CRR DISCUSSION PAPER SERIES A

Discussion Paper No. A-7

# Alteration in Skills and Career-Enhancing in a Frictional Labour Market

Yosuke Oda

September 2013

Center for Risk Research
Faculty of Economics
SHIGA UNIVERSITY

1-1-1 BANBA, HIKONE,
SHIGA 522-8522, JAPAN

# Alteration in Skills and Career-Enhancing in a Frictional Labour Market*

Yosuke Oda †

September 10, 2013

**Abstract**

This article constructs a job-search model in which worker's ability varies over time; a high-ability unemployed might lose her/his skills due to prolonged unemployment whereas a low-ability employed might acquire her/his skills due to (implicit) on-the-job training. I numerically show that both pecuniary reward for short-term unemployed and reduction in unemployment benefits leads to lower unemployment rate, however, the former policy does stimulate career-enhancing of long-term unemployed whereas the latter does not. In addition, numerical analysis suggests that a combination of the two policies can lead to a higher aggregate welfare than when only one of the policies is implemented.

**Keywords:** job-search model; cyclical change in skills; career-enhancing separation

**JEL classification:** J64; J68

*I am grateful to Masaru Sasaki, Jun-ichiro Ishida, Noritaka Kudoh and seminar participants at Kansai university (Search Theory Workshop) and at Osaka university, and JEA Annual Meeting Fall 2006 held at Osaka City University for their helpful comments and discussions. I am particularly indebted to Ryo Horii for his helpful suggestions and comments.

†E-mail: y-oda@biwako.shiga-u.ac.jp Address: 1-1-1, Banba-cho, Hikone, Shiga, JAPAN.

# 1 Introduction

This paper constructs a general equilibrium job-search model in which an individual's their skills vary over time; high-skilled jobless workers lose skills due to prolonged unemployment whereas low-skilled employed workers acquire their skills due to (an implicit) on-the-job training. Using this model, the study aims to consider a policy that would overcome the serious issues below.

OECD (2002) documents that on average 30% of unemployed workers are long-term unemployed who have been unemployed for a year or more in 2000, however, ten countries among these have a proportion exceeding 40%. These proportions are a historical high, and OECD (2002) concludes that "the long-term unemployed appear to be relatively more likely to go on to become very-long-term unemployed in some countries, and more likely to leave the labour force in others". As frequently discussed, a prolonged duration of unemployment discourages workers and/or makes them less skilled, which makes again reemployment of the workers more difficult. Notably, such a negative spiral aggravates social welfare, in the sense that high unemployment rates have an adverse effect on social welfare and that prolonged unemployment deteriorates welfare for long-term jobless workers. Therefore it is worthwhile to consider a policy capable of overcoming these issues.

Earlier studies treat heterogeneity in skills among individuals. For instance, Albrecht and Vroman (2002) studies a job-search model in which distribution of a worker's skill is two-points (high or low). Since their paper focuses on a firm's behavior, however, each individual's skill is assumed to be constant over time. Comparing with the model, Pissarides (1992) analyzes a worker's loss of skills (i.e., a change of the skill) in an overlapping generations framework. In this model, an old who was employed when young has higher productivity than an old who was unemployed when young. Thus an individual's skills change once in her life and the skill is not accumulated at all.

In contrast to these studies, this article assumes that an individual's skill level varies over time.

Following Albrecht and Vroman (2002), we assume that distribution of skills is of two-points (either high or low). Under this assumption, a high-skilled unemployed worker loses her/his skills if she/he cannot find a job within a certain duration, while a low-skilled unemployed worker acquires her/his skills if she/he works at a job for a certain duration. The rationale behind the assumption is that the former is the result of prolonged unemployment while the latter is the result of (implicit) on-the-job training. Given these circumstances, I consider the effects of two labour policies, which are stated below, on unemployment rates and on social welfare.

The policies considered here are quite simple. One is a reduction in unemployment benefits and the other is a reemployment bonus. The former is straightforward. High unemployment benefits lead to a high value of being unemployed, which results in a high unemployment rate. Thus, a cutback in benefits would decrease the unemployment rate. The latter is originally planned as an economic experiment (for a summary, see Meyer, 1995), which is a reward for workers hired within a certain duration after dismissed. It directly increases a worker's incentive to be employed, which results in a lower unemployment rate. In summary, the former is a *stick* while the latter is a *carrot* as an employment-boosting policy.

Theoretically, it is obvious that the two policies have similar effects on the unemployment rate but not on social welfare since the stick policy decreases welfare for jobless workers while the carrot policy one benefits for reward-qualified workers. This paper, however, focuses on another effect of the carrot policy on a worker's behavior in a economy without on-the-job search; so, if a worker wants to change her/his job, she/he must separate from her/his current job and become unemployed to seek a new better job. In that context, a reduction in unemployment benefits would discourage her/him from enhancing her/his career since the value of unemployment decreases. As a consequence, workers those who give up a job-change may arise although, from the point of view of social welfare, it is worthwhile to change their current job.

Recalling that, in my setting, workers are either unemployed or employed and, either high-skilled or low-skilled; a policy that reduces only the rate of unemployment would be insufficient

from the point of view of social welfare. This is because the most socially desirable situation for an economy is to increase the number of high-skilled employed workers. Therefore the ideal is one that not only reduces the rate of unemployment but also allow low-skilled workers to pursue a more productive job. In that context, a reduction in unemployment benefits would discourage a worker from enhancing her/his career since the value of unemployment decreases; consequently, a worker hired for a low-skilled job has no intention to enhance her/his career. The pecuniary reward, however, can give a worker who is employed in a low-skilled job an incentive to enhance her career. In summary, a reduction in unemployment benefits operates as an employment-boosting policy but does not operate as a career-enhancing policy, while the implementation of a pecuniary bonus programme operate as both policies. Notably, since the career-enhancement increases the number of individuals employed at a more productive job, it has a positive effect on social welfare.

