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Von Neumann, Morgenstern, and Theory of Games : Critical Reassessment of Zero-Sum Games

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Abstract This chapter gives a brief yet critical account of the theory of games as jointly developed by the two superstars in the twenty century; namely, gifted mathematician John von Neumann and brilliant economist Oskar Morgenstern. There are two memorial years for game theory -— 1928 as the Year of Birth for Game Theory. and 1944 as the Year of Maturity for Game Theory. Even after the basic mathematical skeleton for game theory was provided by von Neumann in his 1928 paper, giving its real body and substance to the skeleton was a difficult job for Morgenstern. In both his 1928 paper and 1935 paper, Morgenstern paid a special attention on the duel between Sherlock Holmes and Professor Moriarty in Conan Doyle's famous detective story, finding an unsolved puzzle between a infinite chain of "out-thinking" and a finite concept of general equilibrium. In 1944 , the collaboration of von Neumann and Morgenstern finally produced a difficult bulky book *Theory of Games and Economic Behavior*, whose academic reaction was initially rather quiet, requiring many years for its full recognition by social and natural scientists. It is of utmost importance to draw a definite line between "games under risk," represented by "the final problem" in Doyle's detective stories, and "games under uncertainty," shown by "the adventure of the empty house." We believe that the second kind of games ,in which "animal spirits" a la John M Keynes play a key role in decision making, remain to be fairly unexplored, requiring for future research.

Keywords John von Neumann, Oskar Morgenstern, game theory, the duel between Sherlock Holmes and Professor Moriarty, Sherlock Holmes versus Colonel Moran, games under risk, games under uncertainty, the role of animal spirits

I " Specter of Game Theory " : An Introduction

The Manifesto of the Communist Party (German: *Manifest der Kommunistischen Partei*) was an 1848 pamphlet published as the joint work of two giants in philosophy and social science: Karl Marx (1818-1883) and Friedrich Engels (1820-1895). Although it was a rather small pamphlet containing only 100 pages or so, it has been very influential in the economic profession until today. Remarkably, it had a very famous introduction beginning with the following sentence:

> A specter is haunting Europe — the specter of communism.
> (Marx & Engels 1848, page 1)

It seems that a similar phenomenon is taking place in modern economics today. On the analogy of *the Communist Manifesto, we* can metaphorically express such a new phenomenon as follows:

> A specter is haunting the Economics profession — the specter of Game Theory.

Interestingly enough, the theory of games, or more simply game theory, was also the joint product of two superstars in science: John von Neumann (1903-1957) and Oskar Morgenstern (1902-1977). The mathematician Neumann and the economist Morgenstern successfully combined their knowledge and skill to a revolutionary theory of games and economic behavior, based on the interactions of strategies of many players. When the final product of their collaboration appeared as a bulky book *Theory of Games and Economic Behavior* in 1944, namely the year near to the end of the long and dreadful Second World War, an authoritative mathematical journal wrote the following impressive sentence in an ecstasy of joy:

> Posterity may regard this book as one of the major scientific achievements of the first half of the twentieth century. (*American Mathematical Society Bulletin* 1944)

It seems, however, that Mitsuo Suzuki (1999) had an entirely different opinion of the new theory of game. Although he himself was one of outstanding pioneers of game theory in Japan, he once lamented over the effectiveness and applicability as follows:

> When I [Suzuki] started doing research in game theory, my work was exposed to criticism from

> Japanese academia. To be honest, I undergo a neglect, or even attack, from so many people for my "apparently fruitless effort". Then I had to practice my perseverance, sincerely hoping that my lonely effort would be rewarded someday in the not too distance future.
>
> (Suzuki 1999, page 7).

In retrospect, the theory of games was once ignored by so many people, and even by feared by some people. There were a number of reasons for such unjustifiable treatment in the academia. First of all, the name "game theory" per se appeared to be provocative and amusing, thus spreading the wrong impression among the general public. In everyday conversation, if we accuse some persons of "playing games," we mean that they are not serious enough about a difficult situation, or that they are deliberately misleading us or making us do unnecessary things. For example, people can use trump cards to enjoy many games such as poker and bridge. [1)]

It is quite remarkable to see that in commemorating the fifth anniversary of the publication of *Theory of Games and Economic Behavior* (1944), the Princeton University Press made the following announcement:

> A great book often requires so many years for its general recognition. When it is finally recognized as such by the public, its influential range will far-exceed the mere readership, thus appealing to the whole society. (American Scientist 1949, quoted by Poundstone 1992, p.63)

To be honest, only 4000 volumes were sold for 5 years from 1944. The original Neumann-Morgenstern book was not only scarcely read by professional economists, but also not even purchased by many libraries. Interestingly enough, it was enthusiastically bought by some professional gamblers.

Second, the contents of the book was filled with the apparently strange combination of powerful mathematics and special economics, with strange equations and odd figures/tables. As a result, so many researchers who had a habit of sweeping a troublesome problem under the carpet displayed strong risk aversion to game theory.

Furthermore, the book itself was written in a sort of "German English, " definitely not in standard British English. Since von Neumann was born in Budapest, the Kingdom of Hungary and Oskar Morgenstern in a small town of the German Empire, their communications were usually done in German rather than English. [2)]

Because of those reasons aforementioned, game theory was born as "an unfortunate child." We must add, however, that when grown up, such unlucky child became a "great man with varied accomplishments." The purpose of this paper is to briefly

discuss the way of how successfully this great transformation has been accomplished for those long years. As the saying goes, Rome was not built in a day.

The contents of this paper is as follows. In the second section, we will discuss a short history of theory of games, from its lonely birth to popular maturity. It is really important as well as interesting to know how the collaboration between von Neumann and Morgenstern, two distinguished scholars in different fields and from different countries, became possible in the times of hardships and wars. The third section will repeatedly confirm that Arthur Conan Doyle's detective story *The Adventures of Sherlock Homes* gave a strong inducement to the birth and development of the revolutionary theory of strategies and games. It will be shown that the famous duel between Sherlock Holmes and Professor Moriarty gives us the original form of standard zero-sum, two person games. As the saying goes, it is really a "mission impossible" to create something from nothing. Final remarks will be made in the final fourth section.

