CRR DISCUSSION PAPER SERIES A

Discussion Paper No. A-9

A Wage Offer Model with Replacement Hiring and Firing Tax

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September 2013

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September 10, 2013

Abstract

This paper constructs a wage offer model in which firms can employ at most one worker and workers wait for another higher wage offer on the job. In this circumstance, firms hiring an employee must fire the employee if they willing to employ another worker. Given the setting, I examine the effects of the firing tax on the offered wages under (i) workers and firms are identical with no breach, (ii) workers and firms are identical with single breach, and (iii) workers differ in flow utility when unemployed.

Keywords: wage offer; on-the-job search; replacement hiring

JEL classification: J63; J64

* All remaining errors are, of course, my own.

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1 Introduction

In standard job search models assuming two-sided search, many authors have assumed that firms can open at most one vacancy so that they can employ at most one worker. This assumption does not seem to be practical, however, it makes analysis as simple as possible and is not mattered when production function is linear; letting u denotes the measure of unemployed workers and v stands for the measure of vacant jobs, the rate that a worker finds a vacant job and the rate that a firm with vacancy finds an unemployed worker are determined by its ratio, $\theta \equiv v/u$, so-called tightness of the market. Such a setting means that it is equivalent that "one firm employs ten workers" and "ten firms employ one worker in each firm" because the finding rate of each agent, the law of motion of the rate of unemployment, and output are same in the whole of economy.

In contrast to a two-sided search model, Burdett and Mortensen (1998) which is close to the current paper provides a wage posting model. In the model, a continuum of employers choose permanent wage offers and a continuum of workers receive the offers. In such circumstances, the wages offered are continuously distributed because the higher the wage offer and the less profit, the larger applicants whereas the lower the wage offer and the larger profit, the smaller applicants. They shows that there is a positive relation between the labor force size and the wage paid, and that there is negative association between the wage offers and quit rates across employers. The model allows firms to employ more than one worker so that capacity of employee is excluded. This setup suggests that firms do not need to fire the current employee if a new applicant comes.

Burdett et al. (2004) and Kiyotaki and Lagos (2007) advance models in which agents have a limit of the number of partners so that if a new candidate of partner comes to an agent and the agent is willing to match with the new candidate, the agent must break up the partnership and if the agent matches with the new candidate, the partner loses the current agent; so-called *single breach* or *double breach*. The former arises when an agent in existing partnership matches with an agent who have no partner. As a consequence, single breach results in "one partnership and one agent with no partner" to "another partnership and another agent with no partner". The latter arises when an agent in existing partnership matches with an agent who is in another partnership. Hence, double breach results in "two partnerships" results in "one partnership and two agents with no partner". This means that, in a job-worker match framework, if a firm hiring an employee is willing to hire another employee, the firm must fire the current employee and hire the new employee (replacement hiring).

As another related study, Postel-Vinay and Robin (2002) constructs an equilibrium search model in which workers differ in ability and firms differ in the marginal productivity of efficient labor and unemployed workers search for a job and employees search for a better job. In contrast to Burdett and Mortensen (1998), Postel-Vinay and Robin (2002) assumes that firms can vary their wage offers according to the characteristics of a worker they meet and that firms can make the counter offers received by their employees from another firms. Consequently, they make with a source that wage can increase within the firm for the employee who receives it.

This paper constructs a model that combines the wage offer model (but firms cannot make counter offers) with the idea of replacement hiring. To make the analysis as simple as possible, departing from the setting of a continuous wage offer distribution as in Burdett and Mortensen (1998), I assume that the wage offer distribution is two-point; that is, all of the firms offer a wage whether a lower one (which is reservation wage for workers by optimization) or a higher one. [1] In addition, to emphasize and examine the effects of replacement hiring on the equilibrium outcome, I assume that firms must pay their employee a firing tax if they fire the worker.

The main results of this paper is follows. First, when both workers and firms are identical and there is no replacement hiring, the fraction of the lower wage offers is increasing in the arrival rate because when workers frequently receive a wage offer, firms can easily employ a worker so

[1] A model of replacement hiring with continuous wage offers makes analysis quite complex. This is because (i) employed workers have much opportunity that they can receive a new (higher) wage offer according to the wages of workers they receive now, and (ii) there are many potential workers those who come to a particular firm as new candidates according to the wages of they receive now. To avoid this complexity, I assume that a wage offer distribution is two-point.