Given the above, this paper shows examples of policy effects on the unemployment rate and on social welfare. As predicted above, we numerically show that, if the tax that finances unemployment benefits and/or pecuniary bonuses is not too high, (i) both lower unemployment benefits and higher pecuniary rewards lead to a lower unemployment rate, (ii) both higher unemployment benefits and a higher pecuniary bonus result in higher welfare, and (iii) if the two policies are implemented simultaneously, a moderate unemployment compensation and reemployment bonus can achieve higher social welfare than when only one of the two policies is implemented.

The rest of the paper is composed as follows. Section 2 describes the model and defines the equilibrium, section 3 is devoted to numerical analysis, and section 4 concludes.

# 2 The Model

## 2.1 Basic Assumptions

*Workers*

This paper considers a continuous-time job-search model in which workers are infinitely-lived and risk-neutral. I focus only on a steady-state equilibrium. A measure of workers is fixed and nor-

malized to one. Workers are either employed or unemployed, and either high-skilled or low-skilled. Let $u$ be the rate of unemployment and all jobless workers receive unemployment benefits $z$.

Following Albrecht and Vroman (2002), my model assumes that a distribution of skills among individuals is a two-point distribution; a fraction $\gamma$ of the unemployed workers are high-skilled, a fraction $1-\gamma$ of them are low-skilled, a fraction $\phi$ of employed workers are high-skilled, and a fraction $1-\phi$ of them are low-skilled. Note that, unlike Albrecht and Vroman (2002), I assume that workers' skill levels vary over time so that $\gamma$ and $\phi$ are endogenously determined in equilibrium, as described in detail below.

This paper assumes that a high-skilled unemployed worker becomes a low-skilled unemployed at a Poisson rate $\lambda$, which implies that a high-skilled unemployed worker might lose skills if she does not work for certain duration (on average $1/\lambda$). [1] In addition, I presumes that a low-skilled employed acquires skills at a Poisson rate $\mu$. For simplicity, I do not treat on-the-job search. Thus, a low-skilled employed who acquired skills must once become a high-skilled unemployed to improve her career. [2] Suppose that high-skilled unemployed workers (whose unemployment duration is necessarily short as in described in footnote 2) can receive a reemployment bonus $B > 0$ if they are hired.

*Firms*

Jobs are either filled or vacant. For simplicity, unlike Albrecht and Vroman (2002), assume that

[1]Regarding the assumption, it would be appropriate to assume a time-varying unemployment benefit rather than a constant unemployment benefit, however, such an assumption requires a more complicate setup. See footnote 2.

[2]Note that high-skilled unemployed workers are necessarily short-term unemployed worker but low-skilled unemployed workers are not necessarily long-term unemployed worker in the model. This is because low-skilled unemployed workers include a worker who has just lost her job (that is, they are short-term unemployed).

Given the fact, if we assume a time-varying unemployment benefits, the model needs three states of unemployment; high-skilled short-term unemployment, low-skilled short-term unemployment, and low-skilled long-term unemployment. To avoid the complexity, the paper regards all low-skilled unemployed as substantively long-term unemployed even if they are short-term unemployed. Hence, throughout the paper, the words *high-*(*low-*)*skilled* are used rather than *short-*(*long-*)*term*.

there are firms that hire only high-skilled workers and only low-skilled workers. The former is called a type $h$ firm and the latter is a type $l$ firm. Thus there exists practically two labour markets in the model. For convenience, they are called a type $h$ market and a type $l$ market, respectively.

When a job is filled, the job produces output $y_i$ and pays wage $w_i$ in the type $i(= h, l)$ firm. Assume that $y_h > y_l$. The wage is determined by bilateral Nash bargaining, as described below. Filled jobs break up at an exogenous Poisson rate $\delta$. If a job is vacant, the type $i$ firm incurs cost $c_i$ and the cost is assumed as $c_h > c_l$. The markets are assumed to be free entry/exit so that firms enter or exit the market so as to maximize their profits, as described below.

*Government*

In this model, the role of government is to collect taxes to finance unemployment benefit $z$, and reemployment bonus $B$. The tax rate is endogenously determined to hold balanced finance at any moment. The details are in subsection 2.4.

*Matching Technology*

Workers seeking a job and firms recruiting a worker meet randomly through a matching process. The matching technology in each market is specified as follows:

$$M(\gamma u, v_h) = (\gamma u v_h)^{\frac{1}{2}},$$
$$M((1-\gamma)u, v_l) = [(1-\gamma)uv_l]^{\frac{1}{2}},$$

where $v_i (i = h, l)$ denotes a measure of vacancies. Given the matching technology, rates of matching for workers in each market are given by $(\gamma u v_h)^{\frac{1}{2}}/\gamma u = \theta_h^{\frac{1}{2}}$ and $[(1-\gamma)uv_l]^{\frac{1}{2}}/(1-\gamma)u = \theta_l^{\frac{1}{2}}$, where $\theta_h \equiv v_h/\gamma u$ and $\theta_l \equiv v_l/(1-\gamma)u$ are known as labour market tightness. Similarly, matching rates for firms in each market are given by $(\gamma u v_h)^{\frac{1}{2}}/v_h = \theta_h^{-\frac{1}{2}}$ and $[(1-\gamma)uv_l]^{\frac{1}{2}}/v_l = \theta_l^{-\frac{1}{2}}$, respectively.

## 2.2 Asset Value Equations

Before describing asset value equations, recall the assumption that a low-skilled employed acquires skills at a Poisson rate $\mu$. Then, does the worker immediately separate from the current job? In other words, does the worker necessarily become a high-skilled unemployed worker to improve her/his career? The answer is *no*. If the worker behaves rationally, she/he would consider whether or not such a career-enhancing separation is beneficial or not. If so, the worker separates from her/his current job to improve her/his career as soon as she acquirs skills. If not, she/he continues to work at her/his current job until an exogenous job destruction occurs, even if she/he has acquired skills.