II The Two Memorable Years for Game Theory: 1928 as the "Year of Birth" and 1944 as the "Year of Maturity"

This section will outline the birth and development of game theory in a historical perspective. As is seen in Table 1.1, there are the two memorable years for game theory: first, 1928 as the "Year of Birth", and second, 1944 as the "Year of Maturity."

To begin with, we will focus on the first memorable year of 1928. Historically speaking, this is the year in which the world economy reached the height of its prosperity. Although many European countries suffered a great deal from the aftermath of the First World War, they gradually recovered in the late 1920s, so that many people really began to dream of the everlasting capitalist economy. As the old ballade often tells us, however, all things are uncertain and must pass. In 1928, a year earlier than the outbreak of the Great Depression, ordinary people enjoyed their daily lives, being apparently confident of the continuation of their "golden days." [3)]

It is in this 1928 that von Neumann, a native son of Budapest, the Kingdom of Hungary, wrote the following outstanding paper on "social game theory" ;

von Neumann, " Zur Theorie der Gesellschaftsspiele, " (English translation: The theory of social game), *Mathematische Annalen,* Vol. 100, 1928.

There are two things noticeable. First, this was an old technical paper that was .

Table 1.1 John von Neumann and Oskar Morgenstern: Their Lives and Collaboration on Game Theory

John von Neumann	Oskar Morgenstern
1903 Neumann was born in Budapest, Hungary. His father was a wealthy Jewish banker.	**1902** Morgenstern was born in Görlitz, Germany His mother was said to be a noblewoman.
1928 "Theory of social games" (the German original), This was historically the first mathematical paper on game theory.	**1928** "Economic forecasting" (the German original). The Holmes-Moriarty duel was first noticed as a "troublesome game."
(1928 was regarded as the YEAR OF BIRTH for game theory)	
1933 Neumann was appointed as a lifetime professor at the Institute for Advanced Study, Princeton University, New Jersey.	**1935** "Perfect foresight" (the German original). The Holmes-Moriarty duel was again noted as an obstacle to perfect foresight. Then, Edward Chech pointed Morgenstern to Neumann's paper (1928) above.
1937 Neumann became a naturalized citizen of the U.S.	**1938** While Morgenstern was visiting American universities, the Nazis took over in Vienna.
(Both Neumann and Morgenstern stayed at Princeton)	
1939 After giving an after-luncheon talk, Morgenstern talked with Neumann about game theory. Their talk continued for a long time..	
1940 Morgenstern wrote a greatly enlarged version of Neumann's old paper (1928). When Neumann saw it, he suggested to Morgenstern: "Why don't we write this paper together?" Their earnest collaboration began.	
1944 The first edition of *Theory of Games and Economic Behavior* was published by] Princeton University Press.	
(1944 was thus regarded as the YEAR OF MATURITY for game theory)	
1957 Neumann passed away.	**1977** Morgenstern passed away.

written in 1928, with the language used being German, neither English nor French. Second, although it discussed people's social games, it was published in a mathematical journal, thus representing the unique combination of social science and mathematics.

John von Neumann was born in Budapest, Kingdom of Hungary, which was then an integral part of the Austro-Hungarian Empire. He was the eldest son of a wealthy and nonobservant Jewish family, with his father being an influential banker. He himself was a very clever boy with bright mathematical skill, and often called "child prodigy." While by the early age of eight, he was familiar with differential and integral calculus, he found special interest in history as well. He started his lectures as a *Privatdozent* at the University of Berlin in 1928. On New Year's Day in 1930, he married Marietta Kövesi at Budapest University. Von Neumann and Marietta had one child, a daughter, Marina, born in 1935. In the early 1970s, Yasuhiro Sakai became an assistant professor of mathematical economics at the University of Pittsburgh. Very fortunately, Sakai soon got acquainted very well with Maria, who was then highly respected as Professor Marina von Neumann Whitman, eagerly teaching international economics. In hindsight, it was really amazing to trace the existence of "Econ Connection" in mathematician von Neumann's early career thorough both his first wife and his only daughter. [4)]

1n 1933, von Neumann was offered a lifetime professorship at the Institute for Advanced Study, Princeton University. In Princeton, he liked to play loud German march music on his phonograph, annoying his neighbors including Albert Einstein, the creator of the theory of relativity. It seemed that even in the United States, Neumann was fond of German culture and music, but not Nazis , enjoying conversation with his friends in German.

To put it perhaps a little too strongly, Oskar Morgenstern, the hard-working economist whose destiny was to become von Neumann's good collaborator, seemed to be "more German except Nazis" than von Neumann. Morgenstern was born in Görlitz, Germany, a city near the border with Poland. He grew up in Vienna, Austria, and graduated from the University of Vienna and got his Ph.D. in political science, later becoming a professor in economics at the same university in 1928. During his visit to Princeton University in 1938, something he did not expect happened: Adolf Hitler took over Vienna and expanded his political and military influence over the whole Europe. Since Morgenstern was unfairly dismissed as "politically unbearable" from the University of Vienna, he decided to remain in the United States. He became a member of the faculty at Princeton University, and particularly gravitated toward the Institute for Advance Study in which von Neumann already stayed as a lifetime professor. [5)]

In February 1939, Morgenstern gave an after-luncheon talk on the business cycles at the Nassau Club, von Neumann happened to be there with Niels Bohr, a distinguished particle physicist from Denmark. Both von Neumann and Bohr invited Morgenstern that afternoon for tea at Fine Hall, and all of them spent several enjoyable hours talking about games and experiments. Afterward, Morgenstern and von Neumann had many opportunities to talk wide-ranging topics. One day, Morgenstern mentioned to von Neumann that Morgenstern was very much interested in studying von Neumann's 1928 paper on game theory. When Morgenstern had a chance to talk to von Neumann about his pet work on incompatibility between perfect foresight and social games, he had a rather unexpected reply from von Neumann. In all honesty, von Neumann told Morgenstern that von Neumann had done no further work on game theory for those many years after 1928. [6)]

