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Mixed Oligopoly and Monetary Policy in the Financial Market

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Abstract

In this study, existing financial conjectures are used as a base to develop a new theory of mixed oligopoly in financial markets. The main analysis focus of this research is the simple model where one public bank competes in quantity with other private banks. The main hypothesis of this examination is that private banks maximize their profits, while public enterprises behave to maximize its objective function, depending not only on profit but also on social welfare. The models were constructed based on both private and mixed oligopoly. The equilibrium deposit quantities, equilibrium profits of each institution and social welfare were derived in both cases. After that, both are compared and the effects of monetary policies were analyzed.

Keywords: banking sector; mixed oligopoly; privatization policy; reserve ratio operation; entry regulation policy in financial market
JEL classification: E52; E58; G21; L13; L33

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1. Introduction

In December of 1978, a historical decision was made to tackle the adoption of reform and opening policy of China. In response to such a policy shift, Chinese banking industry has also adapted and changed a lot. It is worth nothing that, until then, there was only one bank in China but, after the "Reforms and Opening-up" program was established, four state-owned commercial banks hit the market, together with the first private bank – China Minsheng Bank – in 1996 Since then, many private banks branches have joined the Chinese financial economy. The "wave" of privatization first started from small and medium enterprises, but now extends to large enterprises. One of the most important issues regarding this policy is the privatization of the "big four" state-owned Chinese commercial banks in China. In view of this scenario, Bank of China was listed on the Hong Kong Stock Market in August 2002. Three years later, in 2005, the China Construction Bank was also listed on the Hong Kong Stock Market, followed by Industrial and Commercial Bank of China Limited that was also on the list of Shanghai Stock Market one year later, in 2006. The last branch, the Agricultural Bank of china hit both lists in 2010 and, even now that all the names are, the privatization program is still ongoing. Table 1 shows the 2010 rates of holding stock for all the "big four".

[Table 1 around here.]

In Japan, until the 1980s, the Japanese government had a myriad of restrictions on the market access of the banking industry, and strictly supervised the establishments, the number of institutions, the number of employees, and the business hours of each branch. However, in order to activate its financial market, the Japanese government had significantly relaxed the attention

towards this sector. After that, the privatization program of Japan Post Holdings, discussed in the early 2000s, has gradually been implemented and kicked-off the privatization process of the Japanese financial industry that is still in progress.

Currently, there are many studies on the background of oligopoly in the financial market. For example, in the study conducted by Freixas and Rochet (2008), the Monti-Klein model was used to obtain the effect of interest rate policy under oligopolistic market. This model has been originally analyzed by Klein (1971) and Monti (1972), and the oligopolistic model for financial institutions has been analyzed by using this type of setting. Gunji and Miyazaki (2016) also examined the effect of monetary easing policy on the Monti-Klein model, and proved that the liquidity impact in the model with money creation depends on the ratio of the deposits of borrowers to their loans. They also analyzed the effect of the quantitative monetary easing policy which the Bank of Japan, the Federal Reserve Bank and the Bank of England have been adopting since 2000s.[1]

On the other hand, in the field of industrial organization theory, the analysis of mixed oligopoly has been studied since De Fraja and Delbono (1989). In this research, it was shown that it is desirable, from the viewpoint of the social welfare, that the public enterprises should privatize and behave to maximize their profits even if the number of private companies in the market is large. However, the possibility of partial privatization was not considered in their papers. In real economy, the privatization of public firms is mostly in the form of partially privatized firms, rather than fully privatized ones. Matsumura (1998) was the first one who considered to formalize the possibility of partial privatization; in other words, semi-private branches. In this paper, they assumed a financial market, a mixed duopoly market composed of a Public company and a private company, and it was shown that the private privatization was

[1] Quantitative monetary easing is defined as a policy that the central banks increase the monetary base until interest rates reach the lower bound level. See Gunji and Miyazaki (2016, Section 3) for the detail of quantitative monetary easing policy.

optimum under mild conditions.

Regarding the field of industrial organization and public economics, the competition model between private and public enterprises is a mixed one, recognized as oligopoly model. Since it has been newly developed, there are still very few analyses on the bank sector with the mixed oligopoly. In fact, it can be said that considering the markets in Japan and China, each bank is competing in a mixed oligopoly market today. However, the analysis of mixed oligopoly in existing financial markets has not always been under the standard of behavioral measures between public and private banks. For instance, through the eyes of Yoshino and Fujita (1996), private banks were classified in the interest of maximizing profit, while public banks were labeled to act as zero profit.

The purpose of this study is to verify that, when there is a large privatization phenomenon of public banks what kind of effects and impacts will it bring to the domestic economy, and how will it change the domestic financial policy. Here, the mixed oligopoly model in the banking industry was introduced based on De Fraja and Delbono (1989) model. This study assumes a market where one public bank and n private banks exist. Hence, when these banks compete with each other, they derive two equilibria: the mixed oligopoly (when the public bank is fully nationalized) and the oligopolistic (when the public bank being fully privatized). In addition, to fulfill the aims of this study, it was necessary to compare the amount of deposit, profit, and social welfare.

In standard monetary economic theory, the reserve ratio operation is an important policy tool for financial market adjustment. In particular, People's Bank of China used the reserve requirements policy as a way to control inflation rates and adjust the reserve requirements many times in 2000s. In China, this is still one of the important monetary policies of market regulation. Figure 1 shows China's Reserve Requirement Ratio's changes from 2002 to 2020.

[Figure 1 around here.]

Concerning the entry regulation policy for the banking industry, Gunji, Miura, and Yuan (2009), conducted a study using empirical tools, suggesting that competition in the banking industry reduces the impact of monetary policy on bank lending in the pure oligopolistic setting. On the other hand, Inoue (1995) pointed out that some conditions are necessary to establish both the free entry and the social welfare maximization. Such conditions apply to a scenario where public and private banks conduct Cournot competition, known as a fundamental concept about economic competition that is often used in basic fields such as microeconomics, in the mixed oligopolistic financial market. Moreover, Saha and senarma (2004) conclude that bank entry will only reduce social welfare when all branches are equally efficient in a mixed oligopoly market where one public bank and n private banks coexist.[2]

In this paper, we set up a financial market and establish an oligopoly model consisting of a public bank and n private banks, and we compare the profit of each bank and welfare before and after privatization. as the research object of mixed oligopoly competition in the financial market, which has not been well analyzed in the analysis. However, this paper is that it builds an original model of financial markets that is more in line with reality than the existing mixed oligopoly model of financial markets in which public financial institutions act to maximize their social welfare. Furthermore, the analysis on monetary policy is based on privatization policy, reserve ratio operation and entry regulation policy in financial market in this paper. To date, analyses of privatization policy have often involved empirical analysis; Megginson (2005) documented the size,

[2] Saha and Senarma (2004) model is based on the reaction function of Fershtman (1990). In this model, while the government pursues both social welfare and profit, and maximizes the sum of the total values, in this paper, notes that the government aims to maximize social welfare.

theoretical foundations, and measurement performance of state-owned banks around the world, and assessed why many governments privatize large, usually state-owned banking sectors. As for the reserve ratio operation policy, it has had little application in developed countries in recent years, but it is still an important monetary policy to regulate financial markets in developing countries such as China and Vietnam, where economic growth is attracting attention. Finally, with regard to the analysis of entry regulation policy in the banking industry, Suzumura (1990), who analyzed oligopolistic competition among private banks, derived the excess entry theorem, which states that the number of banks with free entry exceeds the number of banks that maximize social welfare. However, unlike real-world entry/exit regulations, we assume that each bank is not in an economy of scale situation. And this paper is taken into consideration here was regulation policy and its examination was conducted based on the relationship between the number of banks and the interest rates. We assume that this is a new perspective that will be generated in the analysis of industrial organization theory on the banking industry through this paper. This paper is discussed as follows. In Section 2, we develop a mixed oligopoly model which are competed with the public bank and the private banks, oligopolistic model for only private banks because of the privatization of the public bank, and then compare their results. In Section 3, we use the model to analyze one of major financial policies of the central bank, namely, reserve rate policy, and the effect of reserve rate on the other interest rates. Section 4 analyzes the entry regulation policy and describes the effect of private banks' entry into the financial markets on interest rates. Section 5 is conclusion.

