Neumeyer-Yano model. Kondo and Kitaura (2009) theoretically investigated how deflation affected the upper bound in relation to a government's fiscal policy. However, Neumeyer and Yano and Kondo and Kitaura did not consider changes in population size.

Many empirical studies have investigated whether public debt is sustainable. These include studies of the United States by Hamilton and Flavin (1986) and Japan by Fukuda and Teruyama (1994), Ihori et al. (2001) and Broda and Weinstein (2005). Greiner, Koeller and Semmler (2006, 2007) conducted such studies for various countries. However, these empirical studies did not focus on a theoretical relationship between the upper bound and demographic changes.

This note proceeds as follows. The following section builds a base model for the research. Section 3 proposes a set of assumptions on which this study hinges and derives an equilibrium path. The NPG condition is defined and explicitly derived in Section 4. The main analysis of this note is placed in Section 5. Finally, Section 6 provides concluding remarks.

2 Model

Consider a dynamic economy with discrete time points indexed by $t = -1, 0, 1, \cdots$. The period between time $t-1$ and t is called period t. Here, there are many identical consumers and a government. The population size at period t is N_t, and the population growth rate is $1+n = N_t/N_{t-1}$ for $t = 0, 1, \cdots$. The government imposes a lump-sum style tax τ_t that is in a consumption good form on every consumer, and provides an interest-bearing bond B_t^g.[1] The bond generates an interest rate of $1+r_t$ from the period t to $t+1$. The consumers and the government transact goods for consumption and bond at each period.

The representative consumer in period t may have $1+n$ brothers or sisters. The number of this consumer's children is also $1+n$. In this setting, the (gross) population growth rate in our model economy is $1+n$. The representative consumer inherits a bond asset $b_{t-1}/(1+n)$ at time $t-1$ from her parents and transfers b_t to her children. The consumer receives a consumption good endowment y_t, consumes it c_t and pays a tax τ_t to the government. Budget constraints are given by

$$b_t \leq y_t - \tau_t - c_t + (1+r_{t-1})\frac{b_{t-1}}{1+n}, \qquad t = 0, 1, \cdots . \tag{1}$$

The consumer living in period t obtains utility over her consumption $c_t\,(>0)$ and her children's welfare with a discount factor $\beta \in (0,\ 1/(1+n))$. A period-wise utility function is represented in the log-form. Hence, the utility level of the consumer in period t is

$$U_t = \log c_t + \beta \cdot (1+n)\, U_{t+1}, \tag{2}$$

[1] Bräuninger (2005), Greiner (2007, 2008) and Yakita (2008) adopted an income tax in their models instead of a lump-sum tax.

where U_{t+1} is the utility level of the representative consumer in the period $t+1$.[2] The behavior of the representative consumer living in the initial time period 0 is summarized as the following maximizing problem:

$$\begin{aligned} &\max_{\{c_t, b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left[\beta\left(1+n\right)\right]^t \log c_t \qquad (3) \\ &\text{s.t. Equation (1)} \\ &\text{given } \{r_t\}_{t=-1}^{\infty}, \; b_{-1} \end{aligned}$$

The government sets up a stream of policy variables $\{B_t^g, \tau_t, g_t\}$, where g_t is a level of spending per person. Total tax revenue is $T_t = N_t \tau_t$ and total government spending is $G_t = N_t g_t$, $t = 0, 1, \cdots$. The government must be subject to flow budget constraints

$$B_t^g = G_t - T_t + \left(1 + r_{t-1}\right) B_{t-1}^g, \qquad t = 0, 1, \cdots . \qquad (4)$$

The market clearing conditions are as follows:

$$N_t c_t + G_t = N_t y_t; \qquad N_t b_t = B_t^g; \qquad t = 0, 1, \cdots . \qquad (5)$$

A time stream of price and allocation of resources in the economy is determined so that (5) is satisfied. Although the bond market's clearing conditions are redundant according to Walras's law with an initial condition $B_{-1}^g = N_{-1} b_{-1}$, this study focuses on the time evolution of an outcome from the bond market.

3 Equilibrium Path

This section explicitly derives the equilibrium path. (See Lemma 1.) Public debt dynamics are especially focused on in this research. Based on results here, the next section will characterize a condition for the public debt dynamics to satisfy a NPG condition.