This gave Morgenstern to study von Neumann's paper on game theory very seriously. More specifically, Morgenstern referred to the episode of the pursuit of Sherlock Holmes by Professor Moriarty in Conan Doyle's famous detective story. Morgenstern explained to von Neumann in some detail that Holmes and Moriarty could never be resolved on the basis of one of them "out-thinking" the other. The infinite chains of "out-thinking," well-illustrated by the strange sort of sentence " If Mr. H thinks that Mr. M thinks that Mr. H thinks ... , " would guide the reader to a blind alley with no hope to escape. He showed that the strong assumption of "perfect foresight" would lead to inescapable paradoxes, thus being inadmissible for a seemingly powerful general equilibrium theory. [7)]

Morgenstern had then started to investigate many possibilities and further developments of game theory. Morgenstern expanded game theory in a variety of ways, so that their joint research papers was greatly accumulated. Then, von Neumann himself suggested to Morgenstern as follows: Why don't we write this paper together?" Morgenstern solemnly accepted this proposal right away. Initially, their joint work on game theory was prearranged as a mere pamphlet of around 100 pages. As their joint work progressed with the years, however, the number of pages of the pamphlet unexpectedly inflated.

September 18th,1944 — this was really a very historical moment for game theory ! At this particular moment when the long and destructive Second World War was closing at the end, the product of the long and exciting collaboration of von Neumann and Morgenstern was finally released as a quite bulky book with more than 600 pages. So, the year of 1944 should rightly be celebrated as the Year of Maturity for Game Theory..

Ⅲ How to Solve " The Final Problem " : Conan Doyle's Solution and Morgenstern's Objection

If we take a close look at the birth and development of the theory of games, then we can immediately learn that one famous detective story has made a very outstanding contribution to it. That is no less than the short story "the Final Problem" by Conan Doyle, featuring his detective character Sherlock Holmes. It was first published in *The Strand Magazine* in the United Kingdom, under the title "The Adventure of the Final Problem" in December 1893. Then, it reappeared in book form as a part of the collection *The Memoirs of Sherlock Holmes* (1894).

Historically speaking, writer Conan Doyle (1859-1930) was more than 40 years older than mathematician John von Neumann (1903-1957) and economist Oskar Morgenstern (1902-1977). Although the famous detective Sherlock Holmes was a fictitious character created by Doyle himself, it had long been a superstar in the general public, well-known over the British Channel throughout the European Continent. It was Doyle's initial plan that "the Final Problem" was literally no less than "the Final Case," meaning the death of the leading character Sherlock Holmes. In reality, however, the schedule is no more than a schedule, and often apt to be broken. [8)]

As soon as the detective story "The Final Problem" was published together with the implication of the sudden death of Sherlock Holmes, a very strong storm of protest from the reader directed against the author Conan Doyle. "Our beloved detective Holmes should be an invincible man. Mr. Doyle, please let Holmes come back ! " Doyle had constantly been under strong pressure for the revival of Holmes by any possible means. So, after 10 long years since "The Final Problem" (1893), Doyle wrote the new adventure story "The Empty House" in *The Strand Journal* , the October issue, 1903. This was the reason why the third volume of short stories was entitled *The Return of Sherlock Holmes.*

Oskar Morgenstern was then fond of reading Doyle's detective stories, thus becoming a world-wide member of "Sherlockian Group" or the group of Sherlock Holmes fans. It was quite remarkable to see that Morgenstern already introduced Doyle's detective story "The Final Problem" two times before it was again appeared in *Theory of Games and Economic Behavior* (1944), a monumental work with von Neumann. More specifically, as was seen in Table 1.1 above, Morgenstern (1928) first discussed "The Final Problem" in his German book *Wirtschaftsprognose* (English: *Economic Forecasting*) . Interestingly enough, Morgenstern (1935) again referred to the same problem in his article "Vollkommene Voraussicht und wirtschatliches Gleichgewicht "

(English: Perfect foresight and economic equilibrium) in a prestigious German journal. And astonishingly, in von Neumann and Morgenstern (1944), " The Final Problem" was once again taken up for careful discussion, playing a critical role in the development of game theory. As many movie fans know very well, " 007 is alive twice." By using a similar expression, we could say the following : " The Final Problem" seems to be alive three times ! In the following, all the details of this point will be discussed.

As the saying goes, seeing is believing. "The Final Problem" was concerned with the life-or-death struggle between famous detective Sherlock Homes and mathematical evil Professor Moriarty. Holmes described Moriarty as follows:

> He [Moriarty] is the Napoleon of crime, Watson. He is the organizer of half that is evil and of nearly all that is undetected in this great city. He is a genius. a philosopher, an abstract thinker. He has a brain of the order. (Conan Doyle 1894, pp. 335-336)

The struggle between Sherlock Holmes and Professor Moriarty, which was first introduced in Morgenstern (1928), and reproduced in Morgenstern (1935). It was actually Morgenstern's favorite theme long before he met von Neumann and even after their collaboration started at Princeton. First of all, we would like to note the following passage from Morgenstern (1928) , which is later reproduced in Morgenstern (1935) :

> Sherlock Holmes, pursued by his opponent, Moriarty, leaves London for Dover. The train stops at a station on the way, and he alights there rather than travelling on to Dover.. He has seen Moriarty at the railway station, recognizes that he is very clever and expects that Moriarty will take a faster special train in order to catch him in Dover. Holmes' anticipation turns out to be correct. But what if Moriarty had been still more clever, had estimated Holmes' mental abilities better and had foreseen his action accordingly? Then, obviously, he would have traveled to the intermediate station. Holmes, again, would have had to calculate that, and he himself would have decided to go on to Dover. Whereupon, Moriarty would again have "reacted" differently. Because of so much thinking they might not have been able to act at all or the intellectual weaker of the two would have surrendered to the other in the Victoria Station, since the whole flight would have become unnecessary.
>
> (Morgenstern 1928, Morgenstern 1935; quoted in Schotter 1976,pp. 175-176)