2. Mixed oligopoly theory models and monetary policy in the bank

2.1. Mixed oligopoly theory model in the bank

In this paper, it is assumed that an oligopoly market have only two types of financial

institutions owned by n private banks and one public bank. Before the privatization, public bank's objective function is maximizing social welfare. On the contrary, the remaining n banks, since they are private enterprises, they seek to maximize their profits. The hypothesis here is that, after the privatization, both branches will prioritize their incomings. The derivatives of this section were based on the welfare in each case "M" (mixed oligopoly) and case "P" (privatization), and further compared.

In this economic framework, financial institution technology is represented by a cost function, interpreted as the cost of managing a volume d_i of deposits and a volume l_i of loans ($i = 0, 1, \cdots, n$; 0: public bank; $1, \cdots, n$: private banks). The loan market demand function and the deposit market supply function are assumed to be linear as:[3]

$$r_L(L) = a - L, \qquad a > 0, \tag{1}$$

$$r_D(D) = D, \tag{2}$$

where r_L represents the loan rates and r_D the deposits interest rates. L is the total amount of loans in the loan market: $L = l_0 + \sum_{i=1}^{n} l_i$, and D is the total deposit in the deposit market: $D = d_0 + \sum_{i=1}^{n} d_i$. a stands for the market size in the lending market.[4]

Going further, the cost function of financial institution i is assumed as below. Here, it is assumed that each bank has the same cost function regardless of whether it is a public or a private institution.

$$C_i = F + cd_i + \frac{l_i^2}{2}, \qquad F \geq 0, c > 0. \tag{3}$$

In this case, the cost function C_i is an increasing function linked to the amount of deposits and loans. F represents the fixed cost, and cd_i represents the expense for the deposit of bank i.

[3] About models of banks, such as loan market demand function and the deposit market supply function see Dalla and Varelas(2013), VanHouse(2010,ch2) and Saha, and Sensarma (2011).

[4] In this paper, rate on reserve deposits is not considered. Therefore, we assumed that $r_c = 0$.

Also, $l_i^2/2$ goes for the cost of lending to bank i. While the cost of deposit is considered only labor or advertising expenditures, the cost of loan may be related to other aspects including the costs of examination of loan application, managing loanable resources and collection of claims. The loan fee considers that the scale is more significant in comparison with the cost of deposit. Therefore, it is assumed that, while the marginal cost of deposit is constant, that of loan is increasing according to l_i.[5] Moreover, it is established that $F = 0$ because the size of the fixed cost does not greatly affect the conclusion.

The profit of bank i is given by

$$\pi_i = r_L l_i - r_D d_i - \left(c d_i + \frac{l_i^2}{2} \right). \tag{4}$$

Moreover, one of the hypotheses here is that the central bank regulates to keep in its favor a certain proportion of deposits in each bank as cash reserves. Assuming the ratio of cash reserves to deposits as ρ, the following relationship is established for the bank i

$$l_i = (1-\rho) d_i, \qquad 0 < \rho < 1. \tag{5}$$

At this time, the profit of the bank i, Eq. (4), changes to:

$$\begin{aligned} \pi_i &= [(1-\rho) r_L - r_D] d_i - \left(c d_i + \frac{(1-\rho)^2}{2} d_i^2 \right) \\ &= (A - BD) d_i - \left(c d_i + \frac{k}{2} d_i^2 \right), \end{aligned} \tag{6}$$

where $A \equiv (1-\rho)a > 0$, $B \equiv (1-\rho)^2 + 1 > 0$, $k \equiv (1-\rho)^2 > 0$, and A represents the modified market size by the proportion of the volume of loan against the deposit amount. This value is larger with the increase in a in the loan market and is smaller when reserve requirements are boosted.

Further on, surpluses of each economic agent in this economy are defined. First, the surplus of borrowers is given by

[5] See Kopecky and VanHoose (2006), VanHoose (2010) the cost of loaning is set by a quadratic function.

$$LS \equiv \int_0^L r_L(x)\,dx - r_L(L)\cdot L = \left[ax - \frac{x^2}{2}\right]_0^L - (aL - L^2) = \frac{L^2}{2} = \frac{k}{2}D^2.$$

Second, the surplus of the depositors is expressed as:

$$DS \equiv r_D(D)\cdot D - \int_0^D r_D(x)\,dx = D^2 - \left[\frac{x^2}{2}\right]_0^D = \frac{1}{2}D^2.$$

Therefore, the social welfare function is specified as the sum of borrowers' and depositors' surplus and bank profit

$$\begin{aligned} SW &\equiv (LS + DS) + \left(\pi_0 + n\sum_{i=1}^{n}\pi_i\right) \\ &= \frac{B}{2}D^2 + \left[(A - BD)D - \left\{cD + \frac{k}{2}\left(d_0^2 + \sum_{i=1}^{n}{d_i}^2\right)\right\}\right]. \end{aligned} \tag{7}$$

Under these settings, it is possible to assume that public and private banks perform the Cournot competition in this model.[7]

2.2 The case of mixed oligopoly

Before the privatization, public bank selects the deposit d_0 in order to maximize social welfare SW. Here it is presumed that the government representative in the public bank's board does not have the same goals as the government itself, similar to De Fraja and Delbono (1989).[8] On the other hand, each private bank selects the deposit $d_i (i = 0, 1, \cdots, n)$ to maximize its own profit. The aim of this process is to look for Nash equilibrium of this "player's game". Nash equilibrium is therefore obtained as a solution for the following simultaneous equations

$$\frac{\partial SW}{\partial d_0} = 0 \;\Leftrightarrow\; (A - c) - (B + k)d_0 - B\sum_{i=1}^{n} d_i = 0 \tag{8}$$

$$\frac{\partial \pi_i}{\partial d_i} = 0 \;\Leftrightarrow\; (A - c) - Bd_0 - (Bn + B + k)d_i = 0, \qquad i = 1, \cdots, n. \tag{9}$$

It has to be noted that the second order condition of this problem is always satisfied.[9]

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[7] In this paper, the interbank market is not considered. Refer to the Monti-Klein model for the mould that contains the interbank market.

[8] See Bös (1991), De Fraja and Delbono (1990), and Nett (1993) for an overview of this field.

[9] It is proven that the setting of the second order condition is satisfied by the following calculation

Therefore, equilibrium deposit of each financial institution is given by,

$$d_0^M = \frac{(B+k)(A-c)}{(B+k)^2+nkB}, \qquad d_i^M = \frac{k(A-c)}{(B+k)^2+nkB}, \qquad i = 1, \cdots, n, \tag{10}$$

and the total deposit is given by

$$D^M = \frac{(B+k+nk)(A-c)}{(B+k)^2+nBk}. \tag{11}$$

As it can be seen from the equation (10), comparing the equilibrium value of deposits between public and banks, the public deposit is larger in public branches ($d_0^M > d_i^M$). Therefore, the cost function ($C_i = cd_i + l_i^2/2$) shows that public banks tend to choose the amount of deposit with a higher cost.

