## DISCUSSION PAPER SERIES E



## Discussion Paper No. E-4

The Profit-Maximizing and Labor-Managed Firms :
A Unified Approach to the Role of Information

Yasuhiro Sakai

September 2020

# The Institute for Economic and Business Research <br> Faculty of Economics <br> SHIGA UNIVERSITY 

1-1-1 BANBA, HIKONE, SHIGA 522-8522, JAPAN

# The Profit-Maximizing and Labor-Managed Firms : 

## A Unified Approach to the Role of Information

Yasuhiro Sakai<br>Professor Emeritus, Shiga University


#### Abstract

This paper analyzes the welfare implications of acquiring information for profit-maximizing and labor-managed firms (in short, PMF and LMF). We invent a unified method of exploring the role of information in a two-person game under uncertainty on the basis of comparative static analysis, and then apply the method to both the PMF and LMF. It is shown that whereas the LMF 's behavior is analogous to that of the PMF in some circumstances, the former may be entirely different from the latter in others. Because a special status is accorded to labor as a variable factor of production, the informational analysis of the LM economy requires special care for both computation and interpretation. Looking carefully at reality, there are a variety of capitalist firms, presumably forming a sort of spectrum with PMF at one end and the LMF at the other end. We would strongly believe that LMF also matters and should be worthy of due investigation.


This paper is a newly revised version of Sakai (1995). The revision work has been done on the basis of recent development of related areas of research.

## 1 A Variety of Market Economies: An Introduction

This chapter aims to compare the working and performance of profit-maximizing and labor-managed firms from an informational point of view. In what follows, those two different types of firms are respectively shortened to PMF and LMF.

While there are many intriguing problems in modern oligopoly theory , it is unquestionably correct to say that the issue of the impact of the information acquisition and transmission on the activities and welfare of firms operating under uncertainty constitutes a most, if not the most, important problem we have to investigate today. In reality, there exist a variety of institutions through which firms in an industry are able to obtain information about demand, cost or whatever. No doubt, government agencies and trade associations are among those information-gathering organizations. The question of much interest is whether and to what extent the establishment contributes to the welfare of producers, consumers, and the whole society.

Looking at the real world in a historical perspective, we understand that there have been a great variety of market economies. Presumably, at the one end of the spectrum of possible forms, there surely exists the typical American-type economy, which as can often been seen in many standard micro-economics tests, may be well-described by the traditional, neoclassical profit-maximizing firm. Interestingly enough, however, nearly at the other end, there exists the once-admired Japanese-type economy, which seems to be different from the American economy in many respects. First of all, the Japanese firm can often be regarded as a large family, with the company president playing the role of a head of the family. Second, a small group of company managers are no longer thought of mere agents of stockholders, but rather they are more likely the representatives of the whole employees. Third, in Japan, decision making and information flow are not exclusively monopolized by a selected number of top executives but are largely shared by all employees, including even ordinary salaried workers. Fourth, and most importantly, a considerable portion of total profit goes into the pockets of all workers through extra payment of summer and winter bonuses, provision of sport and leisure facilities and the like. In short, as convincingly pointed out by Komiya (1988a, 1988b) and others, the typical Japanese firm seems to have many characteristic of the LMF as opposed to the neoclassical PMF. Since Japan has become a world economic power, it should be worthwhile to carefully compare the PM and LM economies from an informational point of view. 1)

The issue of information transmission and exchange among firms was initiated by Basar and Ho (1974) and Ponssard (1979) as applications of stochastic nonzero-sum games to an oligopoly market with PMFs. While the literature on that issue has been extensive since then, it is quite unfortunate that little attention has been paid to the role of information in oligopoly with LMFs. In fact, although the working and performance of the LM economy under uncertaintyhas been discussed by Fukuda (1980), Hey (1981), Hey and Suckling (1980), Muzondo (1979) and others, it appears that the role of information has not drawn due attention. The purpose of this paper is to fill in such a gap by exploring the effects of information acquisition on LMFs. ${ }^{2)}$

It will well-known that comparative static results for the LMF are usually different from. and sometimes even opposite to, those for the PMFs. For instance, as contrasted with the standard PMF situation, the output of each LMF responds negatively to a rise in the product price and positively to an increase in the final cost. Those and other "perverse results" were first noticed by Ward (1958) and have subsequently provided the focus for much of the literature on the LMF. The question we would like to ask in this paper is whether and to what extent the role of information in the emerging LM economy really differs from that in the traditional PM economy. ${ }^{3)}$