When Morgenstern wrote this passage, he was a professor in economics at the University of Vienna. As a professional economist, he knew very well what "economic equilibrium" was all about. On the one hand, in the duel between Holmes and

Moriarty, each person have to "out-think his opponent" in the form " I think he thinks that I think ... " This is a sort of never-ending sequence of predictions. On the other hand, on ordinary economic equilibrium, the demand and supply of any good are adjusted at the market. Once the market equilibrium is realized, the quantities of trading are determined once and for all, with no need of "out-thinking" and "manipulation." Consequently, it was no wonder that Morgenstern found a sort of "insolvable paradox" between "the Holmes-versus-Moriarty way of out-thinking" and "the traditional way of trading in the market." In this connection, let us quote the following passage from Morgenstern (1935) :

> One may be easily convinced that here lies an insoluble *paradox.* And the situation is not improved, but, rather, greatly aggravated if we assume that more than two individuals — as, for example, in the case with exchange — are brought together into a position, which would correspond to the one brought forward here; Always, *there is exhibited a endless chain of reciprocally conjectured reactions and counter-reactions. This chain can never be broken by an act of knowledge but always only through an arbitrary act — a resolution.* This resolution, again, would have to be foreseen by the two or more persons concerned. The paradox still remains no matter how one attempt to twist or turn things around. *Unlimited foresight and economic equilibrium are thus irreconcilable with one another.*
>
> (Morgenstern 1935, p. 174)

In hindsight, this was really a truly important question that was raised by the still-young and ambitions economist Morgenstern. Unfortunately, it was no more than a question-raising. For a perfect solution for getting out of the paradox, Morgenstern at Vienna needed a mathematical help from von Neumann at Princeton. Even after the collaboration between Morgenstern and von Neumann began at Princeton, the paradox aforementioned gave them a constant headache. And in their joint work *Theory of Games and Economic Behavior,* the Holms-versus-Moriarty duel was once again taken up and very carefully investigated in the following way :

> The game to be considered below is an episode from the Adventure of Sherlock Holmes. Sherlock Holmes desires to proceed from London to Dover and hence to the Continent in order to escape from Professor Moriarty who pursues him. Having boarded the train he observes, as the train pulls out, the appearance of Professor Moriarty on the platform. Sherlock Holmes takes it for granted — and in this he is assumed to be fully justified — that his adversary, who has seen him, might secure a special train and overtake him.

> Sherlock Holmes is faced with the alternative of going to Dover or leaving the train at Canterbury, the only intermediate station. His adversary -— whose intelligence is assumed to be fully adequate to visualize these possibilities — has the same choice. Both opponents must choose the place of their detrainment in ignorance of the other's corresponding decision. If, as a result of these measures, they should find themselves, *in fine*, on the same platform, Sherlock Holmes may with certainty expect to be killed by Moriarty. If Sherlock Holmes reaches Dover unharmed he can make good his escape. (Neumann & Morgenstern 1944, pp,175-176)

The question of much interest is what are the good strategies for Sherlock Holmes , and also what are the good strategies for Professor Moriarty. As is seen in Fig. 1.1 , Holmes, who is leaving London toward Dover, has to choose one out of the following two strategies:

【Strategy D】 Directly going to Dover, the destination,
【Strategy C】 Quitting at Canterbury, only the intermediate station between London and Dover..

Moriarty, who is going in hot pursuit of Holmes, is also supposed to choose one out of the two strategies aforementioned; namely, Strategy D and Strategy C. Consequently, we should have the following four possible combinations of the two players:

【Combination of D and D】 The two players meet at Dover. Holms will certainly be killed by Moriarty,
【Combination of C and C】 The two players meet at Canterbury. Holms will certainly be killed by Moriarty.
【Combination of D and C】 They do NOT meet at all. Holms will quit at Canterbury and safely stay in England.
【Combination of C and D】 They do NOT meet at all. Holms will reach Dover unharmed, and make good his escape to the Continent..

Fig. 1.1 is supposed to summarize the whole picture of the "The Final Problem," in which Moriarty and Holms are the two key players and each one has the two strategies "Going to Dover" and "Quitting at Canterbury." According to Conan Doyle's original story, the "combination of D and C," implying that Moriarty and Holmes do not meet on the same platform and Holmes safely stays in England ", should be the natural consequence of the situation. The question which would immediately occur is whether

		SHERLOCK HOLMES	
		Going to Dover	Quitting at Canterbury
PROFESSOR MORIARTY	Going to Dover	They meet at Dover	They do NOT meet Holmes stays in Engl. 【Doyle 's Story】
	Going to Canterbury	They do NOT meet Holmes goes to the Continent	They meet at Canterbury

Fig. 1 Showdown between Professor Moriarty and Sherlock Holmes : " The final problem " in Conan Doyle (1894)

and to what extent Doyle's solution is justifiable. Is the solution "logically justifiable," and to what extent Doyle's solution is justifiable. Is the solution "logically justifiable," or "emotionally justifiable," or else ?

At first glance, it looks like a game-theoretical situation. Even if we boldly attempt to confuse "the real world where flesh-and-blood persons emotionally interact with each other" with "the hypothetical world where robot-like persons exchange mechanical responses," however, we must take every possible care. Otherwise, a sort of Frankenstein's monster would unfold before our eyes. To this end, we have to make clear the following problems.