In Equations (10) and (11), the profit π_i^M ($i = 0,1, \cdots, n$) of each bank and social welfare is represented by

$$\pi_0^M = \frac{(A-c)^2 k(B+k)^2}{2[(B+k)^2+nBk]^2}, \tag{12}$$

$$\pi_i^M = \frac{(A-c)^2 k^2(2B+k)}{2[(B+k)^2+nBk]^2}, \qquad i = 1, \cdots, n \tag{13}$$

$$SW^M = \frac{(A-c)^2[\mathrm{B}^3 + k^3(1+n) + B^2k(3+2n) + Bk^2(3+4n+n^2)]}{2[(B+k)^2+nBk]^2}. \tag{14}$$

Finally, r_L and r_D are set as

$$r_L^M = a - (1-\rho)D^M = a - (1-\rho)\frac{(B+k+nk)(A-c)}{(B+k)^2+nBk}, \tag{15}$$

$$r_D^M = D^M = \frac{(B+k+nk)(A-c)}{(B+k)^2+nBk}. \tag{16}$$

2.3 The case of private oligopoly

After the privatization policy, the former public bank escalates its own profit. At that time, the analysis in this case is equivalent to the Cournot competition among $n+1$ private banks

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$\frac{\partial^2 SW}{\partial d_0^2} = -(B+k) < 0.$

and, again the attention is restricted to the symmetric Nash equilibria. This can represent an oligopolistic market without any public intervention; so the equilibrium can be found by solving the following equations:

$$\frac{\partial \pi_i}{\partial d_i} = 0 \;\Leftrightarrow\; (A-c) - Bd_0 - (Bn + B + k)d_i = 0, \qquad i = 1, \cdots, n, \qquad j \neq i, \tag{17}$$

therefore, because all banks are completely symmetric; $d_0 = d_i = d$, so it is possible to derive

$$(A-c) - (2B + k + Bn)d = 0. \tag{18}$$

Here, the second order condition is properly satisfied.[10] Following the aforementioned procedure, the values of equilibrium in the oligopolistic market are obtained by

$$d^P = \frac{A-c}{2B + k + Bn}, \tag{19}$$

$$D^P = \frac{(A-c)(1+n)}{2B + k + Bn}, \tag{20}$$

$$r_L^P = a - (1-\rho)D = a - (1-\rho)\frac{(A-c)(1+n)}{2B + k + Bn}, \tag{21}$$

$$r_D^P = D = \frac{(A-c)(1+n)}{2B + k + Bn}, \tag{22}$$

$$\pi^P = \frac{(A-c)^2(2B-k)}{2(2B + k + Bn)^2}, \tag{23}$$

$$SW^P = \frac{(A-c)^2\big(2k + B(5 + 2n + n^2)\big)}{2(2B + k + Bn)^2}. \tag{24}$$

2.4 Privatization and Economic Welfare of Financial Institutions

Here, the process was focused on comparing the equilibrium value of deposit, profit, and social welfare in the two center cases. The results are summarized in Table 2 and made it possible to obtain the following proposition, based on the comparison of both amounts:

[10] It is proven that the second order derivative is negative by the following equation

$$\frac{\partial^2 \pi_i}{\partial d_0^2} = -(3B + k) < 0.$$

Proposition 1:

1. $d_0^M > d_j^P > d_i^M,\ i = 1, \cdots, n,\ \ j = 0,1, \cdots, n,$
2. $D^M > D^P,\ r_L^M < r_L^P,\ r_D^M > r_D^p,$
3. $SW^M > SW^P.$

[Table 2 around here.]

The economic implication of this proposition and its mechanism are as follows: (i) Proposition 1.1 shows the result of comparing the amount of deposits in two cases. It illustrates that there is a relation between them if the deposit amount of public bank in case "M" is larger than the deposit amount of each bank in case "P". In this scenario, the deposit sum in private bank before the process of privatization is the smallest.

(ii) Proposition 1.2 compares the total deposit amount, the deposit interest rate r_D and the loan rate r_L. The link between both can be explained when the deposit amount in case M is larger than the total amount of deposits in case P. On the other hand, the loan rate r_L is a decreasing function in comparison with the total deposit amount, so the loan rate in case P is larger than those in case M. Regarding all the deposit interest rates, it could be stated that the value r_D in case M is larger than that in the case P.

(iii) According to (3), the size of social welfare in the two states depends on the number of private firms participating in the market. In other words, if the number of private firms is small, the social welfare in the mixed state is greater than the social welfare in the privatized state, while if there are many private firms in the market, the social welfare in the mixed state is less than the social welfare in the privatized state. In other words, if the number of private enterprises is large, privatization of public enterprises will bring about an improvement in social

welfare.

3. The Effects of Monetary policy

3.1 Reserve Requirements

To guide the processes included in this study, the reserve rate ρ is considered a policy instrument used by the central bank to try to influence the quantity of money in the economy[11]. According to the usual description of monetary policy that can be found in financial economics textbooks, the reserve deposit system in the financial sector is being introduced as part of monetary policies. When the government decide to raise the reserve requirement, the private branches need to increase their account at the central bank. They will, then, collect loans that have been lending to the company, and put it into the central bank. As a consequence, the amount of loans will decrease together with the money supply, while the interest rates are boosted. Conversely, when the reserve requirement is lowered, the financial market is relaxed, so the interest rate is weakened. This policy is called reserve requirements operation.

In Japan, based on the "Act on Reserve Requirement System" enforced in 1957, it was instituted a requirement for financial branches to separate a certain proportion of deposits held by each of them to the central bank within a period of time. The minimum amount of money required for deposit is referred to as "legal reserve deposit" or just "reserve". The financial institutions subjected to the reserve requirements system are usually deposit handling institution – basically credit unions over and banks of certain scale. It is noteworthy that the reserves rates are determined by the Policy Board of the Bank of Japan. The current reserve rate for banks goes from 0.05 to 1.3%.

[11] Reserve Requirements is a system that manages to deposit the amount of deposits (such as "reserve rate") to the target financial institution to the central bank.

In China, to adjust the economy like Japan did, the market is controlled not only by changing the interest rate (that is, the interbank rate) but also the reserve requirement, as a monetary policy. In fact, Central Bank of China determines both rates accordingly. In 2018, People's Bank of China lowered the reserve requirement by about 2.5% in 2018, consequently dwindling the amount of money that private banks can deposit. This movement makes it for other bank branches to lend easier in the market. As of 2019, the reserve rate was set at 14.5% for major bank and 12.5% for small and medium enterprises.

3.1.1 Effect of changes in Reserve Requirements

This section is mainly focused on introducing the effect of the reserve requirements. When $A = (1-\rho)a$, $B = (1-\rho)^2 + 1$, $k = (1-\rho)^2$ is substituted for Eqs. (8) and (9), the reaction functions in mixed oligopoly case are derived as:

$$\begin{aligned} &[(1-\rho)a - c] - [2(1-\rho)^2 + 1]d_0 - [(1-\rho)^2 + 1]nd_i = 0, \\ &[(1-\rho)a - c] - [(1-\rho)^2 + 1]d_0 - [(n+2)(1-\rho)^2 + 2]d_i = 0. \end{aligned} \tag{25}$$

Here, when the function (25) is solved, d_0^M, d_i^M, and D^M obtained by the functions (10) and (11) can be rewritten as

$$d_0^M = \frac{[2(1-\rho)^2 + 1][(1-\rho)a - c]}{(2\rho^2 + 3 - 4\rho)^2 + n(1-\rho)^2(\rho^2 - 2\rho + 2)}, \tag{10'a}$$

$$d_i^M = \frac{(1-\rho)^2[(1-\rho)a - c]}{(2\rho^2 + 3 - 4\rho)^2 + n(1-\rho)^2(\rho^2 - 2\rho + 2)}, \qquad i = 1, \cdots, n, \tag{10'b}$$

$$D^M = \frac{[(1-\rho)a - c][n(1-\rho)^2 + 2\rho^2 - 4\rho + 3]}{(2\rho^2 + 3 - 4\rho)^2 + n(1-\rho)^2(\rho^2 - 2\rho + 2)}. \tag{11'}$$

The result is represented by Figs. 2.1~2.9, with $c = 0.2$, $n = 10$, and $a = 6$ As can be seen from the figures, by rising ρ, the amount of deposits in public bank increase at first, but goes down a little after. Although this parameter is set in this paper, but if we set a higher value of c, for example, $c = 2.1$, then the amount of deposits in public enterprises will decrease monotonically, but the other figures show that changing the parameter does not change the

result. On the other hand, by boosting ρ, the amount of deposits in private banks will lower sequentially. In addition, it is found that the total deposit amount (the sum from all the branches) drops when the reserve rates expand.