We find quite useful to newly invent a unified method of systematically exploring the role of information in a two-person game under uncertainty. We develop the general framework on the basis of comparative static analysis, and proceed to apply it two specific models, namely the PM and LM duopolies. When there is no information available, the optimal strategy of each player must be a "routine action" because it cannot know any specific value of a stochastic (demand or cost or whatever) parameter. In plain English, an ignorant walking man with no guide maps has no option but simply walk forward on a narrow road in front of his very eyes. If a certain amount of precious information becomes available, however, the optimal strategy of the player becomes no longer routine, but rather a more flexible "contingent action" in the sense that it should be dependent on each realized value. In other words, a cautious and well-equipped climber chooses his best combination of climbing routes taking account of weather and road conditions. For instance, if the mountain whether happens to change, his climbing route might change accordingly. Thus, the important question of whether additional information is beneficial or harmful to a player can simply be reduced to the straightforward one of whether by taking a contingent action rather than a routine action the player is better-off or worse-off. As can easily be expected, in most cases knowledge is indeed valuable. The real world where many imperfect men like us live, the answer should not be simple like that. Indeed, there exist some other circumstances in which less knowledge may be better than more knowledge : possibly,
ignorance produces courage, thus becoming unexpected bliss.
We are really interested in comparing the welfare results of information acquisition and transmission on the LM duopoly with those of its PM twin. By making a sequence of such comparisons, we will succeed in obtaining the following set of comparative static results:
(1) In general, the effect of acquiring demand information on the expected output of both the LMF and the PMF cannot be determined unless a set of restrictions are placed on the form of the demand and (inverse) production function. If the demand function is specifically linear and each (inverse) production is quadratic, however, the expected output of each LMF surely increases by gathering demand information. This specific result may be contrasted with the corresponding PMF case in which the expected output remains unaffected by such information transfer.
(2) Generally speaking, the acquisition of demand information may positively or negatively contribute to both the expected profit per worker of each LMF and the expected profit of each PMF, depending again on the form of demand and production functions. In the specific yet important case of linear demand and quadratic (inverse) production, whereas acquisition of information definitely makes the PMF better off, it may make the LPF better off or worse off, depending on the specific form of the production function of a stochastic (demand or production) parameter. Remarkably, this demonstrates the possibility that in some circumstances, ignorance is bliss to the LMF.

Let us turn to the case when uncertainty is about fixed cost. Then the effects of the acquisition of (fixed) cost information on both the expected output and the welfare of each LMF are indeterminate in sign. These results are definitely positive on the specific case mentioned above, Such welfare results are in sharp contrast to the traditional PMF situation in which both the expected output and the welfare of each PMF remain unscathed by such information transmission.

To summarize, it is a more intricate task to explore the welfare impact of information transmission on the LMF than on the traditional PMF. In the simple yet important case of linear demand and quadratic production, the welfare results for the LFM are clearly different from those for the PMF. This indicates the intriguing properties characteristic of the LM economy.

The paper is organized as follows. In the next section we introduce a unified method of analyzing the role on information in a two-person game under uncertainty. This method is applied to the PM economy on the third section and to the LM economy in the fourth section. A detailed analysis of the simple yet important case where the demand function is linear and
each production function is quadratic is carried out in the fifth section. And conclusions are made in the final section.

## 2 Comparative Statics and the Role of Information: The General Framework and its Applications

This section will introduce a unified method of investigating the role of information in a two person game under uncertainty. The general framework will be developed on the basis of c comparative static analysis, and will be applied in the following sections to two specific models, namely PM and LM duopolies.

### 2.1 The General Framework

Our basic framework is the following two-person game under uncertainty. There are two players in our model - players 1 and 2. Let $Z_{i}\left(y_{i}, y_{j}, \alpha\right)$ the objective function of player $i \quad(i=1,2 ; i \neq j)$, where $y_{i}$ represents the strategic variable of player $i$ and $\alpha$ is a common stochastic parameter, As is usual, it is assumed that $Z_{i}$ is a an increasing and concave function of $y_{i}$.

It should be noted that the parameter $\alpha$ is subject to a certain probability density function $\phi(\alpha)$. Each player may or may not know the realized value of $\alpha$ before making his decision. Concerning the information structure of our model, we are content to limit our attention to the following two opposite cases:
(1) The case of no information whatever, denoted by $\eta^{0}$, in which each player is completely ignorant of $\alpha$.
(2) The case of complete information, written as $\eta^{c}$, in which each player can get information about $\alpha$ presumably through a third party such as a government agency or an independent research association.

In this paper, we are also content to make an additional set of assumptions. First of all, information is neither costly nor noisy. Next, we take account of no possibility of telling a lie or cheating. Moreover, other problems relating to non-symmetric information and risk aversion are not considered here. Although we understand importance of those problems, we dare to omit them here as a first approximation.

Under no information, $\eta^{0}$, each player aims to maximize the expected value of his objective function, where the expectation is taken over $\alpha$. The player makes a Cournot-Nash type of conjecture about the rival, that is, he chooses his best strategy on the assumption that his rival's strategy is fixed, which gives rise to a Cournot-Nash equilibrium.

More specifically, the pair $\left(\mathrm{y}_{1}{ }^{0}, \mathrm{y}_{2}{ }^{0}\right)$ of strategies is said to be an equilibrium pair under $\eta^{0}$ if the following set relations holds:

$$
\begin{align*}
& y_{\mathrm{i}}^{0}=\arg \max E\left[Z_{\mathrm{i}}\left(y_{\mathrm{i}}, y_{\mathrm{j}}{ }^{0}, \alpha\right)\right] \quad \text { for all } y_{\mathrm{i}}  \tag{1}\\
&(i=1,2 ; i \neq j)
\end{align*}
$$

At equilibrium, the optimal strategy of each player stands for a "routine action" since it depends on any specific value of $\alpha$. In plain English, a man sticks to the same action for all possible changes of $\alpha$. In what follows, we will introduce the following linearity assumption:

## Assumption (L)

The objective function $Z_{i}$ of each player is a linear function of $\alpha \quad(\mathrm{i}=1,2)$.