Given that fact, I should consider an economy both with and without career-enhancing separation (hereafter, abbr. CES). Note that whether CES arises or not is endogenously determined because the value of each state is endogenously determined. To conduct the analysis, the paper proceeds in the two steps of guess and verify; the following subsections in this section describe a CES economy supposing that a CES condition holds (the guess), and the next section examines whether the condition is met or not by using numerical calculus (the verify). The economy with no-CES is summarized in Appendix A.1.

*Asset Value Equations for Workers*

I use the following notations: $U_h$ ($U_l$) is the present-discounted value of high- (low-) skilled unemployment, $W_h$ is the value of high-skilled employment and $W_l^h$ ($W_l^l$) is the value of being employed at a type $l$ firm where a worker has acquired skills (where a worker is still low-skilled), respectively. Assume that capitation tax $\tau$ is levied on all individuals, which is used to finance unemployment benefit $z$ and bonus $B$. Suppose also that all jobless workers receive unemployment benefit $z$ regardless of their skills. Recall that a high-skilled unemployed loses skills at the rate $\lambda$ whereas a low-skilled employed acquires skills at the rate $\mu$, and that all jobs face to an exogenous job destruction at the rate $\delta$. Letting $r$ be a discount factor which is common to all individuals, the

value functions are given by the following equations:

$$rU_h = z - \tau + \theta_h^{\frac{1}{2}}(W_h + B - U_h) + \lambda(U_l - U_h), \tag{1}$$

$$rU_l = z - \tau + \theta_l^{\frac{1}{2}}(W_l - U_l), \tag{2}$$

$$rW_h = w_h - \tau + \delta(U_h - W_h), \tag{3}$$

$$rW_l^h = w_l - \tau + \delta(U_h - W_l^h), \tag{4}$$

$$rW_l^l = w_l - \tau + \delta(U_l - W_l^l) + \mu \max\{U_h - W_l^l, W_l^h - W_l^l\}. \tag{5}$$

Note that the fourth term in (5) represents a low-skilled employed worker's decision on whether the worker separates from the current job if she/he acquires skills ($U_h - W_l^l$) or whether the worker remains at her/his current job even if she acquired skills ($W_l^h - W_l^l$). The former corresponds to CES whereas the latter corresponds to no-CES. Using these notations, the CES condition is given by $U_h \geq W_l^h$. [3]

*Asset Value Equations for Firms*

Firms discount the future at the rate $r$ as well as workers. Let $V_i(i = h, l)$ denote a present-discounted value of vacancy for type $i$ firms and let $J_i(i = h, l)$ stand for a present-discounted value of filled job for type $i$ firms. The values of vacancy and filled job are recursively represented as follows:

$$rV_h = -c_h + \theta_h^{-\frac{1}{2}}(J_h - V_h), \tag{6}$$

$$rV_l = -c_l + \theta_l^{-\frac{1}{2}}(J_l - V_l), \tag{7}$$

$$rJ_h = y_h - w_h + \delta(V_h - J_h), \tag{8}$$

$$rJ_l = y_l - w_l + (\delta + \mu)(V_l - J_l). \tag{9}$$

[3]More precisely, the CES condition should be stated as $U_h \geq W_l^l$ since the expression $W_l^h$ does not need in the CES economy, however, to emphasize the fact that CES condition implies that the state value of high-skilled unemployment is higher than (or equal to) the state value of low-skilled employment *after skill acquisition*, we use $W_l^h$.

Eq. (9) shows that type $l$ firms face to an exogenous job destruction at the rate $\delta + \mu$ because the worker-firm match breaks up not only when a shock occurs at the rate $\delta$ but also when the employee acquires skills at the rate $\mu$. Supposing that the labour market is free entry, firms post a vacancy until the expected value of a job offer equals to zero, which implies that $V_h = V_l = 0$ holds in equilibrium (the free entry/exit condition).

## 2.3 Equilibrium

In this subsection I characterize the equilibrium in a CES economy. I begin with a description of flow conditions that determine the distribution of workers.

*Flow Conditions*

In the steady state, the population in each state does not vary over time, implying that an inflow and outflow in each state must be equal. Recall that $u$ represents the rate of unemployment, $\gamma$ denotes the ratio of high-skilled in unemployed workers and $\phi$ indicates the ratio of high-skilled in employed workers in a CES economy.

First, consider the flow condition on the high-skilled unemployment state. The inflow consists of workers from high-skilled employment resulting from an exogenous job destruction, $\phi(1-u)\delta$, plus workers from low-skilled employment resulting from workers' career-enhancing separation, $(1-\phi)(1-u)\mu$, while the outflow consists of workers to low-skilled unemployment due to prolonged unemployment, $\gamma u\lambda$, plus high-skilled employed who find a job, $\gamma u\theta_h^{\frac{1}{2}}$. The flow condition on high-skilled unemployment is thus $\phi(1-u)\delta + (1-\phi)(1-u)\mu = \gamma u\theta_h^{\frac{1}{2}} + \gamma u\lambda$. Similarly, the condition on low-skilled unemployment is given by $\gamma u\lambda + (1-u)(1-\phi)\delta = (1-\gamma)u\theta_l^{\frac{1}{2}}$, which states that the inflow (workers from high-skilled unemployment due to prolonged unemployment plus workers from low-skilled employment due to an exogenous job destruction) equals to the outflow (workers who are employed). Finally, the condition for the high-skilled employment state is $\gamma u\theta_h^{\frac{1}{2}} = \phi(1-u)\delta$, which indicates that the inflow which is high-skilled workers who are employed equals to the outflow

which is composed of high-skilled workers who lose a job resulting from a job destruction shock.