[Figs. 2.1, 2.2, and 2.3 around here.]

The next step was to analyze the relationship between the reserve rate and the profit of each bank. In the same way as in the amount of deposits, A, B and k are substituted for Eqs. (12) and (13) to obtain

$$\pi_0^M = \frac{[(1-\rho)a-c]^2(1-\rho)^2[2(1-\rho)^2+1]^2}{2[(2\rho^2+3-4\rho)^2+n(1-\rho)^2(\rho^2-2\rho+2)]^2}, \tag{12′}$$

$$\pi_i^M = \frac{[(1-\rho)a-c]^2(1-\rho)^4[3(1-\rho)^2+2]}{2[(2\rho^2+3-4\rho)^2+n(1-\rho)^2(\rho^2-2\rho+2)]^2}, \qquad i=1,\cdots,n. \tag{13′}$$

The relationship between profit from public (resp. private) bank (resp. banks) and reserve rate is displayed in the figures, respectively. As shown, when the reserve rate ρ rises, the profit of each bank decreases monotonously. Because of that, the deposit amount (Eqs. $(10'a)$ and $(10'b)$) and the loan amount (Eq. (5)) of each bank is constantly lowering.

[Figs. 2.4 and 2.5 around here.]

Finally, the correlation between social welfare and reserve rate is explained. Substituting A B, and k into Eq. (14),

$$SW^M = \frac{\begin{pmatrix}[(1-\rho)a-c]^2(17-44\rho+50\rho^2-28\rho^2+7\rho^4)\\ \cdot[n^2(1-\rho)^4(\rho^2-2\rho+2)+(2\rho^2-4\rho+3)^3+n(1-\rho)^2]\end{pmatrix}}{[(2\rho^2+3-4\rho)^2+n(1-\rho)^2(\rho^2-2\rho+2)]^2}. \tag{14′}$$

The results show that social welfare and reserve rates and inversely proportional.

[Figure 2.6 around here.]

It is noteworthy that there can be multiple economic implications its mechanism. First, the deposit amount and reserve rate of each bank are explained in Eqs. (8) and (9) or Figs. 2.1–2.3. It can be seen that, due to the rise of ρ, the intersection point between the reaction curve and the horizontal axis may become higher.[12] This means that when the reserve rate is raised within a low domain reserve requirement, the amount of deposit of public bank grows at first, but decreases thereafter. On the other hand, the amount of deposit of private banks is reduced constantly. At this time, it can also be seen that the reserve requirement in the area and the total deposit amount are directly proportional and are boosted together.

As for the individual profit and reserve requirement, the amount of loan l_i of the bank i decreases as the reserve rate goes up with the Eq. (5). Next, as shown in Eq. (4), the decrease in l_i is found also negatively affect the profit of each bank. This result matches with the conclusions in Figs. 2.4 and 2.5.

Next, the attention turns to the relationship between social welfare and reserve rate. Eq. (7) is defined by $SW \equiv (LS + DS) + (\pi_0 + \pi_1)$. The surplus of borrowers and depositors are represented by $kD^2/2$ and $D^2/2$. As shown in Fig. 2.4, although there is a possibility that the surplus of the depositor may be increased with a bigger reserve rate, the surplus of borrowers is surely reduced by numerical analysis. Furthermore, when the reserve rate rises and the profit of each bank decreases, both surpluses loose strength, affecting also the social welfare level (represented by the sum of each surplus) and the profit of each bank. This result is consistent with the characteristics shown in Fig. 2.9. In other words, the central bank's monetary easing

[12] See Li (2019) for explanation of the reaction curve in this model.

policy (monetary tight policy) can generally change the level of profits (surplus and profit) of each economic entity. This major finding goes in accordance with the results of the general macroeconomic theories.

Overall, it is understood by the simple numerical analysis that the relationship here examined happened under the same terms in the case of private oligopoly.

3.1.2 The Effect on Interest Rates

This part is focused on the correlation between the reserve and the interest rate. To further understand that it was necessary to derive the optimal r_L, the optimal r_D, and $r_L - r_D$ in the case of mix oligopoly. The equation obtained throughout the process was:

$$r_L = \frac{a[6 + n(1-\rho)^2 - 14\rho + 15\rho^2 - 8\rho^3 + 2\rho^4] - c(\rho - 1)[3 + n(1-\rho)^2 + 2\rho^2 - 4\rho]}{n(1-\rho)^2(\rho^2 - 2\rho + 2) + (2\rho^2 - 4\rho + 3)^2}, \quad (26)$$

$$r_D = \frac{[(1-\rho)a - c][2\rho^2 + n(1-\rho)^2 - 4\rho + 3]}{n(1-\rho)^2(\rho^2 - 2\rho + 2) + (2\rho^2 - 4\rho + 3)^2}, \quad (27)$$

$$r_L - r_D = \frac{c(2-\rho)[3 + n(1-\rho)^2 - 4\rho + 2\rho^2] + a[3 + \rho(n-7) + (9-2n)\rho^2 + \rho^3(n-6) + 2\rho^4]}{n(1-\rho)^2(\rho^2 - 2\rho + 2) + (2\rho^2 - 4\rho + 3)^2}, \quad (28)$$

by using $A = (1-\rho)a$, $B = (1-\rho)^2 + 1$, and $k = (1-\rho)^2$.

Next, the same derive process was conducted but, this time, in the case of private oligopoly.

$$r_L = \frac{a(2\rho^2 - 4\rho + 4 + n) + (1+n)c(1-\rho)}{n(\rho^2 - 2\rho + 2) + 3\rho^2 - 6\rho + 5} \quad (29)$$

$$r_D = \frac{[(1-\rho)a - c](n+1)}{n(\rho^2 - 2\rho + 2) + 3\rho^2 - 6\rho + 5} \quad (30)$$

$$r_L - r_D = \frac{a(2\rho^2 - 3\rho + 3 + n\rho) + c(1+n)(2-\rho)}{n(\rho^2 - 2\rho + 2) + 3\rho^2 - 6\rho + 5} \quad (31)$$

Both aforementioned scenarios can be represented by Figs. 2.7, 2.8 and 2.9, respectively. As displayed, when the reserve rate rises, the loan rate goes in the same direction. However, the deposit rate grows slightly, decreasing thereafter. Moreover, the optimal difference between deposit interest rate and loan interest rate increases. The reason of this phenomenon can be

interpreted from the equation (5) – as the reserve rate grows, the bank loan l_i lowers. Further, as proved with equation (1), when the amount of loans has downturns, the lending rates go in the opposite direction. Regarding the deposit fares, when the reserve rate rises, the deposit amount of each bank and the deposit rate fairly increase.

[Figs. 2.7, 2.8, and 2.9 around here.]

3.2 Effects of Changes in the Number of Banks

The entry policy for banks has the aim to promote the competition in the market by allowing more financial institutions in and improving the quality of services. This process can be done by eliminating and relaxing public regulation that restricts the phenomenon of free competition.