that firms tend to make the lower wage offer to make a higher profit. In addition, the fraction of the lower wage offers is decreasing in the dissolution rate. Intuitively, this is because when a job-worker match is frequently broken, it is difficult to continue production for a long time on average. In particular, since firms offering the lower wage face a possibility that the employee leaves the current job by receiving a higher wage offer, it makes more difficult to continue production. As a consequence, firms tend to make the higher wage offer. Second, when both workers and firms are identical and there is replacement hiring, the fraction of firms offering the lower wage is increasing in both the arrival rate and the firing tax. The intuition is follows. When the arrival rate is high, all of the firms frequently replace the current worker with a new worker. Even though the firing tax is common to all firms, firms offering the higher wage is less beneficial than firms offering the lower wage because replacement occurs more frequent in firms with the higher wage. This suggests that the firing tax compensates for replaced workers, however, it is at the cost of many of lower wage offers. Third, when workers differ in valuation of utility when unemployed, the fraction of firms offering the lower wage is decreasing (increasing) in the arrival rate when the difference of utility when unemployed is large (small). Finally, the fraction of firms offering the lower wage is decreasing in the dissolution rate whereas increasing in the firm's productivity. In addition, when both of the wage offers is determined as reservation, the firing tax does not affect on the fraction of firms offering the lower or higher wage.

The rest of the paper is organized as follows. Section 2 studies a benchmark case in which both workers and firms are identical. In this section it provides a canonical wage offer model that embodies replacement hiring and a firing tax. Section 3 analyzes the case in which firms are identical but workers differ in valuation on utility when unemployed. Section 4 treats the case in which workers are identical but firms differ in productivity. Section 5 concludes.

2 The Benchmark Case

To see the simplest case that I investigate in later sections, I describe the model in which both workers and firms are identical in this section. Consider an infinite horizon economy with search-frictional labor market. The market operates in continuous time and is populated by a continuum of workers and firms. At a moment in time, workers are either employed or unemployed and firms are in production or vacant. All of the agents discount future at rate r. The measure of workers is normalized to unity. As in Burdett and Mortensen (1998), for the sake of simplicity, I do not consider a matching technology.

Workers receive a wage offer w at the rate λ. I assume that there is on-the-job search so that all of the workers search a job whether they are unemployed or on the job and that the rate of offer is constant over time; that is, the finding rates are same whichever a worker is employed or unemployed and a job is in production or vacant. [2] All of the job-worker match is exogenously dissolved at the rate δ which is assumed to be $\delta < \lambda$. Throughout this paper, I focus on the steady state.

When unemployed, workers receive flow utility b per instant. To make the analysis as simple as possible, I suppose that an offer that firms can send to a worker is either w_1 or w_2 ($w_1 < w_2$). Let ϕ be the fraction of firms that a wage w_1 offers (so the fraction of $1 - \phi$ firms offers w_2). When unemployed workers receive a wage offer $w \in \{w_1, w_2\}$, they decide whether accept or not. If accept, they work and receive a offered wage. If not, they are still unemployed and wait for another wage offer. Let U be the asset value when unemployed, and $W(w_i)$ denotes the asset value when employed with wage w_i ($i = 1, 2$). Given above, the worker's behavior is represented by the

[2] Of course, they might be different. Let λ^e denotes the finding rate when employed and λ^u stands for the finding rate when unemployed. If unemployed workers can seek a job for a longer time than employed workers, it may be $\lambda^e < \lambda^u$. On the other hand, if employed workers can easily obtain information about hiring requirements than unemployed workers, it may be $\lambda^e > \lambda^u$. This possibility, however, makes analysis more complex so that I do not distinguish the offer rate when unemployed from the offer rate when employed. For further details, see Burdett and Mortensen (1998).

following Bellman equations

$$rU = b + \lambda \left[\phi \max\{W(w_1) - U, 0\} + (1-\phi) \max\{W(w_2) - U, 0\}\right], \tag{1}$$

$$rW(w_1) = w_1 + \lambda(1-\phi)[W(w_2) - W(w_1)] + \delta[U - W(w_1)], \tag{2}$$

$$rW(w_2) = w_2 + \delta[U - W(w_2)]. \tag{3}$$

Eq.(1) shows that unemployed workers enjoy flow utility b per instant, and when they receive an offer at the rate λ, which is w_1 (w_2) with probability ϕ $(1-\phi)$, they accept the offer if it is more beneficial than being unemployed. If not, they reject the offer and are still unemployed. Eq.(2) represents that employed workers hired at a firm with wage w_1 has opportunity of job change. Since $w_2 > w_1$, it is more beneficial to work at a firm with wage w_2 so that a worker who receives a wage offer w_2 necessarily changes the current job (so the max operator is omitted). [3] Eq.(3) describes that workers hired at a firm with wage w_2 has no opportunity (or incentive) to change the current job and become unemployed by exogenous dissolution at the rate δ.

The offered wage w_1 is exogenous, however, its level is determined by the firm's optimization problem. From eq.(1), a wage offer that makes $W(w_1) - U < 0$ has no meaning for both workers and firms because the offer is not accepted at all. On the other hand, a wage offer w that makes $W(w_1) - U < W(w) - U < W(w_2) < U$ is also not optimal because the firm can gain more flow profit by reducing the wage level. Consequently, the lower offer, w_1, must hold $W(w_1) - U = 0$. Let $w_R(= w_1)$ be the offered wage that makes $W(w_R) - U = 0$, so-called the reservation wage. From the equations above, it is clear that $w_R = b$; that is, it equals to flow utility per instant when unemployed. [4]

[3] It can be easily shown from subtracting eq.(2) from eq.(3);

$$W(w_2) - W(w_1) = \frac{w_2 - w_1}{r + \delta + \lambda(1-\phi)} > 0.$$

[4] This result comes from the assumption that the rate of offer, λ, is constant over time. If they change over time as discussed in footnote 1, the difference of rates affects the reservation wage. For details, see Burdett and Mortensen (1998).