First, note that the necessary conditions of the consumer's optimization problem (3) yield

$$\frac{c_{t+1}}{c_t} = \beta\left(1 + r_t\right), \qquad t = 0, 1, \cdots . \qquad (6)$$

To solve the equilibrium, the following assumptions are made.

[2]Because Bräuninger (2005) and Yakita (2008) use overlapping generations models, the altruistic preference beyond generations is not considered.

Assumption 1. $y_t/y_{t-1} = g_t/g_{t-1} = \tau_t/\tau_{t-1} = 1+\theta$ for $t = 0, 1, \cdots$.

Assumption 2. $0 \leq g_{-1} < \tau_{-1} < y_{-1}$.

The primary concern of this note is the effects of population growth rate on a government's long-term fiscal standing. Therefore, other conditions—e.g. the growth rate of government spending, tax levies and productivity—are fixed for the sake of simplicity.[3] As a direct result, the GDP at the period t can be simply expressed as

$$Y_t = (1+n)^{t+1}(1+\theta)^{t+1} N_{-1} y_{-1}. \tag{7}$$

The remainder of this note demonstrates that the sustainability of public debt is influenced by population growth rate $1+n$, though the growth rate of the primary fiscal balance per person $(\tau_{t+1} - g_{t+1})/(\tau_t - g_t)$ is assumed to be constant at the rate $1+\theta$.

Given policy parameters (τ_{-1}, g_{-1}), initial conditions (r_{-1}, b_{-1}) and a per person production level y_{-1}, all endogenous variables in the equilibrium are determined.

Lemma 1 *Endogenous variables along the equilibrium path satisfy the following:*

$$\begin{aligned}
(A)\ c_t &= (1+\theta)^{t+1}(y_{-1} - g_{-1}); \qquad (B)\ 1 + r_t = \frac{1+\theta}{\beta}; \\
(C)\ B_t^g &= \left[\sum_{j=0}^{t}\left(\frac{1+\theta}{\beta}\right)^j \left[(1+n)(1+\theta)\right]^{t-j+1}\right] N_{-1}(g_{-1} - \tau_{-1}) \\
&\quad + \left(\frac{1+\theta}{\beta}\right)^t (1+r_{-1}) B_{-1}^g;
\end{aligned}$$

for $t = 0, 1, \cdots$.

Proof. (A) follows from (5) and the assumptions above.
(B) immediately follows from (A) and (6).

[3] Bräuninger (2005) and Yakita (2008) assumed that an increase in public debt $B_t - B_{t-1}$ to GDP ratio is fixed. As a result, the tax rate was determined endogenously. In contrast, this paper assumes that the tax rate per person is exogenous. Its assumption is similar to that used by Neumeyer and Yano (1995) and Kondo (2007). In such a setting, the public debt B_t is endogenously determined.

(C) Because of flow budget constraints on the government (4), the equilibrium debt dynamics must be subject to the difference equation

$$B_t^g = (1+n)^{t+1}(1+\theta)^{t+1} N_{-1}(g_{-1} - \tau_{-1}) + \frac{1+\theta}{\beta} B_{t-1}^g, \quad t = 1, 2, \cdots, \tag{8}$$

with an initial condition

$$B_0^g = (1+n)(1+\theta) N_{-1}(g_{-1} - \tau_{-1}) + (1 + r_{-1}) B_{-1}^g. \tag{9}$$

Note that $1 + r_{-1}$ is exogenously given. The desired result is derived from (8) with (9). ■

Note that in the present economy, the long-term interest rate $1 + r$ ($= 1 + r_t = (1+\theta)/\beta$) is greater than $(1+n)(1+\theta)$ because the condition $(0 <)\, \beta < 1/(1+n)$ is assumed.

4 NPG Condition

This section derives the upper bound of public debt that is sustainable along the equilibrium path. It reveals the way in which the upper bound depends on population growth rate (or the total fertility rate) and population size. (See Theorem 1.) At the outset, the term "sustainability" is formally defined.

Definition 1 *The public debt dynamics $\{B_t^g\}$ is said to be sustainable if the NPG condition*

$$\limsup_{T\to\infty} \left(\prod_{t=1}^{T} \frac{1}{1 + r_{t-1}} \right) B_T^g \leq 0 \tag{10}$$

is satisfied along an equilibrium path.