(i) The first problem to clarify is concerned with the one of how much Holmes's intellectual faculties are in comparison with Moriaty's. On the one hand, it is true that Holmes always devotes himself to chemical experiments and shows flashes of genius in his reasoning as a detective. On the other hand, he is not only a heavy smoker and has little experience of having loved someone, but his criminal investigation has not been 100 percent perfect. For instance, in Doyle's first short story "A Scandal in Bohemia,"

Holmes' skilful project was doomed to failure by the flash of wit of Irene Adler, who earns Holmes' unbounded admiration. [9)]

Professor Moriarty is a mathematical genius and his career has been an extraordinary one. At the age of twenty-one, he wrote a treatise on the Binominal Theorem, and won the Mathematical Chair at a university. But alas, he also had hereditary tendencies: a criminal strain ran in his blood, which was rendered more dangerous by his extraordinary mental powers. According to Sherlock Holms, Moriarty is now "the Napoleon of crime, " and the two men are presumably almost equally intelligent. We must say, however, that Moriarty's crime records have never been perfect, for some of his plans had to suffer considerable setbacks. [10)]

For instance, against Holmes' repeated interferences, Moriarty made the following strong protest: i

> ' You [Holmes] crossed my [Moriarty's] patch on the 4th of January, ' said he [Moriarty. 'On the 23rd you incommoded me; by the middle of February I was seriously inconvenienced by you; at the end March you I was absolutely Hampered in my plans; and now, at the close of April. I find myself placed in such a position through your continual persecution that I am in positive danger of losing my liberty. The situation is becoming an impossible one.'
>
> (Doyle 1894, pp. 336-33'7)

In short, no matter how clever Holms or Moriarty might be, they should never be equivalent to Almighty God. Even if Holmes might think of Moriarty as a person of almost equivalent intelligence, it would be unlikely that Moriarty's intelligence is exactly as the same as Holmes's intelligence. Objectively speaking, he intelligence difference between the two persons might be very small. Subjectively, however, he is likely to believe that there should exist a non-ignorable difference. Holmes must be such a self-assertive person that he could induce the reader to believe in Holmes's final victory over Moriarty.

(ii) The second point is related to the physical strength of Holmes compared with Moriarty. When Holmes happens to fight hand to hand with Moriarty, the former who is a master of Japanese martial art must have an advantage over the latter. In fact, as is seen in Doyle's subsequent story "An Adventure of the Empty House" (1903), Holms takes such a technical advantage that he can knock down Moriarty into the Reichenbach Falls. To tell the truth, however, Moriarty has his own advantage over Holmes in that the remaining gang supporting the former may be strong enough to

overpower the latter. Taking all those points into consideration, there would be no telling what will eventually happen between the two persons.

Now let us take a closer look again at Fig. 1.1. Then, we immediately see the possibility that both Moriarty and Holmes meet face-to-face either at Dover or at Canterbury. If such a face-to-face meeting on the same platform happens, then it is conjectured by von Neumann and Morgenstern that " Holmes may with certainty expect to be killed by Moriarty. " (von Neumann & Morgenstern 1944, p. 177) We do not quite agree with this opinion, however. Generally speaking, assuming the defensive is a harder job than assuming the offensive. Sometimes, however, counterattacking an offensive enemy might be very effective. So, Moriarty's assault on Holms might be successful with high probability, but not with 100 % certainty. Or possibly, Holms' counterattack on Moriarty could produce an unexpectedly good result. No one could tell us in advance which one is more likely than the other.

(iii) The third point involves the realistic validity of the probabilistic combination of any two strategies. Given the matrix of strategies in Fig. 1.1, we have to find "the optimal strategy" of the two players. In this regard, there is a critical gap between Doyle's *practical solution* and von Neumann- Morgenstern's *speculative solution*.

According to Doyle's real story "The Final Case," the combination of D and C represents the one of the optimal strategies of the two players. Holmes who has just quitted at Canterbury is able to see Moriarty's special train passing at Canterburry toward Dover. Then, Holmes let a smile of triumph flash across his face, saying the following:

> "There he goes,." said Holmes, as we watched the carriage swing and rock over the point. "There are limits, you see, to our friend's intelligence. It would have been a coup-de-maître had he deduced what I would deduce and acted accordingly." (Doyle 1894 , p. 340)

We believe that this passage well-describes Holmes's character as a human being and the possible limits of Moriarty's intelligence. It is quite remarkable to see, however, that Morgenstern raised a strong objection against Doyle's practical solution as follows:

> The narrative of *Conan Doyle* — excusably — disregards *mixed* strategies and states instead the actual developments. According to these Sherlock Holmes gets out at the intermediate station and triumphantly watches Moriarty's special train going on to

> Dover. Conan Doyle's solution is the best possible under his limitations (to pure strategies), insofar as he attributes to each opponent the course which we found to be the more probable one (i.e. he replaces 60 % probability by certainty). It is, however, somewhat misleading that this procedure leads to Sherlock Holmes's Complete victory, whereas, as we saw above, the odds (i.e. the value of a play) are definitely in favor of Moriarty. (Our result for ε , η yields that Sherlock Holmes is as good as 48 % dead when his train pulls out from Victoria Station. Compare in this connection the suggestion in *Morgenstern*, loc .cit., p.98, that the whole trip is unnecessary because the loser could be determined before the start.)
>
> (von Neumann & Morgenstern 1944, p.178, footnote 1)

Mitsuo Suzuki (1959) was in strong support of von Neumann & Morgenstern (1944), voicing his own opinion as follows:

> The conclusion would state that Moriarty's best strategy should be going on to Dover with probability 3/5, and Holmes's best strategy, with probability 2/5. In other words, what Moriarty should do at this stage is to make the lottery which contains five tickets, with three for "going on to Dover" and two for "quitting at Canterburry," and then to decide which option to choose on the basis of the result of the lottery. (Suzuki 1959, p. 55)

In the above quotation, we note that the probabilities " 3/5 (60 %)" and " 2/5 (40 %)" are referred. The reason why those specific probabilities matter will be explained in detail later.. Although this may be somewhat important, we do believe that there exists a more critical issue to discussed here. The key question to ask is related to the realistic validity of the concept of "mixed strategies" in the "once and for all game." In "The Final Game," can we possibly imagine that Moriarty, who is desperately pursuing Holmes, quickly makes the relevant lottery with five tickets and simply decides his strategy by drawing lots ? Consider the moment at which Holmes and Moriarty are just going for a one-on-one fight. If they have to decide their next strategies by drawing lots, then the duel scene will be greatly spoiled. No doubt, Holmes as a cool detective should instead adopt a "pure strategy" against Moriarty, a notorious villain.