Using the three conditions, we obtain the rate of unemployment $u$, the ratio of high-skilled in unemployed workers $\gamma$, and the fraction of high-skilled in employed workers $\phi$ in the steady state, which are arranged as follows:

$$u = \frac{\delta(\lambda\delta + \lambda\mu + \mu\theta_l^{\frac{1}{2}})}{\theta_l^{\frac{1}{2}}(\lambda\delta + \mu\theta_h^{\frac{1}{2}}) + \delta(\lambda\delta + \lambda\mu + \mu\theta_l^{\frac{1}{2}})}, \tag{10}$$

$$\gamma = \frac{\mu\theta_l^{\frac{1}{2}}}{\lambda\delta + \lambda\mu + \mu\theta_l^{\frac{1}{2}}}, \tag{11}$$

$$\phi = \frac{\mu\theta_h^{\frac{1}{2}}}{\lambda\delta + \mu\theta_h^{\frac{1}{2}}}. \tag{12}$$

*Wage Determination*

When a match is formed, the wage $w_i(i = h, l)$ is determined so as to maximize a matching surplus: $w_h = \arg\max(W_h + B - U_h)^{\beta}(J_h - V_h)^{1-\beta}$ and $w_l = \arg\max(W_l^l - U_l)^{\beta}(J_l - V_l)^{1-\beta}$, where $\beta$ denotes a bargaining power for workers. The sharing rules are given by $(1-\beta)(W_h + B - U_h) = \beta(J_h - V_h)$ and $(1-\beta)(W_l - U_l) = \beta(J_l - V_l)$. Using these conditions, state values (1)-(9), and free entry/exit conditions $V_h = V_l = 0$, we obtain the following expressions:

$$w_h = \beta y_h + (1-\beta)z - (1-\beta)(r+\delta)B + \frac{\beta(rc_h\theta_h + \lambda c_l\theta_l)}{r+\lambda}, \tag{13}$$

$$w_l = \beta y_l + (1-\beta)z + \frac{\beta[(r+\lambda+\mu)c_l\theta_l - \mu c_h\theta_h]}{r+\lambda}. \tag{14}$$

Eq.(13) shows that $w_h$ is increasing in both $\theta_h$ and $\theta_l$. Since higher $\theta_h$ implies larger vacancies relative to high-skilled jobless workers, it is difficult for firms to recruit a worker, which makes workers more advantageous, which leads to higher wages. In addition, since higher $\theta_l$ implies larger employment opportunity in a type $l$ labour market, high-skilled jobless workers do not care whether or not the bargaining is approved or not, which also makes workers ascendant. In contrast, as (14) shows, $w_l$ is increasing in $\theta_l$ but decreasing in $\theta_h$. Since higher $\theta_h$ indicates that it is easy to find a job in a type $h$ market, this makes the state value of high-skilled unemployment higher. Low-skilled

jobless workers want to move the state, however, since they have to be employed once in a type $l$ firm, they would accept a lower wage. Hence higher $\theta_l$ leads to lower $w_l$. Note that, from (13), $w_h$ is decreasing in bonus level $B$. This implies that since higher $B$ makes workers more eager for the bonus, they are willing to accept much lower wage.

*Job Creation*

As noted above, firms open vacancies until the expected profit equals to zero in the steady state. This fact is represented by the free entry/exit conditions, $V_h = V_l = 0$. Using them and eliminating $J_h$ and $J_l$ from (6)-(9), we have the following expressions:

$$\theta_h = [(y_h - w_h)/c_h(r + \delta)]^2, \tag{15}$$

$$\theta_l = [(y_l - w_l)/c_l(r + \delta + \mu)]^2. \tag{16}$$

By making use of (13)-(16), we can obtain the equilibrium values of $w_h, w_l, \theta_h$, and $\theta_l$.

*Government Budget*

As described in Section 2.1, the expenditure for unemployment benefits $z$ and reemployment bonus $B$ is financed by capitation tax and that tax is determined so as to balance the government budget at any moment. The tax is determined by the following budget constraint:

$$\tau = uz + \gamma u \theta_h^{\frac{1}{2}} B, \tag{17}$$

where the first term of the right hand side indicates the expenditure for unemployment benefit $z$ and the second term represents the expenditure for reemployment bonus $B$.

*Characterization of Equilibrium*

Up to this point, I have obtained all expressions that characterize the equilibrium in our model. The equilibrium consists of 8-tuple, $\{u, \gamma, \phi, \theta_h, \theta_l, w_h, w_l, \tau\}$. They are successively derived as follows. First, the wage $w_i$ and tightness of each market $\theta_i$ $(i = h, l)$ is determined by wage bargaining and

a firm's optimal entry strategy (13)-(16). Second, the rate of unemployment $u$, the fractions of high-skilled in jobless workers $\gamma$ and in employed workers $\phi$ are derived by flow conditions (10)-(12). Finally, the capitation tax $\tau$ is determined so that it stisfies the government budget (17). We can confirm that the equilibrium is uniquely determined, which is shown in Appendix A.2.

# 3 Numerical Analysis

In the previous section, I describe an economy with a career-enhancing separation by assuming that such a behaviour arises: i.e., the CES condition $U_h \geq W_l^h$ is assumed to hold. However, the guess may not be true since the state values in the condition are endogenously determined in a general equilibrium. In other words, the equilibrium stated in the previous section is just a candidate, but not a certifiable equilibrium. Hence, before the analysis, I must rule out candidates of equilibrium if the CES condition does not hold. I examine whether or not the CES condition holds or not with varying policy variables, unemployment benefits $z$, pecuniary rewards $B$ or both since I focus on the policies.