Firms are homogeneous but they offer a wage either $w_1(= w_R = b)$ or w_2. Let $J(w_i)$ $(i = 1, 2)$ stands for the asset value of firms with a wage w_i when production is in operation. Assuming that market entry is free, firms enter the market until the expected value of market entry becomes zero in the steady state. This means that the asset value of a vacancy, V, equals to zero. Given above, firm's behavior is represented by recursive formulae

$$rJ(b) = p - b + \lambda(1 - \phi)[V - J(b)] + \delta[V - J(b)], \tag{4}$$

$$rJ(w_2) = p - w_2 + \delta[V - J(w_2)], \tag{5}$$

where p denotes the flow output per instant.[5] Eq.(4) suggests that, when production is in operation, the firm's flow profit is $p - b$. However, if another firm offers a wage w_2 to the employee, a firm with a wage w_1 loses its employee (i.e., headhunting) and the job becomes vacant. In addition, the job-worker match is broken at the rate δ. On the other hand, eq.(5) shows that a firm with a wage w_2 makes a profit $p - w_2$ and does not face headhunting so that production is in operation as long as the exogenous dissolution does not occur.

Next, I derive the condition that makes firms to be indifferent to offer wages b and w_2. Let u be the rate of unemployment and μ be the fraction of employed workers receiving a wage b. [6] The rate that a firm offering a wage b can employ a worker is λu. This is because all of the unemployed workers accept the wage offer b with Poisson rate λ. On the other hand, the rate that a firm offering a wage w_2 can employ a worker is $\lambda[u + (1 - u)\mu]$. This is because a firm offering a wage w_2 can headhunt an employed worker who is receiving a wage b besides the all of unemployed workers. Therefore by making use of eqs.(4) and (5), the expected profits of each firm, $\pi(w_i)$ $(i = 1, 2)$, are

$$\pi(b) = \frac{\lambda u(p - b)}{r + \delta + \lambda(1 - \phi)} \tag{6}$$

[5]Letting $V(w_i)$ $(i = 1, 2)$ be the asset value of firms offering a wage w_i, it is formally represented as $rV(b) = -k + \lambda[J(b) - V(b)]$ and $rV(w_2) = -k + \lambda[J(w_2) - V(w_2)]$, where k is a cost during a vacancy.

[6]Note that $\phi \neq \mu$ because all of the wage offers is not necessarily send to a worker. Consequently, the fraction of firms offering a wage b (w_2), ϕ, does not coincide with the fraction of workers receiving a wage b (w_2), μ.

and

$$\pi(w_2) = \frac{\lambda[u + (1-u)\mu](p - w_2)}{r + \delta}. \tag{7}$$

In the steady state the rate of unemployment does not vary over time so that the inflow and outflow of unemployment (or employment) coincide. The unemployed workers receive a wage offer at the rate λ while the employed workers lose the job at the rate δ. As a result,

$$\dot{u} = \delta(1-u) - \lambda u = 0.$$

The fraction of employed workers receiving a wage b, μ, is given by a similar procedure. Since the rate that an unemployed worker receives a wage offer b is $\lambda\phi$, the inflow of workers those who receive a wage b is $\lambda\phi u$. On the other hand, the number of workers those who receive a wage b, $(1-u)\mu$, decreases when they lose their job by the exogenous dissolution at the rate δ and when they receive a wage offer w_2 at the rate $\lambda(1-\phi)$. The outflow of workers those who receive a wage b is thus $(1-u)\mu[\delta + \lambda(1-\phi)]$. Since both coincide in the steady state,

$$\lambda\phi u = (1-u)\mu[\delta + \lambda(1-\phi)]. \tag{8}$$

From the two expressions above, I obtain $\mu = \frac{\phi\delta}{\delta+\lambda(1-\phi)}$. Substituting this into the expected profits eqs.(6) and (7), the isoprofit condition, $\pi(b) = \pi(w_2)$, is represented as [7]

$$\frac{\delta}{\lambda} \cdot \frac{p - b}{r + \delta + \lambda(1-\phi)} = \left[\frac{\delta}{\lambda} + \frac{\phi\delta}{\delta + \lambda(1-\phi)}\right] \cdot \frac{p - w_2}{r + \delta}.$$

Solving this equation for ϕ, the isoprofit condition suggests some property of ϕ.