The critical difference between "once and for all games" and "repeated games" must be recognized by any decision makers. We live in the world of uncertainty. What may happen next, nobody can tell. People may not be informed of the exact shape of the distribution, but they have to make decisions on be basis of partial information. Under the conditions of "true uncertainty" a la Frank Knight (1921), we cannot depend on

strict mathematical expectations since the basis per se for making such calculations no longer exist. In this connection, great economist John M. Keynes (1936) once made the following remark:

> Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits — of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities..
>
> (Keynes 1936, p. 161)

If we take those things into consideration, we do not fully endorse the objections of von Neumann & Morgenstern (1944) and Suzuki (1958), but we feel ourselves rather attracted to Doyle's original solution. In short, we are not in a position to imagine the situation under which a gifted person like Holms or Moriarty is drawing lots in his decision making. Any brave man would have animal spirits, always taking a positive attitude and venturing his life.. [11)]

Ⅳ The Adventure of the Empty House: Sherlock Holmes Versus Colonel Moran

As was mentioned above, "The Final Problem" (1893) was intended to be exactly what its name meant. Sherlock Holmes struggled with Professor Moriarty over the Reichenbach Falls, with the result that the two together would fall into the falls. No falling body would possibly escape from inevitable death.

To Doyle's delightful surprise, however, something extraordinary happened. The public reaction to the apparent death of Sherlock Holmes was so great that Doyle was pressured into Holmes's his miraculous revival in "The Adventure of the Empty House" (1903). Contrary to what Watson believed, Holmes won against Professor Moriarty at Reichenbach Falls with the help of Japanese marshal art. Unfortunately, some of Moriarty's confederates knew that Holms was still alive and tried to take every opportunity to kill Holmes. Colonel Sebastian Moran, nicknamed "the second most dangerous man in London" by Holms, was certainly the most risky confederate as a trained sniper firing a specialized air gun. [12)]

One day, Holmes told Watson that they were going to do some dangerous work. They entered an empty house, an abandoned building whose front room overlooked Holmes's room, Baker Street. Very surprisingly, Holmes could be seen silhouetted against the blind: it was a waxwork bust, moved regularly by Mrs. Hudson to simulate

life. Holmes used the dummy, anticipating an attempt on his life that very night. Holmes and Watson waited several hours in the empty house. A sniper fired an air gun to assassinate his foe. To Holmes's surprise, Moran as the sniper chose the empty house as his vantage point. Then, Moran was arrested and disarmed by Inspector Lestrade, the policeman asked to come by Holmes.

Doyle's new story "The Adventure of the Empty House" can be regarded as another masterpiece. Here, we can see a new sort of duel between Colonel Moran and Sherlock Homes. At first glance, this *new duel* seems to similar to the *old duel* between Professor Moriarty and Sherlock Holmes, but these two duels must be entirely different from an intellectual or informational viewpoint. Whereas Moran is a single-minded colonel with imperfect information, Holmes is a very intellectual detective with accurate information, Such a personal difference between the two persons would make a unique game play worthy of close investigation. As is seen in Fig. 1.2, Moran, who is anxious to kill Holmes by air gun, has to choose one out of the following two strategies:

[Strategy OU] Shooting from the outside **[Holmes's prediction]**
[Strategy EH] Shooting from the empty house

Holmes, who is threatened to be shot by an air gun, has to choose the following two strategies:

[Strategy OA] Waiting at the old apartment **[Moran's prediction]**
[Strategy EH] Waiting at the empty house .

Given those strategies, namely two strategies for each player, we must have the following four possible combinations of the two players:

[Combination of OU and OA] This is both predicted by Moran and Holmes
[Combination of EH and EH] This is neither predicted by Moran and Holmes
[Doyle's story]
[Combination of OU and EH] This is NOT predicted by Moran, but surely predicted by Holmes
[Combination of EH and OA] This is predicted by Moran, but NOT so by Holmes

Fig. 2 may summarize the whole picture of "The Adventure of the Empty House." Remarkably, to our surprise, both Moran's prediction and Holmes's prediction are NOT

		SHERLOCK HOLMES	
		(Waiting at) the old apartment 【Moran's prediction】	(Waiting at) the empty house
COLONEL MORAN	(Shooting from) the outside 【Holmes's prediction】	Moran predicts Holmes predicts	Moran does NOT predict Holmes predicts
	(Shooting from) the empty house	Moran predicts」 Holmes does NOT predict	NEITHER Moran NOR Holmes predicts 【Doyle's Story】

Fig. 2 Showdown between Colonel Moran and Sherlock Holmes :
" The adventure of the empty house " in Conan Doyle (1903)

quite correct ! When Moran, a simple-minded person, saw a Holmes-like dummy silhouetted against the blind in the old apartment, he easily misunderstood that the silhouette represented the real Holmes. According to Moran's prediction, however, Holmes should be staying in the old apartment, but not in the empty house. In contrast to Moran, Holmes was a famous detective with high intelligence, so that he rightly predicted that Moran would shoot him by air gun. Holmes's prediction, however, was not quite correct: in fact, Moran attempted to shoot Holmes from the empty house, NOT from the outside. In reality, no man is perfect; even Holmes made a mistake.

We admit that there have been so many Holmes fans around the world. They are constantly reading those Doyle's detective stories which include the first story "The Final Problem" and the second story " The Adventure of the Empty House." Although the first story looks more famous than the second one, we believe that the two stories require equal treatment for scientific investigation.

V Games under Risk versus Games under Uncertainty

As was noticed at the beginning, there are two memorable years for the theory of games — 1928 as the Year of Birth, and 1944 as the Year of Establishment. So, to believe or not, the centenary anniversary of game theory is steadily approaching.

We must bear in mind that game theory is not merely a brainchild of mathematical genius John von Neumann, but rather a product of compassionate economist Oskar Morgenstern as well. Since Morgenstern was a passionate Sherlock Holmes fan, Conan Doyle's detective story, featuring Holmes as its main actor, has produced a far-reaching effect on their joint product *The Theory of Games and Economic Behavior* (1944). We must not forget the fact that from the start, the two fantastic stories "The Final Problem" (1893) and "The Adventure of the Empty House" (1903) were destined to deal with different sorts of game theoretic situations.