At the first glance, this assumption may look rather restrictive. However, it is quite convenient for our analytical purpose. Besides, as will be seen later, it is satisfied in most of standard duopoly models under uncertainty. ${ }^{4)}$

If we make Assumption $(\mathrm{L})$, then $E\left[Z_{\mathrm{i}}\left(y_{\mathrm{i}}, y_{\mathrm{j}}, \alpha\right)\right]$ is mathematically equivalent to $Z_{\mathrm{i}}\left(y_{\mathrm{i}}, y_{\mathrm{j}}, E \alpha\right)$, whence Eq. (1) can be reformulated as the following:

$$
\begin{align*}
&\left.y_{\mathrm{i}}^{0}=\arg \max \quad Z_{\mathrm{i}}\left(y_{\mathrm{i}}, y_{\mathrm{j}}^{0}, E \alpha\right)\right] \quad \text { for all } y_{\mathrm{i}}  \tag{2}\\
&(i=1,2 ; i \neq j)
\end{align*}
$$

Let us assume that the function $Z_{\mathrm{i}}$ is continuously differentiable to a desired degree. Then, a set of sufficient conditions for maximization are given by the following relations:
$\partial Z_{\mathrm{i}} / \partial y_{\mathrm{i}}=0$.
$(i=1,2)$
$\partial{ }^{2} Z_{\mathrm{i}} / \partial y_{\mathrm{i}}{ }^{2}<0 . \quad(i=1,2)$

In the light of (3), it is not a difficult job to derive the following set of reaction functions:

$$
\begin{equation*}
y_{\mathrm{i}}=R_{\mathrm{i}}^{0}\left(y_{\mathrm{j}}, E \alpha\right) \quad(i, j=1,2 ; i \neq j) \tag{5}
\end{equation*}
$$

Clearly, an equilibrium pair $\left(y_{1}{ }^{0}, y_{2}{ }^{0}\right)$, if it exists, is a pair of strategies which satisfy Eq. (5).

Along with some other regularity conditions to be imposed on the objective function, those conditions aforementioned will ensure the existence of a Cournot-Nash equilibrium
under $\eta^{0}$. However, they are not sufficient to ensure the stability of that equilibrium. Following Samuelson (1946), for the sake of comparative static analysis, we have to require stability. This will be guaranteed if the absolute value of the slope of each reaction curve is less than unity, so that the following set of equations must hold:

$$
\begin{equation*}
\left|\mathrm{d} R_{\mathrm{i}} 0 / \mathrm{d} y_{j}\right|<1 . \quad(i, j=1,2 ; i \neq j) \tag{6}
\end{equation*}
$$

where the derivative is evaluated at equilibrium. ${ }^{5)}$
Let us turn to the case of complete information, $\eta^{\mathrm{c}}$. In this case, each player can acquire information about $\alpha$, and for any given $\alpha$ he chooses a best strategy against some optimal strategy of his rival. Therefore, at equilibrium each optimal strategy is regarded as a "contingent action," meaning that it is dependent on each realized value of $\alpha$. Mathematically speaking, the pair $\left(y_{1}{ }^{\mathrm{c}}(\alpha), y_{2^{\mathrm{c}}}(\alpha)\right)$ of strategies is called an equilibrium pair under $\eta^{c}$ if for each given $\alpha$, the following conditions are met:

$$
\begin{align*}
y_{\mathrm{i}}^{\mathrm{c}}(\alpha)=\arg \max Z_{\mathrm{i}}\left(y_{\mathrm{i}}, y_{\mathrm{j}}^{\mathrm{c}}(\alpha), \alpha\right) \quad \text { for all } y_{\mathrm{i}} .  \tag{7}\\
(i, j=1,2 ; i \neq j)
\end{align*}
$$

As is usual, sufficient conditions for maximization are provided by the following equations:

$$
\begin{array}{ll}
\partial Z_{i} / \partial y_{i}=0 . & (i=1,2) \\
\partial^{2} Z_{i} / \partial y_{i}^{2}<0 . & (i=1,2) \tag{9}
\end{array}
$$

By virtue of (8), we obtain a pair of reaction functions as follows:

$$
\begin{equation*}
y_{\mathrm{i}}=R_{\mathrm{i}} \mathrm{c}\left(y_{j}, \alpha\right) \quad(i, j=1,2 ; i \neq j) \tag{10}
\end{equation*}
$$

It is noted here that on appearance, the reaction function $R_{\mathrm{i}}{ }^{\mathrm{c}}$ under $\eta^{\mathrm{c}}$ has the same functional form as the reaction function $R_{\mathrm{i}}{ }^{0}$ under $\eta^{0}$, the only difference being that $\alpha$ is now present instead of $E \alpha$. We would like to stress that such difference is more than mere appearance: it should be very substantial indeed.

Under some regularity conditions on the objective function, there should exist an equilibrium pair $\left(y_{1}{ }^{c}(\alpha), y_{2}{ }^{c}(\alpha)\right)$ such that $y_{\mathrm{i}}=R_{\mathrm{i}}{ }^{c}\left(y_{\mathrm{j}}(\alpha), \alpha\right)$. In addition, stability will be ensured under $\eta^{c}$ whenever, for each given $\alpha$, the following relations hold:

$$
\begin{equation*}
\left|\mathrm{d} R_{\mathrm{i}} \mathrm{c} / \mathrm{d} y_{\mathrm{j}}\right|<1, \quad(i, j=1,2 ; i \neq j) \tag{11}
\end{equation*}
$$

where the derivative is evaluated at equilibrium.
We are in a position to explore the role of information in our two-person game under uncertainty. We can do such a task by making a sequence of comparisons between equilibrium values under $\eta^{0}$ and those under $\eta^{c}$. We are especially interested in comparing $y_{\mathrm{i}}{ }^{0}$ with $E y_{\mathrm{i}}{ }^{c}(\alpha)$, and $Z^{0}$ with $E\left[Z_{\mathrm{i}}(\alpha)\right]$.