The definition of "sustainability" is theoretically appropriate. The reason is under flow budget constraints (4), the NPG condition (10) is equivalent to the following budget constraint on the government:

$$(1 + r_{-1}) B_{-1}^g + \sum_{t=0}^{\infty} \left(\prod_{j=1}^{t} \frac{1}{1 + r_{j-1}} \right) G_t \leq \sum_{t=0}^{\infty} \left(\prod_{j=1}^{t} \frac{1}{1 + r_{j-1}} \right) T_t. \tag{11}$$

Indeed, the definition (10) has been widely adopted in various studies —e.g. Hamilton and Flavin (1986), Greiner (2007, 2008) and Kondo (2007).[4]

By using Lemma 1, the NPG condition (10) can be rewritten as

$$
\begin{aligned}
NPG &\Longleftrightarrow \limsup_{t\to\infty}\left(\frac{\beta}{1+\theta}\right)^t B_t^g \le 0 \qquad (12)\\
&\Longleftrightarrow B_{-1}^g \le \frac{1}{1+r_{-1}}\frac{(1+n)(1+\theta)}{1-\beta(1+n)}N_{-1}(\tau_{-1}-g_{-1})
\end{aligned}
$$

This result is summarized as the following theorem.

Theorem 1 *The upper bound of public debt that is sustainable along an equilibrium path is identified as*

$$
\varphi \equiv \frac{1}{1+r_{-1}}\frac{(1+n)(1+\theta)}{1-\beta(1+n)}N_{-1}(\tau_{-1}-g_{-1}) \qquad (13)
$$

$$
= \frac{1}{1+r_{-1}}\frac{1+n}{1-\beta(1+n)}N_{-1}(\tau_{0}-g_{0}). \qquad (14)
$$

The equation (13) shows how the upper bound φ is influenced by the population growth rate $1+n$, the initial population size N_{-1} and the primary fiscal balance at the per person level $\tau_{-1}-g_{-1}$. Although each of these three factors ($1+n$, N_{-1}, $\tau_{-1}-g_{-1}$) positively correlates with φ, the manner in which each factor affects φ is different. The upper bound φ is linear both in N_{-1} and in $\tau_{-1}-g_{-1}$, but it is convex in $1+n$ under Assumption 2.

As was noted in the Introduction, the present study may be thought of as a special case of Greiner's recent work (2008b). According to Greiner (2008b), who dealt with a continuous time model, the threshold level of the initial debt-to-GDP ratio compatible with the NPG condition is given by

$$
\left(\frac{B_{-1}}{Y_0}\right)_{crit} = m\int_0^\infty e^{-(C_1(\mu)-C_2(\mu))}d\mu, \qquad (15)
$$

where m is the constant upper bound of the primary surplus-to-GDP ratio $(T_t-G_t)/Y_t$. This is smaller than one because the primary surplus must be financed out of the GDP Y_t and where $C_1(\mu)=\int_0^\mu r_s ds$ and $C_2(\mu)=$

[4] Bräuninger (2005) and Yakita (2008) highlighted the conditions guaranteeing that the public debt to private capital ratio does not diverge as time elapses.

$\int_0^{\mu} \left(\dot{Y}_s / Y_s \right) ds$. In the present study, the net interest rate r_t and growth rate GDP $\dot{Y}_s / Y_s$ are constant over time. Their values are $r_t = (1+\theta)/\beta - 1$ and $\dot{Y}_s / Y_s = n + \theta + n\theta$, respectively. Thus, the right hand side of (15) is equal to

$$\frac{\tau_0 - g_0}{y_0} \frac{\beta}{(1+\theta)[1-\beta(1+n)]},$$

and

$$(B_{-1})_{crit} = \frac{\tau_0 - g_0}{y_0} \frac{\beta}{1+\theta} \frac{1}{1-\beta(1+n)} N_0 y_0.$$

This critical value $(B_{-1})_{crit}$ coincides with φ in the present study when $1 + r_{-1} = (1+\theta)/\beta$.