Proposition 1. *(i) Letting $r \to 0$, from the isoprofit condition, the fraction of firms offering a wage b, ϕ, becomes $1 - \frac{\delta}{\lambda}$. (ii) When $r \to 0$, the fraction of firms offering a wage b is increasing in the rate of offer λ, while it is decreasing in the dissolution rate δ.*

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[7] This condition guarantees that both wages $w_1 (= b)$ and w_2 are offered.

Proof. (i) Some algebra yields

$$\phi = 1 - \frac{\delta[(p-b)\delta - (p-w_2)(\lambda+\delta))]}{\lambda[(p-b)(r+\delta) - (p-w_2)(\lambda+\delta))]}.$$

It is derived that $\phi = 1 - \frac{\delta}{\lambda}$ when $r \to 0$. (ii) It is immediately obtained from $\frac{\mathrm{d}\phi}{\mathrm{d}\lambda} > 0$ and $\frac{\mathrm{d}\phi}{\mathrm{d}\delta} < 0$. (iii) This is because μ is increasing in ϕ from $\mu = \frac{\phi\delta}{\delta+\lambda(1-\phi)}$. ■

The intuition of Proposition 1 is as follows. When workers frequently receive a wage offer, firms can easily employ a worker (because the wage offer defined above is necessarily accepted). Consequently firms tend to make the lower wage offer b to make a higher profit. In addition, even though firms offering a wage b face a possibility of loss of worker by headhunting, they can easily employ another worker again and headhunting does not frequently occur since the fraction of firms offering the higher wage w_2, $1-\phi$, is low. In contrast to the rate of wage offer λ, the dissolution rate δ decreases ϕ. When a job-worker match is frequently broken, it is difficult to continue production for a long time on average. In particular, for firms offering a wage b, since they face a possibility of loss of worker by headhunting, it makes more difficult to continue production. As a result, firms tend to make a higher wage offer w_2. The assumption $\lambda > \delta$ excludes a possibility of a corner solution. If $\lambda < \delta$, all of the firms offer a wage w_2 since $\phi < 0$. Finally, the fraction of employed workers receiving a wage w_R, μ, is given by $\mu = \frac{1}{2}(1-\frac{\delta}{\lambda})$ when $r \to 0$. This indicates that the half of wage offer w_R is realized. For example, letting $\frac{\delta}{\lambda} = \frac{1}{2}$ so that the fraction of the wage offer, w_R, is $\phi = \frac{1}{2}$. Then the fraction of the employed worker receiving a wage w_R becomes $\mu = \frac{1}{4}$. Hence, even if the number of wage offers w_R and w_2 are same, the number of employed workers receiving each wages does not coincide.

3 Replacement of Workers

In contrast to Burdett and Mortensen (1998) which analyzes the measure of workers per firm earning a wage w, I consider the case in which firms can employ *at most* one worker hereafter.

That is, if a firm hiring a worker wants to employ a new worker, the previous employee must be fired and becomes unemployed. In other words, it arises a *single breach.* I assume that if a firm fires its employee, the firm must pay the firing tax T to the worker directly. In this section I assume that workers change the current job even if their employment status does not change at all. That is, if an employed worker earning a wage w receives a wage offer w, the worker leaves the current job and is employed by the new firm, and the current employer loses the worker. Of course, since all of the workers are identical in this section, all of the firms have no incentive to replace their worker, they even pay the firing tax. Nevertheless, to compare the case of homogeneous agents with the case of heterogeneous agents later, I briefly describe the model of replacement with homogeneous agents.

Given above, the worker's behavior is represented as

$$rU = b + \lambda \left\{ \phi[W(w_1) - U] + (1-\phi)[W(w_2) - U] \right\}, \tag{9}$$

$$\begin{aligned} rW(w_1) = {} & w_1 + \lambda(1-\phi)[W(w_2) - W(w_1)] + \delta[U - W(w_1)] \\ & + \lambda\phi[u + (1-u)\mu][U - W(w_1) + T], \end{aligned} \tag{10}$$

$$rW(w_2) = w_2 + \delta[U - W(w_2)] + \lambda(1-\phi)[U - W(w_2) + T]. \tag{11}$$

Eq.(9) shows that the asset value of unemployed workers is the same as the case of no replacement. Eq.(10) illustrates that employed workers receiving a wage w_1 have an opportunity to move a higher wage job at the rate $\lambda(1-\phi)$, face a possibility to lose the current job by replacement when an unemployed worker or an employed worker receiving a wage w_1 comes. Eq.(11) represents that employed workers receiving a wage w_2 face a possibility to lose the current job by replacement when an any unemployed or an employed worker comes. If employed workers are fired by replacement, they can receive hiring tax T but if they lose the job by the exogenous dissolution, they cannot receive hiring tax. [8] The reservation wage w_R also corresponds to w_1 here and is given by $U =$

[8] Since if an offered wage is the same as the current wage, the benefit is zero ($W(w_i) - W(w_i) = 0$), such a term is omitted in the above equations.