Under $\eta^{\mathrm{c}}$, both $y_{\mathrm{i}}$ and $Z$ are functions of $\alpha(\mathrm{i}=1,2)$. These functions may be or may not be convex (or concave) in $\alpha$. Suppose that $y_{j}(\alpha)$ is convex (or concave) in $\alpha$. Then, it follows from Jensen's inequality formula that for any possible probability density function $\phi(\alpha)$ of $\alpha, E\left[y_{i}(\alpha)\right]$ is greater than (or less than) $y_{i}(E \alpha)$, where the expectation is taken for $\phi(\alpha)$. Under Assumption (L) above, we find $y_{\mathrm{i}}{ }^{\mathrm{c}}(E \alpha)=$ $y^{\mathrm{i}}{ }^{0}$ and $Z_{\mathrm{i}}{ }^{\mathrm{c}}(E a)=Z_{\mathrm{i}}{ }^{0}$. We can thus establish the following result:

## Proposition 1

Under Assumption (L), information acquisition increases (or decreases) $E\left[y_{\mathrm{i}}\right]$ if $y_{\mathrm{i}}$ is a convex (or concave) function of $\alpha$. A similar result also holds for $E\left[Z_{\mathrm{i}}\right.$ ] ( $\mathrm{i}=1,2$ ) .

In order to investigate the impact of information acquisition on $E\left[Z_{\mathrm{i}}\right]$, it is necessary to examine the convexity (or concavity) of $Z_{\mathrm{i}}$ with respect to $\alpha$. If we differentiate $Z_{\mathrm{i}}=Z_{\mathrm{i}}\left(y_{\mathrm{i}}(\alpha), y_{\mathrm{j}}(\alpha), \alpha\right)$ with respect to $\alpha$, we have the following equation:

$$
\begin{align*}
\mathrm{d} Z_{\mathrm{i}} / \mathrm{d} \alpha= & \left(\partial Z_{\mathrm{i}} / \partial y_{\mathrm{i}}\right)\left(\mathrm{d} y_{\mathrm{i}} / \mathrm{d} \alpha\right)+\left(\partial Z_{\mathrm{i}} / \partial y_{\mathrm{j}}\right)\left(\mathrm{d} y_{\mathrm{j}} / \mathrm{d} \alpha\right) \\
& +\left(\partial Z_{\mathrm{i}} / \partial \alpha\right) . \quad(i, j=1,2 ; i \neq j) \tag{12}
\end{align*}
$$

The first, second and third terms on the right-hand side of (12) respectively denote the "indirect own effect", the "indirect cross effect" and the direct effect" of a rise in $\alpha$ on $Z_{\mathrm{i}}$. Since ( $\partial Z_{\mathrm{i}} / \partial y_{\mathrm{i}}$ ) must vanish by the fist-order condition, (12) can be simplified to the following:

$$
\begin{equation*}
\mathrm{d} Z_{\mathrm{i}} / \mathrm{d} \alpha=\left(\partial Z_{\mathrm{i}} / \partial y_{\mathrm{j}}\right)\left(\mathrm{d} y_{\mathrm{j}} / \mathrm{d} \alpha\right)+\left(\partial Z_{\mathrm{i}} / \partial \alpha\right) . \quad(i \neq j) \tag{13}
\end{equation*}
$$

If we further differentiate both sides of (7.13) with respect to $\alpha$, we can determine the sign of the second-order derivative $\mathrm{d}^{2} Z_{\mathrm{i}} / \mathrm{d} \alpha^{2}$. In fact, we have the following equation:

$$
\begin{aligned}
\mathrm{d}^{2} Z_{\mathrm{i}} / \mathrm{d} \alpha^{2}= & 【\left(\partial^{2} Z_{\mathrm{i}} / \partial y_{\mathrm{i}} \partial y_{\mathrm{j}}\right)\left(\mathrm{d} y_{\mathrm{i}} / \mathrm{d} \alpha\right)+\left(\partial^{2} Z_{\mathrm{i}} / \partial y_{\mathrm{j}}^{2}\right)\left(\mathrm{d} y_{\mathrm{j}} / \mathrm{d} \alpha\right) 】\left(\mathrm{~d} y_{\mathrm{j}} / \mathrm{d} \alpha\right) \\
& +\left(\partial Z_{\mathrm{i}} / \partial y_{\mathrm{j}}\right)\left(\mathrm{d}^{2} y_{\mathrm{j}} / \mathrm{d} \alpha^{2}\right)+\left(\partial^{2} Z_{\mathrm{i}} / \partial \alpha^{2}\right) . \quad(i \neq j)
\end{aligned}
$$

If the game in question is symmetric，then the resulting Cournot－Nash equilibrium is also symmetric，so that the equilibrium value of $y_{i}$ and $y_{j}$ should be just equal． Consequently，（7．13）can be rewritten as follows：

$$
\begin{align*}
\mathrm{d}^{2} Z_{\mathrm{i}} / \mathrm{d} \alpha^{2}= & 【\left(\partial^{2} Z_{\mathrm{i}} / \partial y_{\mathrm{i}} \partial y_{\mathrm{j}}\right)+\left(\partial^{2} Z_{\mathrm{i}} / \partial y_{\mathrm{j}}^{2}\right) 】\left(\mathrm{~d} y_{\mathrm{i}} / \mathrm{d} \alpha\right)^{2} \\
& +\left(\partial Z_{\mathrm{i}} / \partial y_{\mathrm{j}}\right)\left(\mathrm{d}^{2} y_{\mathrm{j}} / \mathrm{d} \alpha^{2}\right)+\left(\partial^{2} Z_{\mathrm{i}} / \partial \alpha^{2}\right) . \quad(i \neq j) \tag{14}
\end{align*}
$$