Before the examination, I can confirm that two employment policies, one is an increase in pecuniary bonus $B$ and the other is a decrease in unemployment benefits $z$, have a different effect on the decision about career-enhancing separation. Regarding the CES condition, $U_h \geq W_l^h$, the former policy directly increases the left-hand side whereas the latter one directly decreases it (of course, both $B$ and $z$ indirectly affect the state values). This suggests that the both policies can operate as employment-boosting programmes, however, the policy that cuts unemployment benefits is less apt to be a career-enhancing policy. Since career-enhancing separation increases the number of employed in a high-skilled job (which is the most valuable state in our economy), consequently, a pecuniary reward policy seems to be more desirable from the point of view of social welfare.

*Social Welfare Function and Parameters*

As preparation for our analysis, define the measure of social welfare and set parameter values. Following Cahuc and Lehmann (2000), we use an expected utility for each type of individual as a measure of welfare (for example, the measure of welfare for a high-skilled unemployed is $rU_h$). In line with this manner, I define a measure of aggregate welfare by the weighted sum of each individual's welfare:

$$\Omega = r\{(1-u)[\phi W_h + (1-\phi)W_l^l] + u[\gamma U_h + (1-\gamma)U_l]\}. \tag{18}$$

It is difficult to obtain some of the parameter values we need, in particular, we do not have decisive evidence on the rate of loss and acquirement of an individual's ability, $\lambda$ and $\mu$. In consideration of plausibility, we set $\lambda = 1.0$ and $\mu = 0.4$. This implies that the average duration of being high-skilled when unemployed is 12 months ($12 \times (1/1.0) = 12.0$) and that the average duration of being low-skilled when she works at type $l$ firm is 30 months ($12 \times (1/4.0) = 30.0$). [4] In other words, a high-skilled worker, on average, loses her/his skills if she/he cannot find a job within one year after being fired, and a low-skilled worker acquires her/his skills if she works at a certain job for, on average, two and a half years. Regarding the rest of the parameter values, we set $y_h = 5.0$, $y_l = 3.0$, $\beta = 0.5$, $r = 0.05$, $c_h = 1.0$, $c_l = 0.5$, $\delta = 0.2$, $z \in [0, 3.5]$, and $B \in [0, 10]$. [5]

In the rest of the paper, we focus on the CES economy and show results under (i) a pecuniary reward policy where $B$ is the policy variable given $z$, (ii) a reduction in unemployment benefits where $z$ is the policy variable given $B$, and (iii) a mixture of the two policies where both $B$ and $z$ are policy variables in order.

*Pecuniary Reward*

Here, I examine effects of a reemployment bonus programme on the unemployment rate and on social welfare. To focus on this policy and since the level of unemployment benefit is taken as given

[4] These calculations are followed from Albrecht and Vroman (2002).

[5] Note that since the elasticity of the matching technology with respect to vacancy is 0.5, $\beta = 0.5$ implies that I focus on an efficient economy in the sense that Hosios condition holds (see Hosios, 1990).

here, we assume that $z = 3.5$. This implies that a policy that reduces the unemployment benefit is not implemented at all.

Numerical results are placed in Figure 1. (1-i) represents social welfare defined in (18) with varying bonus levels. It is hump-shaped, which implies that welfare improves as the bonus increases and worsens after that. This is because of higher reward benefits for bonus-qualified workers, however, it heavily burdens them as taxes that finance the benefits increase. (1-ii) and (1-iii) represents the unemployment rate for high- and low-skilled, respectively. These are monotonically decreasing in bonus level as predicted. Comparing (1-ii) to (1-iii), one can see that the number of low-skilled jobless workers decreases more than that of high-skilled. This is caused through two channels. First, since a higher bonus makes bonus-qualified workers more apt to get a job, inflow to low-skilled unemployment (i.e., prolonged unemployment) is reduced. Second, since low-skilled jobless workers are also induced to get a job, as discussed before, the outflow from low-skilled unemployment increases and part of these workers move to high-skilled unemployment as a result of career-enhancing separations, which results in an increase in high-skilled unemployed. Consequently, these two effects significantly decrease the number of low-skilled unemployed and moderately decrease the number of high-skilled unemployed. This result suggests that, if the tax burden is not too heavy, a pecuniary bonus programme seems to be quite an employment-boosting and career-enhancing policy.

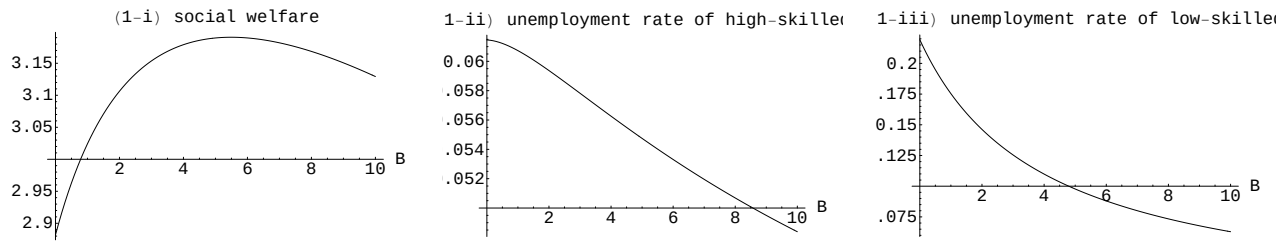


Figure 1: reemployment bonus ($z = 3.5$)

*Reduction in Unemployment Benefits*

Here, Iexamine effects of a reduction in unemployment benefits on social welfare and the unemployment rate for high- and low-skilled workers. To focus on this policy, I assume that no bonus programme exists ($B = 0$). The results are summarized in Figure 2, and are quite similar to the case of the reemployment bonus. Social welfare (2-i) has a hump-shaped result, suggesting that moderate levels of unemployment benefit maximizes social welfare. If the levels are too high, a heavy tax burden worsens an individual's welfare. If the level is too low, an individual's welfare is directly reduced. (2-ii) and (2-iii) show the unemployment rate for high- and low-skilled unemployed, respectively. As discussed above, a lower benefit leads to a lower unemployment rate because jobless workers are more apt to get a job to escape from the current state. [6] Note that a generous benefit decreases the number of high-skilled unemployed, which results in a drastic increase in low-skilled unemployed as Figure (2-iii) indicates.