$W(w_1)$. Some algebra yields

$$w_R(=w_1) = b - \lambda\phi[u + (1-u)\mu]T.$$

This suggests that when there is replacement of workers, the reservation wage reduces because workers are compensated by the firing tax T.

Similarly, the firm's behavior is written as follows

$$rJ(w_R) = p - w_R + (\delta+\lambda)[V - J(w_R)] - \lambda\phi[u + (1-u)\mu]T, \tag{12}$$

$$rJ(w_2) = p - w_2 + [\delta + \lambda(1-\phi)][V - J(w_2)] - \lambda(1-\phi)T. \tag{13}$$

Eq.(12) represents that when the exogenous dissolution occurs and when the current employee receives a wage offer, the job-worker match is broken and the job becomes vacant. In addition, when an unemployed worker or an employed worker receiving a wage w_R comes, the firm replaces the current employee with the newcomer and must pay the hiring tax T. Eq.(13) shows that, in addition to the exogenous job destruction, the job-worker match is broken when the current worker receives a wage offer w_2. Finally, when an unemployed worker or an employed worker comes, replacement occurs and firm must pay the firing tax T.

The rate that a firm offering a wage w_i is filled is as follows. For a firm offering a wage w_R, the job is filled when any of unemployed workers or employed workers earing a wage w_R comes. So the rate is $\lambda[u + (1-u)\mu]$. Similarly, for a firm offering a wage w_2, since the job is filled when any of unemployed or employed workers comes, the rate is λ. Consequently, the expected profit for a firm offering a wage w_i, π_i, is

$$\pi(w_R) = \lambda[u + (1-u)\mu] \cdot \frac{p - w_R - \lambda\phi[u + (1-u)\mu]T}{r + \delta + \lambda} = \frac{\lambda[u + (1-u)\mu](p-b)}{r + \delta + \lambda}$$

and

$$\pi(w_2) = \frac{\lambda[p - w_2 - \lambda(1-\phi)T]}{r + \delta + \lambda(1-\phi)},$$

where I use $w_R = b - \lambda\phi[u + (1-u)\mu]T$. Next, I derive the flow condition. Since all of the unemployed workers receive a wage offer at the rate λ, the outflow from unemployment pool is λu.

On the other hand, the inflow to unemployment is given by two channels; the first is the exogenous dissolution and the second is replacement of workers. An employed worker receiving a wage w_R is replaced if an unemployed worker or an employed worker earning a wage w_R receives a wage offer at the rate $\lambda\phi$. In addition, an employed worker receiving a wage w_2 is replaced if any of unemployed or employed worker comes at the rate $\lambda(1-\phi)$. In aggregate, the condition that the outflow equals to the inflow is given by

$$\dot{u} = (1-u)[\delta + \lambda\mu\phi + \lambda(1-\mu)(1-\phi)] - \lambda u = 0.$$

The number of employed worker receiving a wage w_R is calculated as follows. On the one hand, it increases when an unemployed worker receive a wage offer w_R so that the number is $\lambda\phi u$. On the other hand, it decreases when an employed worker earning a wage w_R exogenously loses job, and when he receives a wage offer w_2 so that $[\delta + \lambda(1-\phi)](1-u)\mu$. Note that, since replacement of workers receiving a wage w_R does not change the number of them, it does not matter how many workers are replaced. Consequently, in the steady state, the number of employed worker receiving a wage w_R is

$$\lambda\phi u = [\delta + \lambda(1-\phi)](1-u)\mu.$$

From these two equations, I have $\mu = \frac{\delta+\lambda(1-\phi)}{\delta+\lambda(1-2\phi^2)}\phi$. By making use of this the isoprofit condition $\pi(w_R) = \pi(w_2)$ becomes

$$\frac{[\delta+\lambda(1-\phi)]^2}{\lambda[\delta+\lambda(1-2\phi^2)]} \cdot \frac{p - w_2 - \lambda(1-\phi)T}{r+\delta+\lambda(1-\phi)} = \left\{ \frac{[\delta+\lambda(1-\phi)]^2}{\lambda[\delta+\lambda(1-2\phi^2)]} + \frac{\delta+\lambda(1-\phi)}{\delta+\lambda(1-2\phi^2)}\phi \right\} \cdot \frac{p-b}{r+\lambda+\delta}.$$

Taking the limit $r \to 0$, the isoprofit condition gives some properties of ϕ.

Proposition 2. *(i) If $b + w_2 < \lambda T$, an interior solution $\phi \in (0,1)$ exists. (ii) In that range, the fraction of firms offering the lower wage b is increasing in the arrival rate λ and the firing tax T in the limit $r \to 0$.*

Proof. (i) Taking the limit $r \to 0$, the isoprofit condition becomes

$$\phi = 1 - \frac{b + w_2}{\lambda T}.$$

As a consequence, $\phi > 0$ is guaranteed by $b + w_2 < \lambda T$ and $\phi < 1$ is guaranteed by $b + w_2 > 0$ which holds by assumption. (ii) It is immediately from $\frac{\mathrm{d}\phi}{\mathrm{d}\lambda} > 0$ and $\frac{\mathrm{d}\phi}{\mathrm{d}T} > 0$.