It is noted that the right－side of（14）consists of three terms．The first term is ambiguous in sign，depending on the value and sign of the two second－order derivatives $\left(\partial^{2} Z_{\mathrm{i}} / \partial y_{\mathrm{i}} \partial y_{\mathrm{j}}\right)$ and $\left(\partial^{2} Z_{\mathrm{i}} / \partial y_{\mathrm{j}}{ }^{2}\right)$ ．While the second term is positive（or negative） if
$y_{i}$ is convex（or concave）in $\alpha$ ，the sign of the third term is indeterminate．Needless to say，the total impact of information acquisition is a combination of these three components each of which might go either directions．Therefore，we should be extremely careful before reaching any definite conclusion．As the saying goes，Rome was not built in a day．Likewise，a unified approach to oligopoly and information cannot be established so easily，requiring a detailed case－by－case analysis．

Let us safely get out of such a＂blind ally of ambiguity．＂We believe that a diagrammatic explanation would be very instructive in understanding the meaning of Proposition 1．For simplicity，as is indicated in Fig．1，let us assume that the stochastic parameter $\alpha$ takes on one of two equally likely values－high $(H)$ or Low $(L)$ ．It is noted that $E \alpha=(H+L) / 2$ ．

In this simple two－valued distribution case，the relevant reaction curves may be positively or negatively sloping，depending on the functional form of $Z$ ．For $a=\mathrm{H}$ ， ＂player 1＇s reaction curve for player 2＇s choice y2＂is shown as $R_{1} \mathrm{H}$ ，whereas for $\alpha=\mathrm{L}$ ，it is drawn as $R_{1} \mathrm{~L}$ ．Note that $R_{1} \mathrm{~L}$ lies west of $R_{1}{ }^{\mathrm{H}}$ because $L$ is numerically less than $H$ ．A dotted curve $\quad R_{1} 0$ denotes the average of these two reaction curves for player 1．In a similar fashion，we are able to draw the reaction curves $R_{2}{ }^{\mathrm{H}}$ and $R_{2} \mathrm{~L}$ together with their average $R_{2} \mathrm{O}$ for player 2.


Fig. 1 The Simple Uniform Distribution : $\phi(\alpha)$
Remark. $\quad \phi(H)=\phi(L)=1 / 2 ; \phi(\alpha)=0$ otherwise


Fig. 2 Equilibrium under Uncertainty: $\eta^{0}$ versus $\eta^{\text {c }}$
Remark. $Q^{\mathrm{O}}=\left(y_{1}{ }^{0}, y_{2}{ }^{0}\right) ; Q^{\mathrm{H}}=\left(y_{1}(H), y_{2}(H)\right), Q^{\mathrm{L}}=\left(y_{1}(L), y_{2}(L)\right)$

In Fig. 2, we may easily find Cournot-Nash equilibriums under $\eta^{0}$ and those under $\eta^{c}$. On the one hand, when both players are ignorant of $\alpha$, the point represents a stable equilibrium under $\eta^{0}$, with $\left(y_{1}{ }^{0}, y_{2}{ }^{0}\right)$ being a pair of equilibrium strategies. On the other hand, in ca both players can know $\alpha$, a pair of the two points, $Q^{\mathrm{H}}$ and $Q^{\mathrm{L}}$, show stable equilibriums. In this latter case, the vector of two pairs, $\left(\left(y_{1}(H), y_{2}(H)\right.\right.$ $) ;\left(y_{1}(L), y_{2}(L)\right)$ stands for a pair of equilibrium strategies of players 1 and 2 under $\eta^{c}$.

We are in a position to see by means of a diagram how information acquisition by both players affects $E y_{\mathrm{i}}$, the average of $y_{\mathrm{i}}$. Suppose that $y_{\mathrm{i}}(\alpha)$ is a convex function of $\alpha$ and that the three points, $\left(H, y_{i}(H)\right),\left(L, y_{i}(L)\right)$ and $\left(E \alpha, y_{i}{ }^{0}\right)$, are located as in Fig. 3, where $E \alpha=(H+L) / 2$. Ten, we can easily find the following relations:

$$
\left(y_{\mathrm{i}}(H)+y_{\mathrm{i}}(L)\right) / 2=E y_{\mathrm{i}}(\alpha)>y_{\mathrm{i}}(E \alpha)=y_{\mathrm{i}}^{0} .
$$

Consequently, information acquisition, on average, makes both players more active and lively.


Fig. 3 The convexity of $y_{i}(\alpha)$

### 2.2 Applications to Profit-Maximizing Duopoly

We are ready to apply the general framework developed in the previous section to two important classes of two-person games under uncertainty, namely, profit-maximizing and labor managed duopolies facing demand or cost risk. This section will deal with the impact of information acquisition on the PM economy.