Bank regulations have been greatly relieved since late 1980s in Japan. In the latter half of the 90s, bank services could be requested according to its own management strategy owning to some new strategies like the abolition of the bank store notification and diversification of services (e.g. Internet banking and convenience store ATM). The deregulation of banks' stores can be roughly divided into three categories. The first period ranges from the 1980's until the abolition of the store service in 1997. In the latter half of the 1990s, the number of branch offices in urban areas lowered significantly and this downgrading process reached a peak in 1993 due to the Bank for International Settlements regulation and the issue of bad debts for strengthening capital. During the second period, the diversification of existing stores and bank channels was carried out during the large-scale financial reform "big bang" from 1996 to 2001[13]. While store regulations became more relaxed, it was possible to open a bank ATM in the form

[13] The Big Bang is the second large-scale reform of the financial system in Japan in November 1996, after the collapse of the bubble economy in November 1996, which was published by the cabinet of ministers in 2001.

of a consultation window in commercial facilities including supermarkets and convenience stores, so bank services got more accessible to customers. The third stage is represented by the flow to the stand-how continuation from 2001 to present days. In other words, the first and the second period were stamped by the changes for store regulation, and the third period is marked by the sales expansion of financial products and the new entry of the finance industry.

3.2.1 The Effect on Interest Rates

As part of this study, it is important to also understand the change in number n of banks. By increasing n under both mixed and private conditions, the total amount of deposits also increases. However, the profit rates of each bank start lowering, while social welfare features get stronger. This might be due to an intensified competition, reassured by a larger number of banks and certain loss in their profit acquaintances.

The result is represented by Figs. 3.1~3.7, with $c = 0.2$, $\rho = 0.01$, and $a = 6$. Figs. 3.1, 3.2, and 3.3 show the correlation between total deposits, public bank profits, private bank profits and the total number of branches in a scenario of mix oligopoly. As can be seen when there are a larger number of banks included, the total amount of deposits increases, but the profit of each one is reduced. In addition, the relationship between social welfare and the number of banks is shown in Fig. 3.4, showing that they are directly proportional. Incidentally, the same analytical results are obtained when private cases are considered.

[Figs. 3.1, 3.2, 3.3, and 3.4 around here.]

Lemma 1: When the number of banks $n \to +\infty$ the social welfare SW^M, SW^P is represented by

$$\lim_{n\to\infty} SW^M = \frac{A^2Bk^2 - 2ABck^2 + Bc^2k^2}{2B^2k^2}$$

$$\lim_{n\to\infty} SW^P = \frac{A^2B - 2ABc + Bc^2}{2B^2}$$

More information about the link between the number of banks and the respective interest rates are shown in Figs. 3.5, 3.6 and 3.7.

[Figs. 3.5, 3.6, and 3.7 around here.]

As can be seen from above, when are more banks in the market, the optimal loans rate falls and the optimal deposit rate rises. The reason can be extracted from Eqs. (1) and (2). Therefore, it can be concluded that, when the banks enter the market, intensifying the competition, the interest rate differential is weakened as a consequence.

4. Conclusion

In this paper, we analyzed the present state of mixed oligopoly market in the world and investigated the structural change of banks sector under such situation. The analysis of the model showed that the If there are many private banks entering the market, social welfare in the full privatization was higher than that in the complete nationalization. About the financial policy, we analyzed the effect of the reserve policy and the effect of the number of banks.

It is observed that when the reserve rate rises, that is, when the financial tightening policy is carried out, the total deposit amount, profit and social welfare of each bank decrease. In other words, it will lead to higher social welfare by conducting a monetary easing policy to reduce the

reserve rate. The second policy taken into consideration here was regulation policy and its examination was conducted based on the relationship between the number of banks and the interest rates. The increase in the number of banks makes the deposit interest rate rise. Then, the loan interest rate and the difference between deposit interest rate and loan interest rate decrease.

However, the interbank market is not considered in the construction model of this study. Capital flows between city banks or between city banks and central banks are not taken into account. Moreover, as for cases closer to reality, such as the model of partial privatization of public and financial institutions, the optimal privatization ratio, and the case when foreign financial institutions enter the domestic market to compete with domestic financial institutions in the mixed oligopoly framework, the topic worth exploring in the future involves the relationship between privatization ratio and deposit reserve ratio[14].

Appendix

proof of Proposition 1

(1):

$$\frac{(B+k)(A-c)}{(B+k)^2+nkB}-\frac{A-c}{2B+k+Bn}$$

$$=\frac{(B+k)(A-c)(2B+k+Bn)}{((B+k)^2+nkB)(2B+k+Bn)}-\frac{(A-c)((B+k)^2+nkB)}{((B+k)^2+nkB)(2B+k+Bn)}$$

$$=\frac{(A-c)(B^2+Bk+nB^2)}{((B+k)^2+nkB)(2B+k+Bn)}>0$$

$$\frac{A-c}{2B+k+Bn}-\frac{k(A-c)}{(B+k)^2+nkB}$$

$$=\frac{(A-c)(B+k)^2+nkB}{(2B+k+Bn)(B+k)^2+nkB}-\frac{k(A-c)(2B+k+Bn)}{(2B+k+Bn)(B+k)^2+nkB}$$

[14] See Matsumura (2003), Fjell and Heywood (2002), and Chao and Yu (2006) for the model of competition with foreign companies which enter domestic market.

$$= \frac{(A-c)B^2}{(2B+k+Bn)(B+k)^2+nkB} > 0$$

(2):

$$\frac{(B+k+nk)(A-c)}{(B+k)^2+nBk} - \frac{(A-c)(1+n)}{2B+k+Bn}$$

$$= \frac{(B+k+nk)(A-c)(2B+k+Bn)}{(2B+k+Bn)(B+k)^2+nkB} - \frac{(A-c)(1+n)((B+k)^2+nkB)}{(2B+k+Bn)(B+k)^2+nkB}$$

$$= \frac{(A-c)(B^2+Bk)}{(2B+k+Bn)(B+k)^2+nkB} > 0$$

$$a-(1-\rho)\frac{(B+k+nk)(A-c)}{(B+k)^2+nBk} - (a-(1-\rho)\frac{(A-c)(1+n)}{2B+k+Bn})$$

$$= -(1-\rho)\frac{(A-c)(B^2+Bk)}{(2B+k+Bn)(B+k)^2+nkB} < 0$$

(3):

$$\frac{(A-c)^2[\mathrm{B}^3+k^3(1+n)+B^2k(3+2n)+Bk^2(3+4n+n^2)]}{2[(B+k)^2+nBk]^2}$$

$$- \frac{(A-c)^2\big(2k+B(5+2n+n^2)\big)}{2(2B+k+Bn)^2}$$

$$= \frac{\begin{matrix}(A-c)^2(k^5(n-1)+B^5(2n-1)+2Bk^4(2n+n^2-3)\\ +B^4k(4n+3n^2-6)+B^2k^3(6n+7n^2+n^3-13)+B^3k^2(6n+8n^2+2n^3-13))\end{matrix}}{2(k+B(2+n))^2(B^2+k^2+Bk(2+n))^2}$$