■

The results above hold as long as $w_2 < b + \lambda(1 - T)$ (recall that it is required that $\lambda > \delta$ when there is no replacement). Notice that a higher offer rate λ given hiring tax T and a larger hiring tax T given an offer rate λ generally have the same effects because both implies a higher cost of replacement. Given this fact, the intuition of the proposition is as follows. When λ is large, all of the firms frequently replace the current worker with a new comer. Even though the firing tax T is common to all firms, firms offering a wage w_2 is less beneficial than firms offering a wage w_1 because replacement occurs more frequent in firms with wage offer w_2 than w_1. Or equivalently, given the rate of offer λ, a higher firing tax also makes firms with wage offer w_2 less beneficial. Therefore, a higher λ and T increases the number of firms offering the lower wage. This suggests that the firing tax compensates for replaced workers, however, it is at the cost of many of lower wage offers.

4 Heterogeneous Workers

In this section I treat the case in which workers differ in their characteristics. To make the analysis as simple as possible, I consider that the difference of characteristics is two types in the same manner as in the previous section.

Suppose that workers are different in the flow utility when unemployed (for example, the value of leisure) and that all of firms are identical. Let $b_j > 0$ be the flow utility when unemployed ($j = 1, 2$ and $b_1 < b_2$), and $U(b_j)$ denotes the asset value of unemployed workers whose valuation of leisure is b_j. Firms are assumed to choice a wage offer $w_i \in \{w_1, w_2\}$ $(w_1 < w_2)$. Let $W(w_i, b_j)$ stands for the asset value of employed workers receiving a wage w_i with valuation on leisure is b_j. As in the previous section, firms would set a wage w_1 to satisfy $U(b_1) = W(w_1, b_1)$. In addition, in this circumstance, it would be plausible that firms set a wage w_2 to satisfy $U(b_2) = W(w_2, b_2)$. This is because, if a firm offers a wage $w > w_2$ such that $U(b_2) < W(w, b_2)$, they can make more profit by reduction in the wage without a possibility that the wage offer is not accepted.

The asset values for workers are thus represented as

$$rU(b_1) = b_1 + \lambda \left\{ \phi[W(w_1, b_1) - U(b_1)] + (1-\phi)[W(w_2) - (b_1)] \right\}, \tag{14}$$

$$rU(b_2) = b_2 + \lambda \left\{ \phi \max\{W(w_1, b_2) - U(b_2), 0\} + (1-\phi)[W(w_2) - (b_2)] \right\}, \tag{15}$$

$$\begin{aligned} rW(w_1, b_j) = {} & w_1 + \lambda(1-\phi)[W(w_2, b_j) - W(w_1, b_j)] + \delta[U(b_j) - W(w_1, b_j)] \\ & + \lambda\phi[u + (1-u)\mu][U(b_j) - W(w_1, b_j) + T], \end{aligned} \tag{16}$$

$$rW(w_2, b_j) = w_2 + \delta[U(b_j) - W(w_2, b_j)] + \lambda(1-\phi)[U(b_j) - W(w_2, b_j) + T]. \tag{17}$$

These equations are almost same as the previous section except for a possibility that unemployed workers with b_2 may accept a wage offer w_1, as shown in eq.(15). This means that it may exist the case $W(w_1, b_2) - U(b_2) > 0$, however, it does not hold which I check below. To see this, I first derive the reservation wages w_1 and w_2 that satisfy $W(w_1, b_1) = U(b_1)$ and $W(w_2, b_2) = U(b_2)$;

$$w_1 = b_1 - \lambda\phi[u + (1-u)\mu]T,$$

and

$$w_2 = b_2 - \lambda(1-\phi)T + \lambda\phi \max\{W(w_1, b_2) - U(b_2), 0\}.$$

Next, from the Bellman equations above, I obtain

$$W(w_1, b_2) - U(b_2) = \frac{w_1 - b_2 + \lambda\phi[u + (1-u)\mu]T}{r + \lambda + \delta + \lambda\phi[u + (1-u)\mu]} = \frac{b_1 - b_2}{r + \lambda + \delta + \lambda\phi[u + (1-u)\mu]},$$

where I use $b_1 = w_1 + \lambda\phi[u + (1-u)\mu]T$. Since $b_1 < b_2$ by assumption, $W(w_1, b_2) - U(b_2) < 0$ so that an unemployed worker whose valuation on leisure is b_2 does not accept a wage offer w_1. As a consequence,

$$w_2 = b_2 - \lambda(1-\phi)T.$$

Let ζ denotes the fraction of workers those who have utility b_1 when unemployed. Here, since a wage offer w_1 is not accepted by workers with b_2, the Bellman equations for firms are

$$rJ(w_1) = p - w_1 + (\delta+\lambda)[V - J(w_1)] - \lambda\phi\zeta[u + (1-u)\mu]T, \tag{18}$$

$$rJ(w_2) = p - w_2 + [\delta + \lambda(1-\phi)][V - J(w_2)] - \lambda(1-\phi)T. \tag{19}$$