Let us consider an industry with two firms producing a homogeneous product. Firm $i$ produces output $x_{\mathrm{i}}$ with the help of labor $l_{\mathrm{i}}$ and other unspecified fixed factors. The relation between $x_{\mathrm{i}}$ and $l_{\mathrm{i}}$ is described by the inverse production function $l_{\mathrm{i}}=$
$g\left(x_{\mathrm{i}}\right)$, in which $g^{\prime}\left(x_{\mathrm{i}}\right)>0$ and $g^{\prime \prime}\left(x_{\mathrm{i}}\right)>0 \quad(\mathrm{i}=1,2)$.
Each firm faces a price $p_{\mathrm{i}}$ for its product, being given by the inverse demand function $p=b-h(X)$, where $h^{\prime}(X)>0$ and $X=x_{1}+x_{2}$. Let ${ }_{W}$ be the competitively given wage rate and $k$ the fixed cost. Then, firm $i$ 's profit is defined by the following: 7)

$$
\begin{equation*}
\Pi_{\mathrm{i}}=(b-h(X))_{x_{\mathrm{i}}}-W g\left(x_{\mathrm{i}}\right)-k . \quad(i=1,2) \tag{15}
\end{equation*}
$$

The profit function $\Pi_{i}$ is a linear function of $b$ or $k$, thus satisfying Assumption (L) stated above. Under some circumstances, the parameter $b$ may be a stochastic parameter showing demand uncertainty. Under others, the parameter $k$ may be a stochastic parameter representing fixed cost uncertainty. For instance, if $x_{i}$ stands for the amount of beer production by the $i$ th brewery, then $b$ may represent the fluctuations of GDP or the state of the weather whereas $k$ may be related to the variations of various rents or the breakdown of machinery.

Sufficient conditions for profit maximization are given as follows:

$$
\begin{array}{ll}
\partial \Pi_{\mathrm{i}} / \partial x_{\mathrm{i}} \equiv b-h-h^{\prime} x_{\mathrm{i}}-w g^{\prime}=0, & (i=1,2) \\
\partial^{2} \Pi_{\mathrm{i}} / \partial x_{\mathrm{i}}{ }^{2} \equiv-\left(2 h^{\prime}+h^{\prime \prime} x_{\mathrm{i}}+w g^{\prime \prime}\right)<0 . & (i=1,2) \tag{17}
\end{array}
$$

In the light of (16), we find that, at equilibrium, $x_{i}$ is a function of $x_{j}$, which is nothing but firm $i^{\prime} s$ reaction function for firm $j$, being simply denoted by $R_{\mathrm{i}}\left(x_{\mathrm{j}}\right)$. Indeed, it is not a difficult job to derive the first-order derivatives of $R_{\mathrm{i}}\left(x_{\mathrm{j}}\right)$ as follows:
$\mathrm{d} R_{\mathrm{i}} / \mathrm{d} x_{\mathrm{j}}=-\left[h+h " x_{\mathrm{i}}\right] /\left[2 h+h " x_{\mathrm{i}}+W g^{\prime \prime}\right]$

$$
(i, j=1,2 ; i \neq j)
$$

Now, recall the stability condition (6) implies the following: equation,

$$
\left|\mathrm{d} R_{\mathrm{i}} / \mathrm{d} x_{\mathrm{j}}\right|<1 . \quad(i, j=1,2 ; i \neq j)
$$

This together with the second-order condition (7.17) above implies that [2h+h" $x_{\text {i }}$ $\left.+W g{ }^{\prime \prime}\right]>\left|h+h{ }^{\prime} x_{i}\right|$, which in turn implies the following:

$$
\begin{equation*}
3 h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}+w g^{\prime \prime}>0 \tag{19}
\end{equation*}
$$

Let us carry out comparative static analysis and explore the impact of information acquisition by PMF on equilibrium values. If we differentiate (16) with respect to $b$, then we obtain the following first derivative:

$$
\begin{equation*}
\mathrm{d} x_{\mathrm{i}} / \mathrm{d} b=1 /\left(3 h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}+w g^{\prime \prime}\right), \tag{20}
\end{equation*}
$$

which is positive by virtue of (7.19). Further differentiation of (20) with respect to $b$ results in the following second derivative:

$$
\begin{equation*}
\mathrm{d}^{2} x_{\mathrm{i}} / \mathrm{d} b^{2}=-\left(8 h^{\prime}+4 h^{\prime \prime} x_{\mathrm{i}}+w g^{\prime \prime \prime}\right) /\left(3 h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}+w g^{\prime \prime}\right)^{3 .} \tag{21}
\end{equation*}
$$

Mathematically speaking, the second derivative $\mathrm{d}^{2} \boldsymbol{X}_{\mathrm{i} /} \mathrm{d} b^{2}$ may go in either direction, depending on the sign of the quantity $\left(8 h^{\prime}+4 h^{\prime \prime} x_{i}+w g^{\prime \prime \prime}\right)$. Therefore, we can establish the following proposition:

```
Proposition 2
```



In the light of Proposition 1 above, this proposition tells us that information acquisition about $b$ may increase or decrease the expected output of each PMF, depending on the sign and value of $h$ and $g$.

Now, let us focus on the special yet interesting case in which the demand function is
linear, and the production function is linear or quadratic, implying that $h "=h "=0$ and $g^{\prime \prime \prime}=0$. Then, since the quantity $\left(8 h^{\prime \prime}+4 h^{\prime \prime \prime} x_{\mathrm{i}}+w g^{\prime \prime \prime}\right)$ vanishes in this special case, it follows from Proposition 2 that $\mathrm{d}^{2} X_{\mathrm{i}} / \mathrm{d} b^{2}=0$, so that the expected output of each PMF remains unscathed by information acquisition about $b$. Interesting enough, such a special and simple case has long been a focal point of investigation in the literature on oligopoly with uncertainty (see Ponssard (1979), Sakai (1990, 1991), for instance). Besides, a much more detailed analysis of this simple case will be given in the next section.