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Tables and Figures

Table1: Holding stock rate of China's four largest commercial banks in 2020

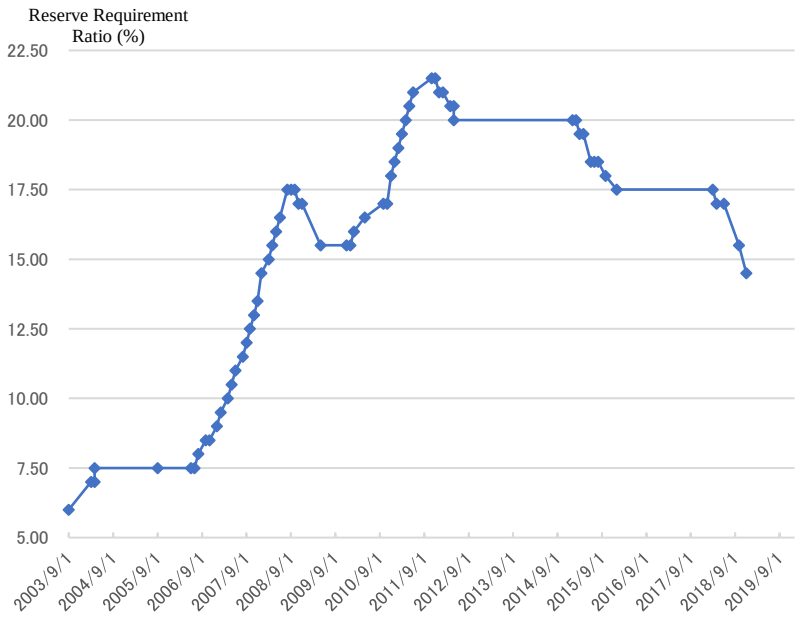
	Holding stock rate by Ministry of Finance of China	Holding stock rate by Chinese Government	Private holding stock rate
Bank of China	0%	64.02%	35.98%
China Construction Bank	0%	57.11%	42.89%
Industrial and Commercial Bank of China Limited	31.14%	34.71%	34.15%
Agricultural Bank of China	35.29%	40.03%	24.68%

Source: See each bank's annual report for 2020 and news releases for this year on their Homepages[15].

Table 2: the values of equilibrium in the two case

	Case M	Case P
d_0	$\frac{(B+k)(A-c)}{(B+k)^2+nkB}$	$\frac{A-c}{2B+k+Bn}$
d_i	$\frac{k(A-c)}{(B+k)^2+nkB}$	$\frac{A-c}{2B+k+Bn}$
D	$\frac{(B+k+nk)(A-c)}{(B+k)^2+nBk}$	$\frac{(A-c)(1+n)}{2B+k+Bn}$
$r_L(L)$	$a-(1-\rho)\frac{(B+k+nk)(A-c)}{(B+k)^2+nBk}$	$a-(1-\rho)\frac{(A-c)(1+n)}{2B+k+Bn}$
$r_D(D)$	$\frac{(B+k+nk)(A-c)}{(B+k)^2+nBk}$	$\frac{(A-c)(1+n)}{2B+k+Bn}$
π_0	$\frac{(A-c)^2k(B+k)^2}{2[(B+k)^2+nBk]^2}$	$\frac{(A-c)^2(2B-k)}{2(2B+k+Bn)^2}$
π_i	$\frac{(A-c)^2k^2(2B+k)}{2[(B+k)^2+nBk]^2}$	$\frac{(A-c)^2(2B-k)}{2(2B+k+Bn)^2}$
SW	$\frac{(A-c)^2[\mathrm{B}^3+k^3(1+n)+B^2k(3+2n)+Bk^2(3+4n+n^2)]}{2[(B+k)^2+nBk]^2}$	$\frac{(A-c)^2\left(2k+B(5+2n+n^2)\right)}{2(2B+k+Bn)^2}$

[15] Homepage of Bank of China: URL https://www.boc.cn/
Homepage of China Construction Bank: URL http://www.ccb.com/cn/home/indexv3.html
Homepage of Industrial and Commercial Bank of China Limited: URL http://www.icbc.com.cn/icbc/
Homepage of Agricultural Bank of China: URL http://www.abchina.com/cn/



Source: Homepages of People's Bank of China and National Bureau of Statistics.[16]

Figure 1 China's Reserve Requirement Ratio from 2003 to 2019

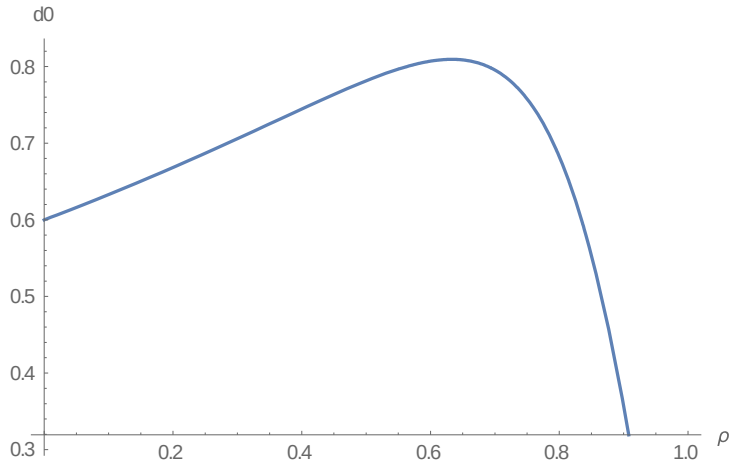


Figure 2.1 relationship between d_0^M and ρ

[16] People's Bank of China URL http://www.pbc.gov.cn/zhengcehuobisi/125207/125213/125434/125798/17085/index1.html
National Bureau of Statistics URL http://www.stats.gov.cn/