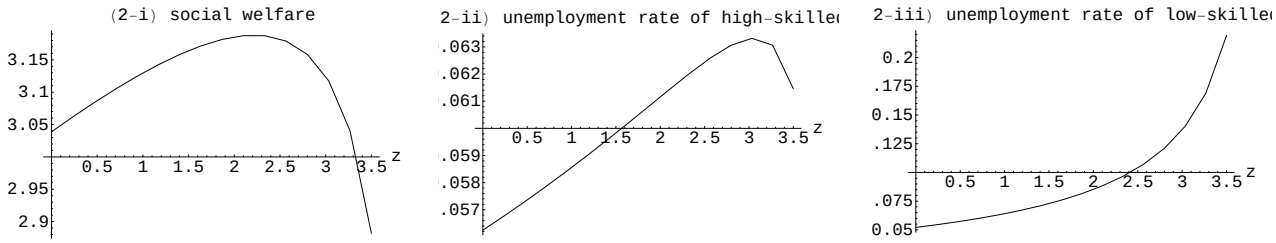


Figure 2: reduction in unemployment benefits ($B = 0$)

*A Combination of the Two Policies*

Finally I analyze effects of mixture of the two policies on social welfare and on the unemployment rate. To capture the shape of the plane by numerical calculation, levels of pecuniary reward and unemployment benefit are divided into 10 grids, as shown in Figure 3. Figure (3-i) shows welfare levels under arbitrary pairs of bonus $B$ and unemployment benefit $z$.

[6] In a model with search efforts of workers, it can be easily confirmed that lower unemployment benefits leads to higher job-search efforts, which results in lower unemployment rate.

When $z$ is low, welfare is monotonically decreasing in bonus level. Such a counterintuitive situation occurs because bonus-qualified workers (high-skilled) can gain the bonus if they are hired, whereas no-bonus-qualified workers (low-skilled) are heavily imputed by the tax burden although they do not gain benefits from a pecuniary reward. In aggregate, a reduction in the welfare of low-skilled workers dominates the increase in the welfare of high-skilled workers, which results in the worsening of social welfare. When $z$ is high, however, welfare is hump-shaped with respect to bonus level. Consider an extreme case, $z = 3.5$. Under such a generous unemployment compensation, there are many unemployed workers. Given the situation, an increase in the pecuniary reward has two opposite effects on the tax level. First, it straightforwardly increases the level , which worsens welfare. Second, since a large number of jobless workers receive generous unemployment compensation, a reduction in the number of jobless workers by bonus programme drastically decreases the tax burden as a result of unemployment compensation, which improves welfare (even if the pecuniary reward level is decent). When the bonus level is not too high, the second effect dominates the first one, which improves aggregate welfare, and vice versa.

Note that, by comparing Figure (3-i) with Figures (1-i) and (2-i), the maximized level of welfare under mixture of the two policies is higher than the welfare level under a sole policy. The reason is straightforward. Since the tax burden resulting from the unemployment benefit is reduced if the number of unemployed decreases, the two have positive effects on social welfare. A reduction in a jobless worker's welfare due to curtailed unemployment benefit is dominated by the positive effect, resulting in an improvement in aggregate welfare.

Figures (3-ii) and (3-iii) show the unemployment rate of high- and low-skilled workers, respectively. Both rates are almost increasing in unemployment benefit and decreasing in the pecuniary reward. From Figure (3-ii), a high unemployment benefit and a low bonus lead to a reduction in the number of high-skilled unemployed, because this drastically increases the number of low-skilled jobless workers as Figure (3-iii) indicates. Note that, by comparing Figure (3-iii) with Figures (1-iii) and (2-iii), the number of low-skilled unemployed workers under a combination of the policies

is much lower than under a single policy, which suggests the usefulness of the mixture of the two policies.

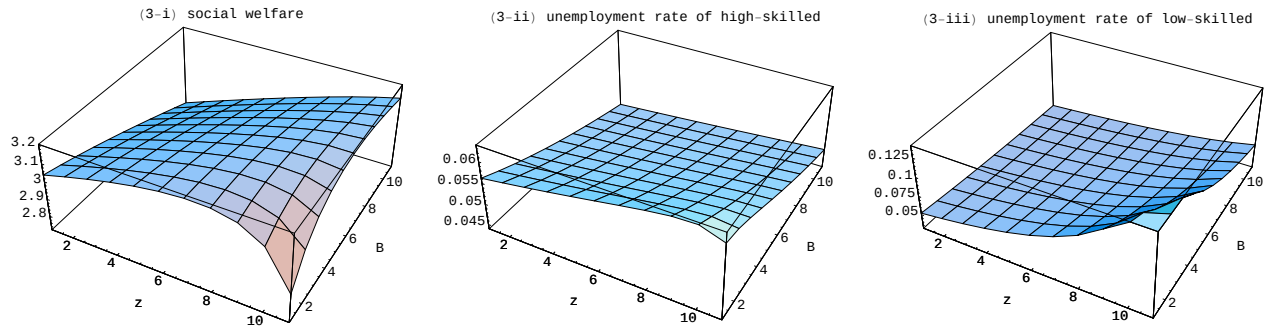


Figure 3: mixture of the two policies

# 4 Conclusion

This study constructs a job-search model in which an individual's skills vary over time and examines the effects of an employment-boosting policy on the unemployment rate and on social welfare. I show that, (i) a lower unemployment benefit and/or higher pecuniary bonus results in a lower unemployment rate, (ii) a higher unemployment benefit and/or higher reward leads to higher social welfare, and (iii) an implementation of the two policies can achieve higher social welfare than an implementation of a single policy, if the tax that finances these transfers is not too high. These result because the pecuniary bonus programme induces workers to seek a more productive job, which is socially beneficial. In summary, a reduction in the unemployment benefit decreases unemployment rate at the cost of an individual's welfare, however, the pecuniary bonus operates as both an employment-boosting policy and a career-enhancing policy, resulting in much higher social welfare.