The last term of the RHS in eq.(18) describes that when an unemployed worker or an employed worker whose valuation on leisure is b_1 comes, the firm replaces the current worker with a new worker. Since a wage offer w_2 is accepted by all unemployed and employed workers, the difference of utility when unemployed does not affect the behavior of firms offering a wage w_2. Eq.(19) is thus same as the previous section. The rate that a firm offering a wage w_2 is also same as the previous section, however, the rate a firm offering a wage w_1 is now $\lambda\zeta[u + (1-u)\mu]$ since a wage offer w_1 is accepted by unemployed and employed workers with b_1. Hence, by making use of $w_1 = b_1 - \lambda\phi[u + (1-u)\mu]T$ and $w_2 = b_2 - \lambda(1-\phi)T$, the expected profit for each firms is

$$\pi(w_1) = \frac{\lambda\zeta[u + (1-u)\mu](p - w_1 - \lambda\phi[u + (1-u)\mu]T)}{r+\delta+\lambda} = \frac{\lambda\zeta[u + (1-u)\mu](p - b_1)}{r+\delta+\lambda},$$

and

$$\pi(w_2) = \frac{\lambda(p - w_2 - \lambda(1-\phi)T)}{r+\delta+\lambda(1-\phi)} = \frac{\lambda(p - b_2)}{r+\delta+\lambda(1-\phi)}.$$

These equations show that the expected profit for each firm does not depend on the firing tax T when both of offered wages w_1 and w_2 are determined as reservation wage. This is because these wages embed a potential payment of the hiring tax as shown in $w_1 = b_1 - \lambda\phi[u + (1-u)\mu]T$ and $w_2 = b_2 - \lambda(1-\phi)T$. In other words, workers accept a lower wage than income when unemployed b_i because they expect that they can receive the firing tax if they are fired. This makes the firm's flow profit constant so that it does not depend on the firing tax.

Next, the flow condition is rewritten as

$$(1-u)[\delta + \lambda\phi\mu\zeta + \lambda(1-\phi)(1-\mu)] = \lambda u[\zeta + (1-\zeta)(1-\phi)].$$

The LHS represents the inflow to unemployment pool; all of the employed workers exogenously lose the current job at the rate δ, employed workers receiving a wage w_1 is replaced at the rate $\lambda\phi\zeta$, and employed workers earning a wage w_2 is replaced at the rate $\lambda(1-\phi)$. On the other hand, the RHS illustrate the outflow from unemployment pool; unemployed workers with b_1 is hired at the rate λ and unemployed workers with w_2 is employed at the rate $\lambda(1-\phi)$. Similarly, the number of employees with wage w_1 is given by

$$\lambda\phi u\zeta = [(\delta + \lambda(1-\phi)](1-u)\mu\zeta \quad \Rightarrow \quad \lambda\phi u = [\delta + \lambda(1-\phi)](1-u)\mu.$$

Here, the increment of workers receiving a wage w_1 is $\lambda\phi u\zeta$ which indicates the number of unemployed workers with b_1 those who receive a wage offer w_1. On the other hand, the decrement of them is $[\delta + \lambda(1-\phi)](1-u)\mu\zeta$ which represents the number of workers those who lose their job exogenously at the rate δ and those who move to a new job with w_2. As the above equation indicates, the fraction ζ does not affect the condition that determines the number of employed workers earning a wage w_1. Consequently, the fraction of employed workers those who receive a wage w_1, μ, is

$$\mu = \frac{[\delta + \lambda(1-\phi)]\phi}{(\delta + \lambda)[1-(1-\zeta)\phi] - 2\lambda\zeta\phi^2} \equiv \mu(\phi),$$

which coincides to the value in the previous section when $\zeta = 1$ (all of the unemployed workers enjoy flow utility b). To make firms indifferent to offer a wage w_1 or w_2, the isoprofit condition becomes

$$\left[\frac{\phi + \lambda(1-\phi)}{\lambda\phi} + \mu(\phi)\right]\frac{p - b_1}{r + \delta + \lambda} = \frac{\delta + \lambda(1-\phi)}{\lambda\phi} \cdot \frac{p - b_2}{r + \delta + \lambda(1-\phi)},$$

where $\mu(\phi)$ is defined above. The property of ϕ when workers differ in valuation on leisure is summarized as follows.

Proposition 3. *In the limit $r \to 0$, the fraction of firms offering a wage w_1, ϕ, is increasing (decreasing) in the arrival rate λ when the difference of valuation of leisure $b_2 - b_1$ is small (large). In addition, ϕ is decreasing in the dissolution rate δ whereas increasing in the productivity p.*

Proof. See Appendix.