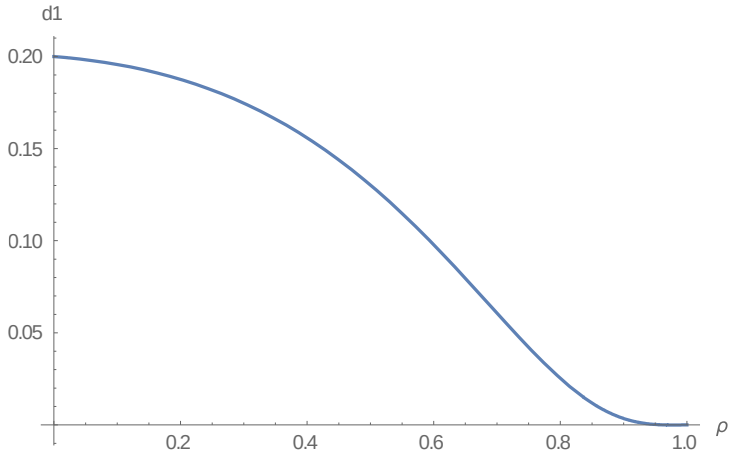


Figure 2.2. relationship between d_1^M and ρ

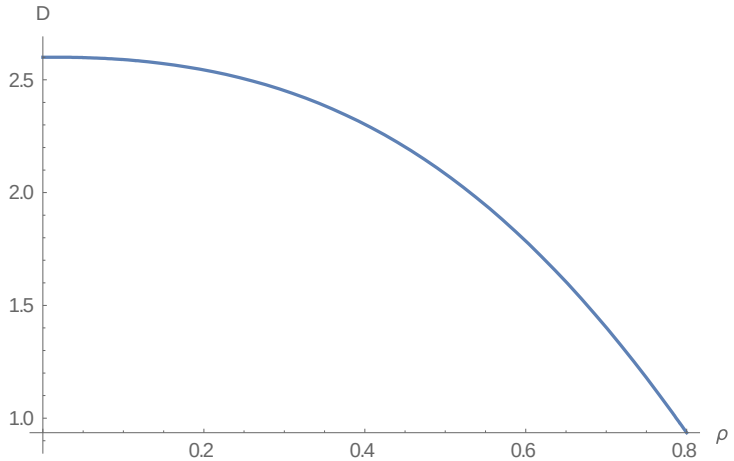


Figure 2.3 relationship between D^M and ρ

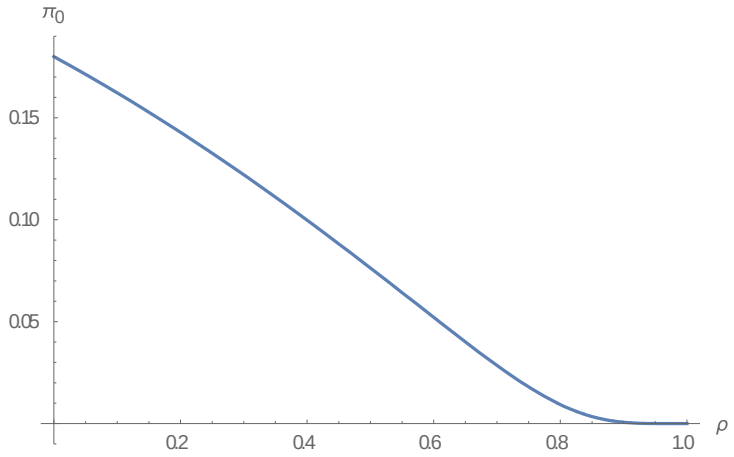


Figure 2.4 relationship between π_0^M and ρ

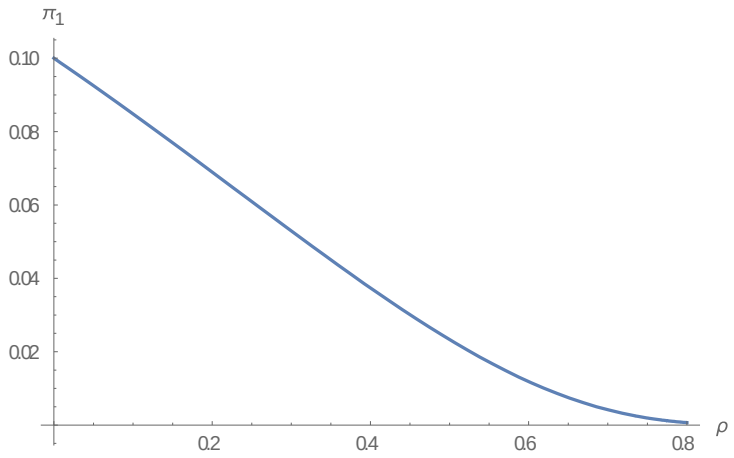


Figure 2.5 relationship between π_i^M and ρ

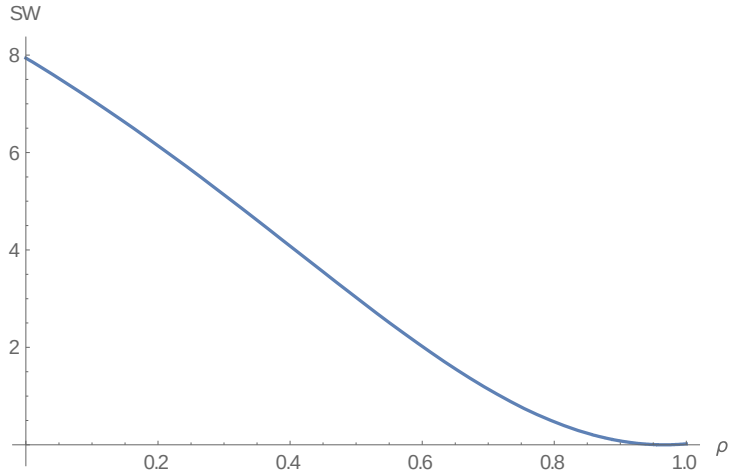


Figure 2.6 relationship between SW^M and ρ

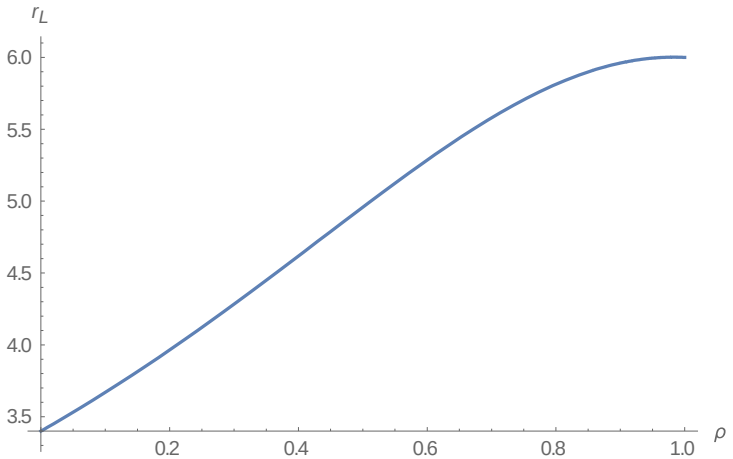


Figure 2.7 relationship between r_L and ρ

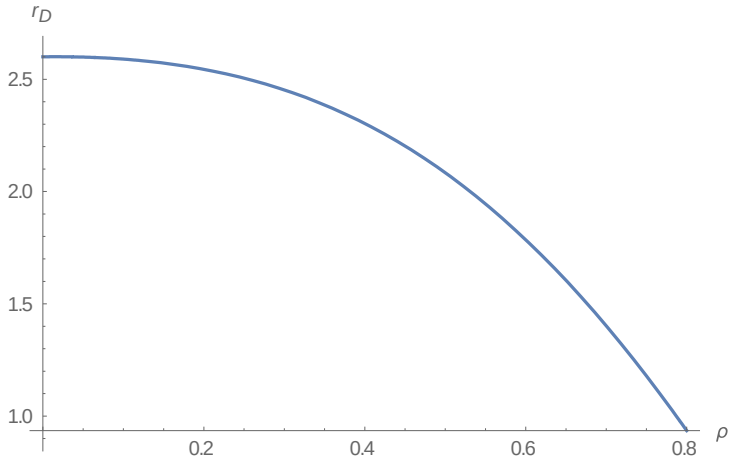


Figure 2.8 relationship between r_D and ρ

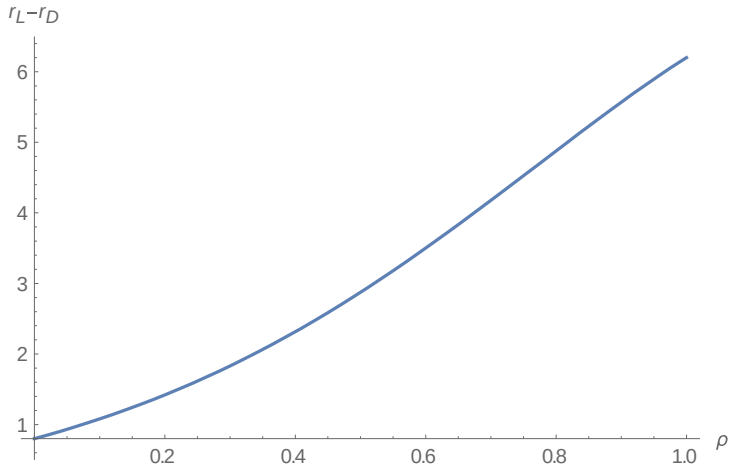