Next, let me explain roles of productivity growth rates and population growth rates for the sustainability of the public debt. Although, as shown in (7), the population growth rate $1+n$ and the productivity growth rate $1+\theta$ contribute to GDP growth in the same manner, how these influence the upper bound φ is markedly different. (See (13) or (14).) Given interest rate $1 + r_{-1}$, an unexpected improvement to the productivity growth rate $1+\theta$ at the beginning of the period 0 positively affects the upper bound because it generates additional tax revenue for government. Meanwhile, (14) shows that an anticipated change of $1+\theta$ does not raise the upper bound. High productivity results in a high interest rate because it implies that future supply of goods become more abundant relative to current goods supply, and a high interest rate results in an additional interest payment burden on government. On the other hand, the population growth rate $1+n$ does not affect interest rates. If per person productivity is fixed, a high population growth rate does not change the value of future goods relative to current goods. (See Lemma 1-(B).) That is the reason why $1+n$ and $1+\theta$ affect the upper bound φ differently.

In what follows below, the productivity growth rate $1+\theta$ will be fixed, and (14) will be used instead of (13). The reason is this note highlights population growth rate, population size and budget surplus in relation to public debt sustainability.

5 Effects of Demographic Changes

This section investigates the effects of demographic changes on the sustainable upper bound of public debt φ, which was derived in the previous section.

5.1 Marginal Effects and Elasticities

This subsection highlights the relative degrees of dependence of the upper bound on the net population growth rate n, the population size N_{-1} and tax increases (or spending cuts) $\tau_0 - g_0$. To assess these dependencies, they are compared with the elasticities of the upper bound.

The marginal effects are calculated using (14):

$$\frac{\partial \varphi}{\partial n} = \frac{1}{1+r_{-1}} N_{-1}(\tau_0 - g_0) \frac{1}{[1-\beta(1+n)]^2}, \tag{16}$$

$$\frac{\partial \varphi}{\partial N_{-1}} = \frac{1}{1+r_{-1}} (\tau_0 - g_0) \frac{1+n}{1-\beta(1+n)}, \tag{17}$$

$$\frac{\partial \varphi}{\partial(\tau_0 - g_0)} = \frac{1}{1+r_{-1}} \frac{1+n}{1-\beta(1+n)} N_{-1}. \tag{18}$$

The elasticities are as follows:

$$\frac{\partial \varphi}{\partial n}\frac{n}{\varphi} = \frac{n}{(1+n)[1-\beta(1+n)]}, \tag{19}$$

$$\frac{\partial \varphi}{\partial N_{-1}}\frac{N_{-1}}{\varphi} = \frac{\partial \varphi}{\partial(\tau_0 - g_0)}\frac{(\tau_0 - g_0)}{\varphi} = 1. \tag{20}$$

With (19) and (20), the following holds

$$\frac{\partial \varphi}{\partial n}\frac{n}{\varphi} \gtreqless \frac{\partial \varphi}{\partial N_{-1}}\frac{N_{-1}}{\varphi} \left(\text{or } \frac{\partial \varphi}{\partial(\tau_0 - g_0)}\frac{(\tau_0 - g_0)}{\varphi}\right) \tag{21}$$

$$\iff \sqrt{\beta^{-1}} \lesseqgtr 1+n \iff \sqrt{\frac{1+r}{1+\theta}} \lesseqgtr 1+n,$$

where $1 + r \,(= (1+\theta)/\beta)$ is the long-term interest rate.

The elasticity of the upper bound φ with respect to the net population growth rate n depends on the subjective discount factor of consumers β and n itself, while those with respect to N_{-1} and $\tau_0 - g_0$ are always one. The condition (21) shows that if the net population growth rate n is relatively large in comparison to the ratio of interest rate $1+r$ and productivity growth rate $1+\theta$, then the upper bound depends more on n than on N_{-1} or $\tau_0 - g_0$.

The result obtained in this subsection can be summarized as the following theorem:

Theorem 2 *If* $(0 <)1+n < \sqrt{(1+r)\,/\,(1+\theta)}$ *is satisfied, then the upper bound of the public debt that is compatible with the NPG condition* φ *is more elastically dependent on the population size* N_{-1} *(or the government's budget surplus* $\tau_0 - g_0$*) than the net population growth rate* n *in the sense that* $\frac{\partial\varphi}{\partial n}\frac{n}{\varphi} < \frac{\partial\varphi}{\partial N_{-1}}\frac{N_{-1}}{\varphi}$ $\left(\text{or } \frac{\partial\varphi}{\partial(\tau_0-g_0)}\frac{(\tau_0-g_0)}{\varphi}\right)$*, where* $1+r$ *is the long-term interest rate. If* $\sqrt{(1+r)\,/\,(1+\theta)} < 1+n\,(< (1+r)\,/\,(1+\theta))$ *is satisfied, then the upper bound* φ *is more elastically dependent on the net population growth rate* n *than the population size* N_{-1} *(or* $\tau_0 - g_0$*) in the sense that* $\frac{\partial\varphi}{\partial n}\frac{n}{\varphi} > \frac{\partial\varphi}{\partial N_{-1}}\frac{N_{-1}}{\varphi}$ $\left(\text{or } \frac{\partial\varphi}{\partial(\tau_0-g_0)}\frac{(\tau_0-g_0)}{\varphi}\right)$.

Policy implications may help to interpret this theorem. As pointed out in the introduction, a government that attempts to improve its fiscal standing should assess the benefits of its policy instruments carefully. To raise the upper bound φ, what type of policy regime is most effective? Theorem 2 implies that when the net population growth rate n is low, relative to the ratio of the long-term interest rate and productivity growth rate, then the government should actively pursue population size N_{-1} enlargement policies such as immigration or foreign labor policies or try to improve its primary per person fiscal balance $\tau_0 - g_0$. Once the population growth rate recovers to within the range $\sqrt{(1+r)\,/\,(1+\theta)} < 1+n < (1+r)\,/\,(1+\theta)$, the government should increase the population growth rate using policies, such as child-care support or subsidies for childbirth, to effectively raise the upper bound φ. The reason why such a reversal occurs is that the upper bound is convex with respect to $1+n$, while it linearly depends on N_{-1} and $\tau_0 - g_0$.

5.2 Marginal Rate of Substitution

This subsection investigates the marginal rates of substitution with respect to the "indifferent curve" of φ to answer questions (a) and (b) posed in the introduction.

By implicitly differentiating Equation (14), the following holds:

$$\frac{\partial\left(\tau_0-g_0\right)}{\partial n} = -\frac{\tau_0-g_0}{\left(1+n\right)\left[1-\beta\left(1+n\right)\right]}, \tag{22}$$

$$\frac{\partial N_{-1}}{\partial n} = -\frac{N_{-1}}{\left(1+n\right)\left[1-\beta\left(1+n\right)\right]}. \tag{23}$$

Suppose that the population growth rate is rectified slightly downward at $t = -1$, meaning a reduction in the expected net tax revenue $\{T_t - G_t\}$.[5] This loss can be compensated for in two ways: (a) adjust tax (or cut spending) or (b) increase population size to yield additional tax revenue in the future. Equations (22) and (23) estimate to what extent the other variables ($\tau_0 - g_0$ and N_{-1}, respectively) need to be adjusted to avoid the government's fiscal bankruptcy.

The following theorem is established:

Theorem 3 *Assume that the expected population growth rate $1+n$ declines slightly at the beginning of period 0. To maintain the upper bound φ at a constant level $\frac{1}{1+r_{-1}}\frac{1+n}{1-\beta(1+n)}N_{-1}(\tau_0-g_0)$, two types of measures can be considered: (a) increase taxes by $\left(\tau_0-g_0\right)/\left(1+n\right)\left[1-\beta\left(1+n\right)\right]$ or (b) recover population size by $N_{-1}/\left(1+n\right)\left[1-\beta\left(1+n\right)\right]$.*

6 Concluding Remarks

This note explicitly derives the upper bound of public debt that satisfies a NPG condition in a DGE model of population growth. Although the issues examined here are of sufficient interest to economists and policy makers in the many developed countries experiencing both a declining population and fiscal crisis, the model used in the present study is somewhat simple. To obtain more exact guidelines for economic policies, the extensions mentioned below should be conducted as possible future works.

This note has analyzed public debt dynamics without regard to physical capital. Although this assumption outstandingly simplifies the analysis, it is desirable to take capital accumulation into account. How are debt-capital ratio dynamics compatible with public debt sustainability and influenced by

[5] The long-term population growth rate is difficult to forecast accurately. Indeed, the expected total fertility rate has often been adjusted.

population growth? This presents an interesting research topic. Another direction for further research is an examination of the relationship between public debt sustainability and population growth within endogenous growth settings. Finally, the assumption regarding a lump-sum style tax should be changed to take account of a real-world complex tax system.