# Appendix

*A.1. No-CES Economy*

Here we describe the economy with no career-enhancing separation (no-CES). To distinguish the endogenous variables in this economy from CES economy, we denote the variables with tilde in no-CES economy. Matching technologies are given by $M(\tilde{\gamma}, \tilde{u}) = (\tilde{\gamma}\tilde{u}\tilde{v}_h)^{\frac{1}{2}}$ and $M((1-\tilde{\gamma}), \tilde{u}) = [(1-\tilde{\gamma})\tilde{u}\tilde{v}_l]^{\frac{1}{2}}$. So the rate of matching for type $i$ $(i = h, l)$ workers (firms) is $\tilde{\theta}_i^{\frac{1}{2}}$ $(\tilde{\theta}_i^{-\frac{1}{2}})$.

Asset value equations for workers and for firms are represented as follows:

$$r\tilde{U}_h = z - \tilde{\tau} + \tilde{\theta}_h^{\frac{1}{2}}(\tilde{W}_h + B - \tilde{U}_h) + \lambda(\tilde{U}_l - \tilde{U}_h), \tag{1'}$$

$$r\tilde{U}_l = z - \tilde{\tau} + \tilde{\theta}_l^{\frac{1}{2}}(\tilde{W}_l^l - \tilde{U}_l), \tag{2'}$$

$$r\tilde{W}_h = \tilde{w}_h - \tilde{\tau} + \delta(\tilde{U}_h - \tilde{W}_h), \tag{3'}$$

$$r\tilde{W}_l^h = \tilde{w}_l - \tilde{\tau} + \delta(\tilde{U}_h - \tilde{W}_l^h), \tag{4'}$$

$$r\tilde{W}_l^l = \tilde{w}_l - \tilde{\tau} + \delta(\tilde{U}_l - \tilde{W}_l^l) + \mu(\tilde{W}_l^h - \tilde{W}_l^l), \tag{5'}$$

$$r\tilde{V}_h = -c + \tilde{\theta}_h^{-\frac{1}{2}}(\tilde{J}_h - \tilde{V}_h), \tag{6'}$$

$$r\tilde{V}_l = -c + \tilde{\theta}_l^{-\frac{1}{2}}(\tilde{J}_l - \tilde{V}_l), \tag{7'}$$

$$r\tilde{J}_h = y_h - \tilde{w}_h + \delta(\tilde{V}_h - \tilde{J}_h), \tag{8'}$$

$$r\tilde{J}_l = y_l - \tilde{w}_l + \delta(\tilde{V}_l - \tilde{J}_l). \tag{9'}$$

Flow conditions are given by:

$$\tilde{u} = \frac{\delta(\lambda\delta + \lambda\mu + \mu\theta_l^{\frac{1}{2}})}{\tilde{\theta}_l^{\frac{1}{2}}(\lambda\delta + \lambda\mu + \mu\tilde{\theta}_h^{\frac{1}{2}}) + \delta(\lambda\delta + \lambda\mu + \mu\tilde{\theta}_l^{\frac{1}{2}})}, \tag{10'}$$

$$\tilde{\gamma} = \frac{\mu\tilde{\theta}_l^{\frac{1}{2}}}{\lambda\delta + \lambda\mu + \mu\tilde{\theta}_l^{\frac{1}{2}}}, \tag{11'}$$

$$\tilde{\phi} = \frac{\mu\tilde{\theta}_h^{\frac{1}{2}}}{\lambda\delta + \lambda\mu + \mu\tilde{\theta}_h^{\frac{1}{2}}}. \tag{12'}$$

Wage equations and job-creation conditions that determine equilibrium wage and market tightness

are derived as follows:

$$\tilde{w}_h = \beta y_h + (1-\beta)z - (1-\beta)(r+\delta)B + \frac{\beta(rc_h\tilde{\theta}_h + \lambda c_l\tilde{\theta}_l)}{r+\lambda}, \tag{13'}$$

$$\tilde{w}_l = \beta y_l + (1-\beta)z + \frac{\beta\{[(r+\lambda)(\delta+\mu)+\delta\mu]c_l\tilde{\theta}_l - \delta\mu c_h\tilde{\theta}_h\}}{(r+\lambda)(\delta+\mu)}, \tag{14'}$$

$$\tilde{\theta}_h = [(y_h - \tilde{w}_h)/c_h(r+\delta)]^2, \tag{15'}$$

$$\tilde{\theta}_l = [(y_l - \tilde{w}_l)/c_l(r+\delta)]^2. \tag{16'}$$

The capitation tax is determined so as to balance the following government budget:

$$\tilde{\tau} = \tilde{u}z + \tilde{\gamma}\tilde{u}\tilde{\theta}_h B. \tag{17'}$$

Welfare function in no-CES economy is given by:

$$\tilde{\Omega} = r\{(1-u)[\phi W_h + \tfrac{(1-\phi)\delta}{\delta+\mu}W_l^l + \tfrac{(1-\phi)\mu}{\delta+\mu}W_l^h] + u[\gamma U_h + (1-\gamma)U_l]\}. \tag{18'}$$