In "The Final Problem," there are the outstanding two players, i.e., famous detective Sherlock Holms and notorious mathematician Professor Moriarty. Each player, who lives up to his reputation as a genius in his own field respects each other, can perfectly foresee the *set* of possible strategies of his opponent. It is true that he has no power to exactly anticipate which action his enemy really takes. Whichever action the enemy takes, however, it should be only *one element* of the whole anticipated set, so that there would no surprises whatever. This is no less than the world of games under *risk*, with no *black swans* a la Taleb (2007) being present.

Turning to another story "The Adventure of the Empty House," we get into the new world of unexpected results and surprises. Here again, there are the two players, namely super detective Sherlock Holmes and simple-minded Colonel Moran. Although their intelligence levels are poles apart, they are just human beings who are apt to make some mistakes. Although Holmes is thought of as a man of super brain, he should be no way free of making mistakes. Here, each player cannot even foresee the set of possible strategies of his opponent, so that unexpected events, which do not belong to the anticipated set, are likely to take place. This is really the world of games under *true uncertainty* , in which the concept per se of an equilibrium of the game ceases to be effective. [13)]

Now, if we getting back to the real world today, it should be worth questioning which world we live, the world under risk or the world under uncertainty. We do think that the real events we observe in our daily lives bear more resemblance to the second story "The Adventure of the Empty House" than to the first story "The Final Problem."

Close examination of the real world may tell us that we live in the WORLD OF UNCERTAINTY in which every thing including human beings are uncertain and unreliable. It is true that some persons are very clever and resourceful. Cleverness and rich resource, however, do not guarantee correct judgment and perfect forecasting. Besides, we should not forget that sad fact that both intelligence and information are not fairly distributed. It seems that the majority of the people behave like mediocre Colonel Moran, not like superman Sherlock Holmes or Professor Moriarty.

In conclusion, it is high time for us to go beyond the "clean and beautiful world of von Neumann and Morgenstern." We have to find a better way, not even the best way of life, in such an "unclean and dirty world" as we see it.

REFERENCES

Aumann RJ, Hart S (eds.) (1992) Handbook of game theory and economic applications. North Holland

Dasgupta P, Gale D, Hart O, Maskin E (eds.) (1992) Economic Analysis of Markets and Games. The MIT Press, Massachusetts.

Dixit AK, Nalebuff BJ (1991) Thinking strategically: the competitive edge in business, politics and economic life. Norton, New York

Doyle, Sir AC (1892) The adventures of Sherlock Holmes. London

Doyle, Sir AC (1894) The memoirs of Sherlock Holmes. London

Doyle, Sir AC (1905) The return of Sherlock Holmes. London

Gamow G (1047, revised 1961) One Two Three ... Infinity . Viking Press

Keynes JM (1936) The General Theory of Employment, Interest and Money. Macmillan, London

Knight HF (1921) Risk, Uncertainty and Profit. Univ. of Chicago Press

Kuhn HW, Nasar S (eds.) (2002) The essential John Nash. Princeton Univ. Press, Princeton

Luce RD, Raiffa H (1957; 1989) Games and decisions: introduction and critical survey . John Wiley, New York; Dover, New York

Marx K, Engels F (1848) Manifest der Kommunistischen Partei). (English: The manifesto of the communist party, or simply the communist manifesto), a pamphlet. London.

Morgenstern O (1928) Wirtschaftsprognose, einer Untersuchung ihrer Voraussetzungen und Mölichkeiten (English: Economic forecasting: a study of its difficulties and impossibilities). Springer Verlag, Vienna

Morgenstern O (1935) Vollkommene Voraussicht und wirtschaftliche Gleichgewicht (English: Perfect foresight and economic equilibrium). Zeitschrift für Nationalökonomie 6-3 : 337-357

Morgenstern O (1958) John von Neumann, 1883-1957. Econ J 68: 499-503

Morgenstern O (1972) Thirteen critical points in contemporary economic theory: an interpretation.

J Econ Literature 10-4 : 1163-1189

Morgenstern O (1976) The collaboration between Oskar Morgenstern and John von Neumann. J Econ Literature 14-3 : 805-816

Nakayama C (1997) The process of collaboration between von Neumann and Morgenstern. History of Econ Review 26: 40-50

Nakayama M, Muto S, Funaki Y (eds.) (2000) Thinking in Game theory. Keiso, Tokyo

Nasar S (1998) A beautiful mind: the life of mathematical genius and Nobel laureate John Nash. Touchstone, New York

Okada A (1996; 2nd ed. 2011) Game theory. Yuhikaku, Tokyo

Poundstone W (1992) Prisoner's dilemma: John von Neumann, game theory, and the puzzle of the bomb. Doubleday, New York

Sakai Y (1982) The economics of uncertainty (in Japanese). Yuhikaku, Tokyo

Sakai Y (1996) The economics of risk: information and culture (in Japanese). Yuhikaku, Tokyo.

Sakai Y (2000) The concept of risk in economics: the impact of sympathy and animal spirits (in Japanese). ESP, Economic Panning Agency, Tokyo

Sakai Y (2001) Von Neumann , Morgenstern, and the world of game theory (in Japanese). Tsukuba Econ Studies, Spring: 1-42, and Summer : 81-122

Sakai Y (2005) On risk society (in Japanese). Iwanami, Tokyo

Sakai Y (2010) The economic thought of risk and uncertainty (in Japanese). Minerva, Kyoto

Sakai Y (2019) J.M. Keynes versus F.H. Knight: risk, probability, and uncertainty. Springer.