Now, let us consider the impact of changes in $b$ on profits. By making use of (15), (20) and (21) above, we have the following first and second derivatives:

$$
\begin{align*}
\mathrm{d} \Pi_{\mathrm{i}} / \mathrm{d} b= & x_{\mathrm{i}}\left(1-h^{\prime}\left(\mathrm{d} x_{\mathrm{i}} / \mathrm{d} b\right)\right) \\
= & x_{\mathrm{i}}\left(2 h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}+w^{\prime}\right) /\left(3 h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}+w g^{\prime \prime}\right)  \tag{22}\\
\mathrm{d}^{2} \Pi_{\mathrm{i}} / \mathrm{d} b^{2}= & \left(\mathrm{d} x_{\mathrm{i}} / \mathrm{d} b\right)-\left(h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}\right)\left(\mathrm{d} x_{\mathrm{i}} / \mathrm{d} b\right)^{2}-h^{\prime} x_{\mathrm{i}}\left(\mathrm{~d}^{2} x_{\mathrm{i}} / \mathrm{d} b^{2}\right) \\
= & \left(2 h^{\prime}+w g^{\prime \prime}\right) /\left(3 h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}+w g^{\prime \prime}\right)^{2} \\
& +h^{\prime} x_{\mathrm{i}}\left(8 h^{\prime \prime}+4 h^{\prime \prime \prime} x_{\mathrm{i}}+w g^{\prime \prime \prime}\right) /\left(3 h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}+w^{\prime \prime \prime}\right)^{3} \tag{23}
\end{align*}
$$

Consequently, in view of (23), we can establish the following proposition:

## Proposition 3

$$
\begin{aligned}
& \mathrm{d}^{2} \Pi_{\mathrm{i}} / \mathrm{d} b^{2} \text { そ } 0 \\
& \quad \Leftrightarrow\left(2 h^{\prime}+w g^{\prime \prime}\right)\left(2 h^{\prime}+w g^{\prime \prime}\right)+h^{\prime} x_{\mathrm{i}}\left(8 h^{\prime \prime}+4 h^{\prime \prime \prime} x_{\mathrm{i}}+w g^{\prime \prime \prime}\right) \text { そ } 0 . \quad(i=1,2)
\end{aligned}
$$

The sign of the second derivative $\mathrm{d}^{2} \Pi_{\mathrm{i}} / \mathrm{d} b^{2}$ cannot be determined unless some specific conditions are imposed on demand and production. In general, the acquisition of demand information may positively or negatively contribute to the expected profit of each PMF, depending to the sign and value of $h$ and $g$. In the special case of linear demand and quadratic production, (23) is reduced to the following:

$$
\mathrm{d}^{2} \Pi_{\mathrm{i}} / \mathrm{d} b^{2}=\left(2 h^{\prime}+w g^{\prime \prime}\right)\left(3 h^{\prime}+w g^{\prime \prime}\right)^{2}
$$

which is clearly positive. Therefore, in this simple case, the acquisition of demand
information makes each PMF better off, which agrees common sense. ${ }^{8)}$
Now, suppose that there is uncertainty about $k$, the fixed cost. Since there are no terms associated with $k$ present in (16), it follows that $\mathrm{d}^{2} \mathrm{x}$ i/ $\mathrm{d} k^{2}=0$. Moreover, by means of (15), we have $\mathrm{d}^{2} \Pi_{\mathrm{i}} / \mathrm{d} k^{2}=0$. We can summarize these observations as follows:

## Proposition 4 <br> $\mathrm{d}^{2} \mathrm{x}_{\mathrm{i}} / \mathrm{d} k^{2}=0 \quad$ and $\quad \mathrm{d}^{2} \Pi_{\mathrm{i}} / \mathrm{d} k^{2}=0 . \quad(i=1,2)$

This proposition says that, as can be expected, the acquisition of fixed cost information does not affect the expected output and expected profit of each PHF in any way.

### 2.3 Applications to Labor-Managed Duopoly

In this section, we will examine the role of information in the LM economy, the main theme of this chapter. It is well-known that the comparative static analysis of the LMF leads to some "perverse results." We employ the term "perverse" to indicate behavior opposite to that of the PMF. The question of interest is how and to what extent the LMF' s response to information acquisition is different from that of the PMF. 9)

Using the same notation as in the previous sections and following the tradition of Ward (1958), we assume that the LMF 's objection is to maximize its profit per worker rather than profit per se. Specifically, firm $i$ 's profit per worker is given as follows:

$$
\begin{equation*}
S_{\mathrm{i}} \equiv \Pi_{\mathrm{i}} / l_{\mathrm{i}}=\left[(b-h)_{X_{\mathrm{i}}-W} g-k\right] / g . \quad(i=1,2) \tag{24}
\end{equation*}
$$

We can write the sufficient first-order and second-order conditions as follows:

$$
\begin{align*}
\partial S_{\mathrm{i}} / \partial x_{\mathrm{i}} \equiv\left[b-h-h^{\prime} x_{\mathrm{i}}-\left(w+S_{\mathrm{i}}\right) g^{\prime}\right] / g=0 & ,(i=1,2)  \tag{25}\\
\partial^{2} S_{\mathrm{i}} / \partial x_{\mathrm{i}}{ }^{2} \equiv-\left[\left(2 h^{\prime}+h^{\prime \prime} x_{\mathrm{i}}+\left({ }_{\mathrm{i}}+S_{\mathrm{i}}\right) g^{\prime \prime}\right] / g<0\right. & (i=1,2)
\end{align*}
$$

Observation of (25) tells us that, at equilibrium, $x_{i}$ is a function of $x_{j}$, which is nothing but firm i's reaction function for firm $j$, being simply denoted by $R_{\mathrm{i}}\left(x_{\mathrm{j}}\right)$.