As Proposition 2 suggests, when valuation on leisure b is small, a higher arrival rate λ increases the fraction of firms offering the lower wage because a higher λ means a higher cost of replacement. Consequently, when λ is large, all of the firms frequently replace the current worker with a new comer. If b is common to all of the workers, this makes firms offering the higher wage w_2 less beneficial because (i) the flow profit is small and (ii) frequent replacement hiring reduces the expected benefit to offer the higher wage. However, if b is different among workers, the intuition becomes more complicate. To see the intuition as easy as possible, consider the case that b_2 is not too high whereas b_1 is low and constant; that is, the difference $b_2 - b_1$ is small. When the difference $b_2 - b_1$ is small, the difference of the flow profit $p - b_2$ and $p - b_1$ is also small. This makes advantage to offer the higher wage w_2 less attractive since the flow profit is almost same even if firms offer the lower wage. Consequently, firms tends to make the lower wage offer. The intuition that ϕ is decreasing in the dissolution rate δ is straightforward. When the dissolution rate δ is high, a job-worker match is frequently broken. This means that it is difficult to continue production for a long time. In particular, for firms offering the lower wage w_1, since they are prone to lose their employee because the employee may move to another firm (in addition to the exogenous dissolution). Hence, a higher δ decreases the fraction of firms offering the lower wage w_1. Finally, a higher productivity p increases ϕ because it makes the flow profit larger. Even though firms offering the lower wage w_1 frequently lose the current employee, they have the incentive to offer the lower wage because of a large profit $p - w_1$.

5 Conclusion

This paper have investigated a wage offer model in which there is on-the-job search and replacement hiring. Assuming that the offer wage is two-point distribution, the paper provides some properties of the fraction of firms offering the lower wage. When both workers and firms are identical and there is replacement hiring, the fraction of firms offering the lower wage is increasing in both the arrival rate and the firing tax. This is because when the arrival rate is high, all of the firms frequently replace the current worker with a new worker. So firms offering the higher wage is less beneficial than firms offering the lower wage because replacement occurs more frequent in firms with the higher wage even though the firing tax is common to all firms. In the whole of economy, this result highlights that the firing tax compensates for replaced workers, however, it is at the cost of many of lower wage offers.

Appendix

Proof of Proposition 3.

The isoprofit condition is

$$\left[\frac{\phi+\lambda(1-\phi)}{\lambda\phi}+\mu(\phi)\right]\frac{p-b_1}{r+\delta+\lambda}=\frac{\delta+\lambda(1-\phi)}{\lambda\phi}\cdot\frac{p-b_2}{r+\delta+\lambda(1-\phi)},$$

where

$$\mu(\phi)=\frac{[\delta+\lambda(1-\phi)]\phi}{(\delta+\lambda)[1-(1-\zeta)\phi]-2\lambda\zeta\phi^2}.$$

Differentiating this with respect to ϕ, I obtain

$$\mu'(\phi)=\frac{(\delta+\lambda)\{(\delta+\lambda)[2-(1-\phi)\zeta]+2\lambda\phi^2\}}{\{(\delta+\lambda)[1-(1-\zeta)\phi]-2\lambda\zeta\phi^2\}^2}$$

Taking the limit $r\to 0$, the isoprofit condition becomes

$$[\delta+\lambda(1-\phi)+\lambda\phi\mu(\phi)](p-b_1)-(\delta+\lambda)(p-b_2)=0,$$

and define

$$F(\phi)\equiv[\delta+\lambda(1-\phi)+\lambda\phi\mu(\phi)](p-b_1)-(\delta+\lambda)(p-b_2). \tag{A.1}$$

Differentiating this with respect to ϕ, the derivations is $F'(\phi)=-\lambda(p-b_1)[1-\mu(\phi)-\phi\mu'(\phi)]$ so that $1-\mu(\phi)-\phi\mu'(\phi)\lesseqgtr 0$ means $F'(\phi)\gtreqless 0$. From eq.(A.1) I have

$$\begin{aligned}\frac{\mathrm{d}F(\phi)}{\mathrm{d}\lambda}&=b_2-b_1-(p-b_1)[1-\mu(\phi)]\phi,\\ \frac{\mathrm{d}F(\phi)}{\mathrm{d}\delta}&=b_2-b_1>0,\\ \frac{\mathrm{d}F(\phi)}{\mathrm{d}p}&=-\lambda\phi[1-\mu(\phi)]<0.\end{aligned}$$

By making use of the implicit function theorem and assuming that $1-\mu(\phi)-\phi\mu'(\phi)>0$, I obtain $\frac{\mathrm{d}\phi}{\mathrm{d}\lambda}>0$ ($\frac{\mathrm{d}\phi}{\mathrm{d}\lambda}<0$) if b_2-b_1 is small (large), $\frac{\mathrm{d}\phi}{\mathrm{d}\delta}<0$, and $\frac{\mathrm{d}\phi}{\mathrm{d}p}>0$, which is summarized in Proposition 3.

■

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