Figure 2.9 relationship between $r_L - r_D$ and ρ

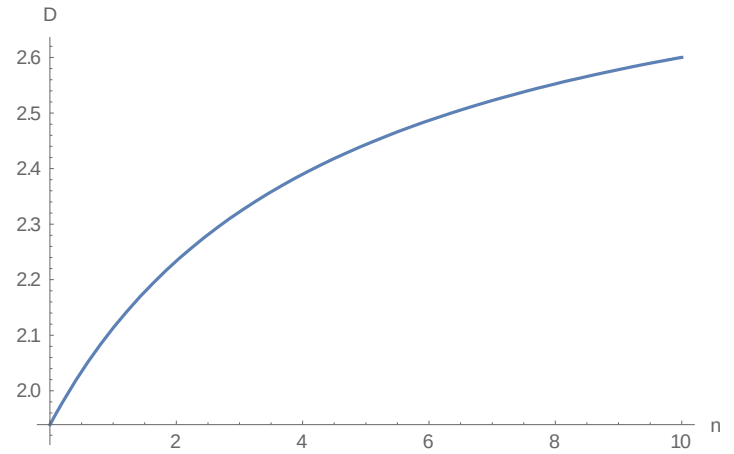


Figure 3.1 relationship between D^M and n

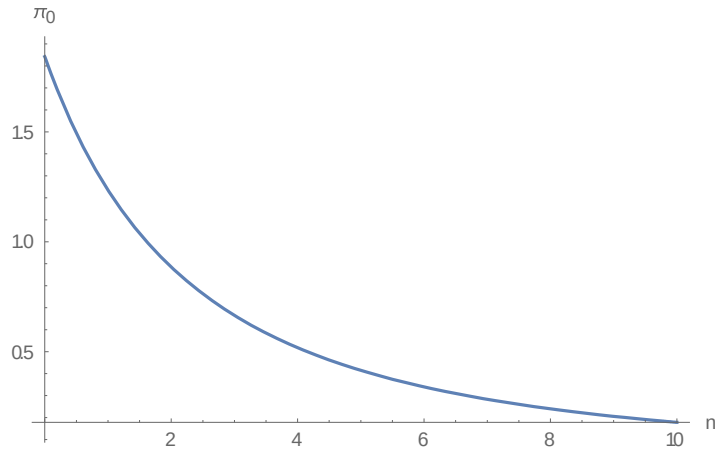


Figure 3.2 relationship between π_0^M and n

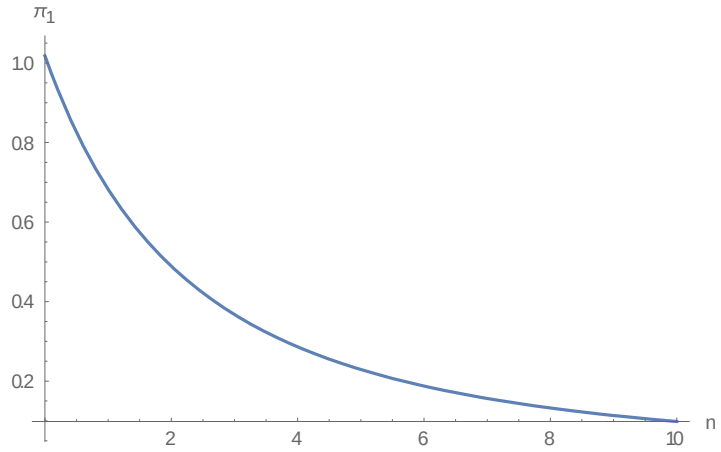


Figure 3.3 relationship between π_1^M and n

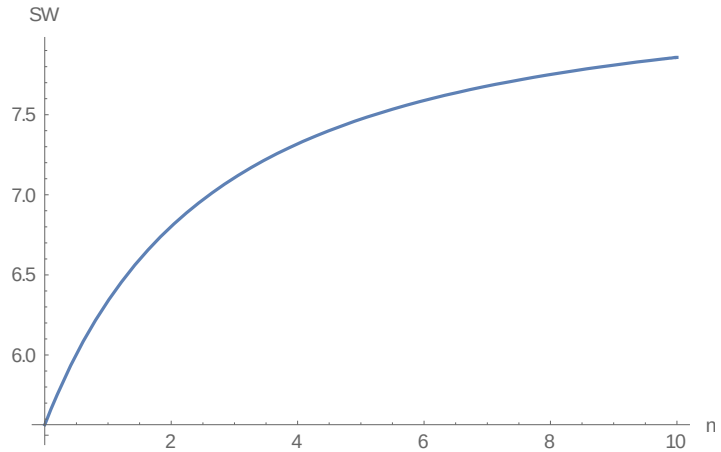


Figure 3.4 relationship between SW and n

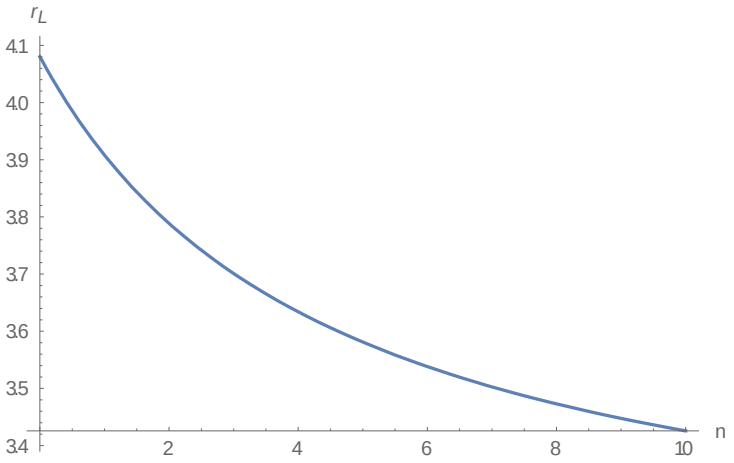


Figure 3.5 relationship between r_L and n

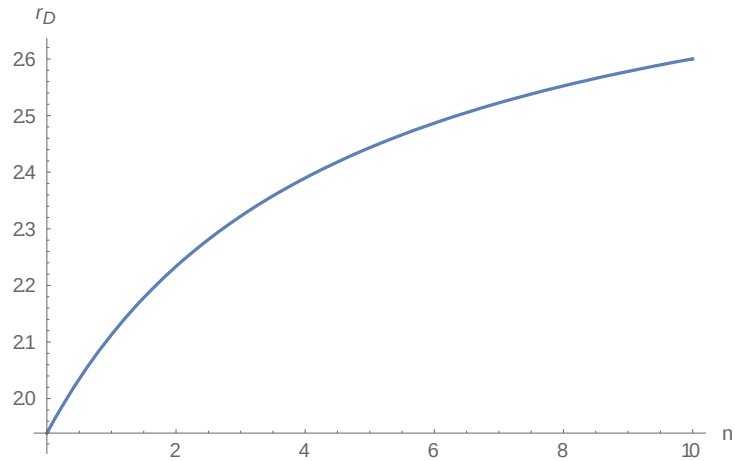


Figure 3.6 relationship between r_D and n

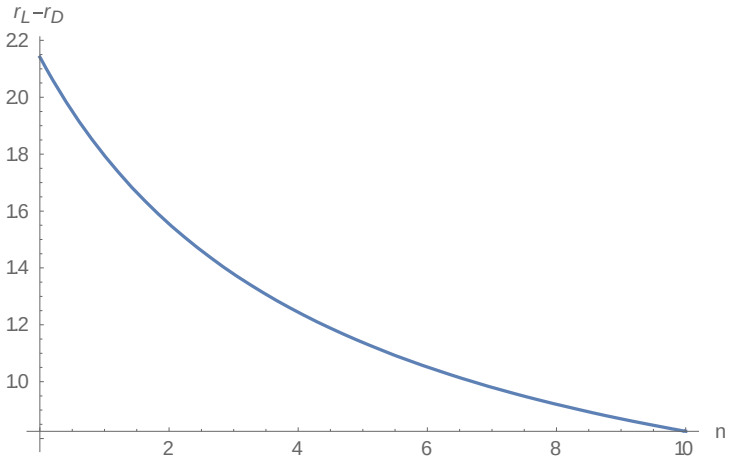


Figure 3.7 relationship between $r_L - r_D$ and n