Schotter A (ed.) (1976) Selected economic writings of Oskar Morgenstern. New York Univ. Press, New York

Shubik M (1967) Essays in mathematical economics: in honor of Oskar Morgenstern. Princeton Univ. Press,, Princeton

Shubik M (1982) Game Theory in the Social Sciences: Concepts and Solutions. The MIT Press, Massachusetts

Soros G (2008) The crash of 2008 and what it means. Public Affairs, New York

Suzuki M (1959) The theory of games (in Japanese). Keiso, Tokyo

Suzuki M (1994) The new theory of games (in Japanese). Keiso, Tokyo

Suzuki M (1999) The world of game theory (in Japanese). Keiso, Tokyo

Taleb NN (2007) The black swan: the impact of the highly improbable. Random House, New York

Thurow LC (1980) The zero-sum society. Basic books, New York

Varoufakis Y (ed.) (2001) Game theory: critical concepts in the social sciences. Routledge, London and New York

von Neumann J, Morgenstern O (1944; 2nd ed. 1946; 3rd ed. 1953) Game theory and economic behavior. Princeton Univ. Press, Princeton

Yoshioka S (1996)　Poe's short stories and mathematics (in Japanese). Contained in Iyanaga S　et al. (ed.) (1966) Mathematics and human life, Gakuseisha, Tokyo

FOOTNOTES

1)　For the sad history of game theory in its early days, Suzuki (1991) once lamented it as follows: "The theory of games was born and regarded long as heretical.　Its wide recognition as an important contribution to science took really long years." (Suzuki 1991, page 7)　Suziki has been a lone pioneer in game theory in Japan, and worked hard as a professor at Tohoku University and Tokyo Institute of Technology.　Fortunately, his long and lonely struggle is now amply rewarded now.

2)　The original book *Theory of Games and Economic Behavior* (1944) was a very bulky book, full of mathematical equations and strange notations.　It was treated with "fear and respect. " In commemorating the fifth anniversary of its publication, the Princeton University Press voiced the following announcement in *The American Sciencist*：　" Recognition of first-rate books often requires so many years.　Once it is rightly recognized as such, however, its influence will far-exceed the narrowed scope of readers, probably reaching the general public in the world."　To tell the truth, the volume of the first edition was only 4000.　Understandably, it was hardly read by economists, and purchased by a limited number of libraries.　Remarkably, however, it was bought by some curious gamblers.　For details, see Poundstone 1992, page 63.

3)　For the history and international relations of Europe, see *World History Illustrated*　(1996).

4)　For the life and work of John von Neumann, see Poundstone (1992).　Neumann has only one daughter, whose name is Marina von Neumann Whitman.　When Marina was very young, she was a very clever girl, and said to George Gamow, a well-known science writer, that other than pure mathematics, she was much clever than her father, John von Neumann.　And shed added that even in pure mathematics, she was almost as clever as John (see Preface, Gamow 1947, revised 1961).　In the early 1970s, Sakai was an assistant professor of mathematical economics at the University of Pittsburgh.　Sakai still has a very fond memory of Marina, already a noted professor of international economics, who once told Sakai that she had lost much interest in pure and applied mathematics.　.

5)　For the life of Oskar Morgenstern and his collaboration with von Neumann, see Morgenstern (1976) and Suzuki (1994).

6)　By chance, the year of 1928 was the Year of the Dragon in terms of the Japanese animal calendar.　In that memorial year, we saw the birth of many famous Japanese economists including Hirofumi Uzawa as a world-wide pioneer of general equilibrium theory, growth theory and the economics of global warming, and Mitsuo Suzuki as an outstanding promoter of game theory and its applications.　Twelve years later than Uzawa and Suzuki, Sakai was born in 1940, another Year of the Dragon.

7)　There were so many economists who got quite impressed by Morgenstern's German paper

(1935). Among those people were Frank H. Knight, a pioneer of the economics of uncertainty at Chicago, who took pains to translate it to English and fondly use it as a material in his lecture. Its official publication, however, was done far later in Schotter (ed.) (1976) *Selected Economic Writings of Oskar Morgenstern.*. Its English paper was titled "Perfect Foresight and Economic Equilibrium." As the saying goes, life is short but art is long ! A further investigation between Knight and Morgenstern should be eagerly awaited.

8) Writer Conan Doyle was not born in England, but in Scotland, an archrival of England. Interestingly enough, he graduated from the Faculty of Medicine, University of Edinburgh. By chance, Adam Smith, called the Father of Economics, was also a son of Scotland, being a graduate of the University of Glasgow. Scotland used to be the home of Celtic culture, which seems to do some contribution to the Study of Man.

9) Although Sherlock Holmes is a brilliant detective, he is no more than a human being, sometimes making some human mistakes. In the story "A Scandal in Bohemia," a lovely and cool female, called Irene Adler, is one of those women who have cleverly escaped from the pursuit of Holmes and London Police.

10) According to Conan Doyle's detective story, "Monna Lisa," the famous masterpiece of Leonardo da Vinci, was once stolen from the musee du Louvre (English: Louvre Museum). Sherlock Holmes, who was secretly asked to do criminal investigation from the French Government, suspected that Professor Moriarty and his criminal gang were working behind the scene and succeeded in getting back the masterpiece to the Museum. Such an outstanding achievement by Holmes, however, meant a devastating damage to Moriarty, who was then planning to kill Holms. "the unbearable person"

11) In recent times, "animal spirits" a la John M Keynes and "courage to deal with uncertainty" a la Frank H. Knight have attracted the attention of economists again. For this point, see Sakai (2000).

12) In my opinion, Colonel Moran plays an important role in Conan Doyle's detective stories. Moran is not a superman like Sherlock Holmes or Professor Moriarty, but very good at air rifle shooting. Although the importance of the duel between Moran and Holmes has been underestimated in the academia, we should pay more attention to it. In our opinion, another dual between Professor Moriarty and Sherlock Holmes should not be overestimated. A balanced view of the two duels would be required for full understanding of Holmes drama.

13) According to Frank H. Knight (1921), "non-measurable uncertainty" should be distinguished from "measurable risk," In the measurable world, while risk is a valid concept, uncertainty tends to lose its validity. We believe that there is an incurable gap between game theory and uncertainty theory. It is high time to establish a NEW theory of unexpected surprises, going beyond the narrowed scope of OLD theory of games.