*A.2. Uniqueness of the Equilibrium*

As stated in section 2.3, the endogenous variables which construct the equilibrium are successively determined. Recall that wage $w_i$ and market tightness $\theta_i$ $(i = h, l)$ are firstly determined by (13)-(16). Eliminating $w_h$ and $w_l$, we have the following expressions:

$$(r+\delta)(r+\lambda)c_h\theta_h^{\frac{1}{2}} + \beta(rc_h\theta_h + \lambda c_l\theta_l) - (1-\beta)(r+\lambda)[y_h - z + (r+\delta)B] = 0, \tag{A.1}$$

$$(r+\delta+\mu)(r+\lambda)c_l\theta_l^{\frac{1}{2}} + \beta[(r+\lambda+\mu)c_l\theta_l - \mu c_h\theta_h] - (1-\beta)(r+\lambda)(y_l - z) = 0, \tag{A.2}$$

which are simultaneous equations with respect to $\theta_h$ and $\theta_l$. By implicit function theorem, we can easily obtain the shape of these expressions:

$$\frac{\partial\theta_h}{\partial\theta_l} = -\frac{\beta\lambda c_l}{\frac{1}{2}(r+\delta)(r+\lambda)c_h\theta_h^{-\frac{1}{2}} + \beta r c_h} < 0, \tag{A.1'}$$

$$\frac{\partial\theta_h}{\partial\theta_l} = \frac{\frac{1}{2}(r+\lambda)(r+\delta+\mu)c_l\theta_l^{-\frac{1}{2}} + \beta c_l(r+\lambda+\mu)}{\beta\mu c_h} > 0. \tag{A.2'}$$

Thus in $\theta_l$-$\theta_h$ plane, (A.1) is monotonically decreasing while (A.2) is monotonically increasing, which guarantees uniqueness of the solution. Given the values of $\theta_h$ and $\theta_l$, wages are determined by (15) $w_h = y_h - (r+\delta)c_h\theta_h^{\frac{1}{2}}$ and by (16) $w_l = y_l - (r+\delta+\mu)c_l\theta_l^{\frac{1}{2}}$, which is obvious that these values are also uniquely determined. Since the rest of endogenous variables are evidently unique, uniqueness of the equilibrium is proved.

# References


[1] Albrecht, James. and Vroman, Susan. (2002). "A Matching Model with Endogenous Skill Requirements", *International Economic Review*, vol.43, No.1, pp.283-305.

[2] Albrecht, James. and Vroman, Susan. (2005). "Equilibrium search with time-varying unemployment benefits", *Economic Journal*, vol.115 (*July*), pp.631-48.

[3] Anderson, Patricia M. (1992). "Time-varying Effects of Recall Expectation, a Reemployment Bonus, and Job Counseling on Unemployment Durations", *Journal of Labor Economics*, vol.10, No.1, pp.99-115.

[4] Cahuc, Pierre. and Lehmann, Etienne. (2000). "Should unemployment benefits decrease with the unemployment spell?" *Journal of Public Economics*, vol.77, pp.135-53.

[5] Corson, Walter, Paul Decker, Shari Dunstan, and Stuart Kerachsky. (1992). "Pennsylvania Reemployment Bonus Demonstration: Final Report", Unemployment Insurance Occasional Paper 92-1. Washington D.C.: U.S.Department of Labor.

[6] Davidson, Paul, and Stephen A. Woodbury. (1993). "The Displacement Effects of Reemployment Bonus Programs", *Journal of Labor Economics*, vol.11, No.4, pp.575-605.

[7] Davidson, Paul, and Stephen A. Woodbury. (1997). "Optimal unemployment insurance", *Journal of Public Economics*, vol.64, No.3, pp.2.5-87.

[8] Decker, Paul T., and Christopher J. O'Leary. (1995). "Evaluating Pooled Evidence from the Reemployment Bonus Experiments", *Journal of Human Resources*, vol.30, No.3, pp.534-50.

[9] Decker, Paul T. (1994). "The Impact of Reemployment Bonuses on Insured Unemployment in the New Jersey and Illinois Reemployment Bonus Experiments", *Journal of Human Resources*, vol.29, No.3, pp.718-41.

[10] Hosios, Arthur J. (1990). "On the Efficiency of Matching and Related Models of Search and Unemployment", *Review of Economic Studies*, vol.57, No.2, pp.279-98.

[11] Meyer, Bruce D. (1990). "Unemployment Insurance and Unemployment Spells", *Econometrica*, vol.58, No.4, pp.757-82.

[12] Meyer, Bruce D. (1995). "Lessons from the U.S. Unemployment Insurance Experiments", *Journal of Economic Literature*, vol.33, No.1, pp.91-131.

[13] OECD. (2000). "OECD Employment Outlook".

[14] O'Leary, Christopher J., Spiegelman, Robert G., and Kline, Kenneth J. (1995). "Do Bonus Offers Shorten Unemployment Insurance Spells? Results from the Washington Experiment", *Journal of Policy Analysis and Management*, vol.14, No.2, pp.245-69.

[15] O'Leary, Christopher J., Decker, Paul T., and Wander, Stephen A. (2005). "Cost-Effectiveness of Targeted Reemployment Bonuses", *Journal of Human Resources* 40, No.1, pp.270-79.

[16] Petrongolo, Barbara, and Pissarides Christopher A. (2001). "Looking into the Black Box: A Survey of the Matching Function", *Journal of Economic Literature*, vol.39, No.2, pp. 390-431.

[17] Pissarides, Christopher A. (1992). "Loss of Skill During Unemployment and the Persistence of Employment Shocks", *Quarterly Journal of Economics*, vol.107, No.4, pp.1371-91.

[18] Pissarides, Christopher A. (2000). " Equilibrium Unemployment Theory", 2nd edition, Cambridge, MA: MIT Press.

[19] Woodbury, Stephen A., and Robert G. Spiegelman. (1987). "Bonuses to Workers and Employers to Reduce Unemployment: Randomized Trials in Illinois", *American Economic Review*, vol.77, No.4, pp.513-30.