References

Barro, R. J. and Becker, G. S., (1989): "Fertility Choice in a Model of Economic Growth," *Econometrica* 57, 481–501.

Becker, G. S. and Barro, R. J., (1988): "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics* 103, 1–25.

Benhabib, J. and Nishimura, K., (1989): "Endogenous Fluctuations in the Barro-Becker Theory of Fertility," in *Demographic Change and Economic Development*, Wening, A. and K. F. Zimmermann (eds.), Springer-Verlag.

Bohn, H., (1998): "The Behavior of U. S. Public Debt and Deficits," *Quarterly Journal of Economics*, 113, 949-963.

Bräuninger, M., (2005): "The Budget Deficit, Public Debt, and Endogenous Growth," *Journal of Public Economic Theory*, Vol. 7, No.5, 827-840.

Broda, C. and Weinstein, D.E., (2005): "Happy News from the Dismal Science: Reassessing the Japanese Fiscal Policy and Sustainability," in Takatoshi Ito, Hugh Patrick and David E. Weinstein(eds.), Reviving Japan's Economy, The MIT Press, p.39-78.

Domar, E. D., (1944): "The "Burden of the Debt" and the National Income," *American Economic Review*, Vol. 34, No. 4, 798-827.

Fukuda, S. and Teruyama, H., (1994): "The Sustainability of Budget Deficits in Japan," *Hitotsubashi Journal of Economics*, 35, 109-119.

Greiner, A., (2007): "An Endogenous Growth Model with Public Capital and Sustainable Government Debt," *Japanese Economic Review*, Vol. 58, No.3, 345-361.

Greiner, A., (2008a): "Human Capital Formation, Public Debt and Economic Growth," *Jounal of Macroeconomics*, Vol. 30, Issue 1, 415-427.

Greiner, A., (2008b): "Sustainability of Public Debt: Some Theoretical Considerations," SSRN Working Paper, available at
http://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=22496

Greiner, A., Koeller, U. and Semmler, W., (2006): "Testing the sustainability of German fiscal policy: evidence for the period 1960–2003," *Empirica*, 33, 127-140.

Greiner, A., Koeller, U. and Semmler, W., (2007): “Debt Sustainability in the European Monetary Union: Theory and Empirical Evidence for Selected Countries,” *Oxford Economic Papers*, Vol. 59, 194–218.

Hamilton, J. D. and Flavin, M. A. (1986): “On the Limitations of Government Borrowing: A Framework for Empirical Testing,” *American Economic Review*, 76, 808-816.

Ihori, T., Doi, T. and Kondo, H., (2001): “Japanese Fiscal Reform: Fiscal Reconstruction and Fiscal Policy,” *Japan and the World Economy*, 13, 351-370.

Kondo, A., (2007): “On the Sustainability of Government Borrowing in a Dynamic General Equilibrium,” *Pacific Economic Review*, 12, No. 5, 565-576.

Kondo, A., (2008): “On the Inefficacy of Temporary Policy in a Dynamic General Equilibrium with Money,” *Japanese Economic Review*, 59, 324-344.

Kondo, A. and Kitaura, K., (2009): “Does Deflation Impinge on a Government’s Fiscal Standing?,” *Pacific Economic Review*, 14, No. 5, 651-656.

Neumeyer, P., and Yano. M., (1995): “Cross Border Nominal Asset and International Monetary Interdependence,” University of Southern California, Los Angeles, California, manuscript.

Solow, R., (1956): “A Contribution to the Theory of Economic Growth: A Contribution to the Theory of Economic Growth,” *Quarterly Journal of Economics*, 70, 65-94.

Yakita, A., (2008): “Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting,” *Journal of Public Economics* 92, 879-914.

Yano, M., (1984): “The Turnpike of Dynamic General Equilibrium Paths and its Insensitivity to Initial Conditions,” *Journal of Mathematical Economics*, 13, 235-254.

Yano, M., (1998): “ On the Dual Stability of a von Neumann Facet and the Inefficacy of Temporary Fiscal Policy,” *Econometrica*, 66, 427-451.

Atsumasa Kondo
Shiga University, Department of Economics
1-1-1 Banba, Hikone, Shiga, 522-8522
Japan
e-mail: a-kondo@biwako.shiga-u.ac.jp