Indeed, it is a rather routine task to derive the first-order derivatives of each LMF as follows:
$\mathrm{d} R_{\mathrm{i}} / \mathrm{d} X_{\mathrm{j}}=$

$$
\begin{array}{r}
{\left[g^{\prime} h^{\prime} x_{\mathrm{i}}-g\left(h^{\prime}+h^{\prime \prime} x_{\mathrm{i}}\right)\right] / g \cdot\left[\left(2 h^{\prime}+h^{\prime \prime} x_{\mathrm{i}}+\left(w^{2}+S_{\mathrm{i}}\right) g^{\prime \prime}\right] .\right.} \\
(i, j=1,2 ; i \neq j)
\end{array}
$$

The stability condition (6) above in conjunction with the second-order condition (26) implies that $\left|g^{\prime} h^{\prime} x_{\mathrm{i}}-g\left(h^{\prime}+h^{\prime \prime} x_{\mathrm{i}}\right)\right|<\quad g \quad\left[\left(2 h^{\prime}+h^{\prime \prime} x_{\mathrm{i}}+\left(W^{2}+S_{\mathrm{i}}\right) g^{\prime \prime}\right]\right.$, which in turn leads to the following : 10)

$$
\begin{equation*}
D_{\mathrm{i}} \equiv g\left[3 h^{\prime}+2 h^{\prime \prime} x_{\mathrm{i}}+\left(W+S_{\mathrm{i}}\right) g^{\prime \prime}\right]-g^{\prime} h^{\prime} x_{\mathrm{i}}>0 \tag{28}
\end{equation*}
$$

The question to ask is how and to what extent the acquisition of information of demand or cost affects the expected output and expected profit per worker of each LMF. First, we wish to explore the LMF 's response to information acquisition about $b$, the demand intercept. By differentiating (25) with respect to $b$, we obtain the first-order and second order derivatives:

$$
\begin{align*}
& \mathrm{d} x_{\mathrm{i}} / \mathrm{d} b=-\left(g^{\prime} x_{\mathrm{i}}-g\right) / D_{\mathrm{i}},  \tag{29}\\
& \mathrm{~d}^{2} \mathrm{x}_{\mathrm{i}} / \mathrm{d} b^{2}=\left(g^{\prime} x_{\mathrm{i}}-g\right) 【 2 g^{\prime \prime x_{\mathrm{i}}} D_{\mathrm{i}}-\left(g^{\prime} x_{\mathrm{i}}-g\right) E_{\mathrm{i}} 】 / D_{\mathrm{i}}^{3}, \tag{30}
\end{align*}
$$

where the quantity $\quad E_{\mathrm{i}}$ is defined as follows:

$$
\begin{aligned}
E_{\mathrm{i}}= & 2 h\left(g^{\prime}-g^{\prime \prime} x_{\mathrm{i}}\right)+\left(g g^{\prime \prime \prime}+g^{\prime} g^{\prime \prime}\right)\left(w+S_{\mathrm{i}}\right) \\
& +4 g\left(2 h^{\prime}+h^{\prime \prime} x_{\mathrm{i}}\right) .
\end{aligned}
$$

Note that the quantity $\left(g^{\prime} x_{i}-g\right)$ is positive whenever $g$ is convex in $x_{i}$. Therefore, it is seen in (29) that an increase in demand really decreases the output level, confirming a famous "perverse behavior" of the LMF. In the light of (30), we immediately establish the following proposition:

## Proposition 5

$$
\mathrm{d}^{2} \mathrm{x}_{\mathrm{i}} / \mathrm{d} b^{2} \gtreqless 0 \quad \Leftrightarrow \quad 2 g^{\prime \prime} x_{\mathrm{i}} D_{\mathrm{i}}-\left(g^{\prime} x_{\mathrm{i}}-g\right) E_{\mathrm{i}} \gtreqless 0 . \quad(i=1,2) .
$$

If we compare this proposition with Proposition 2 above，then we readily see that it is generally a more demanding job to analyze the informational implications for the LM duopoly than those for the PM duopoly．Indeed，it is not easy to determine whether the quantity $2 g^{\prime \prime} x_{i} D_{i}$ is greater or less than the quantity $\left(g^{\prime} x_{i}-g\right) E E_{i}$ ．Therefore， the effect of obtaining demand information on the expected output of each LMF is ambiguous unless further restrictions are placed on the form of the demand and production functions．It is in the next section，we will carry out a detailed analysis for the simple yet important case where $h$ is linear and $g$ is quadratic．

The impact of acquiring demand information on the welfare of each LMF can be measured by the following first－order and second－order derivatives：

$$
\begin{align*}
\mathrm{d} S_{\mathrm{i}} / \mathrm{d} b= & \left(x_{\mathrm{i}} / g\right)\left[1-h^{\prime}\left(\mathrm{d} x_{\mathrm{i}} / \mathrm{d} b\right)\right] \\
= & x_{\mathrm{i}}\left[D_{\mathrm{i}}+h^{\prime}\left(g^{\prime} x_{\mathrm{i}}-g^{\prime}\right)\right] / g D_{\mathrm{i}} ;  \tag{31}\\
\mathrm{d}^{2} S_{\mathrm{i}} / \mathrm{d} b^{2}= & {\left[\left(g^{\prime} x_{\mathrm{i}}-g^{\prime}\right) / g^{2}\right]\left(-\mathrm{d} x_{\mathrm{i}} / \mathrm{d} b\right) \quad\left[1-h^{\prime}\left(\mathrm{d} x_{\mathrm{i}} / \mathrm{d} b\right)\right] } \\
& -\left(x_{\mathrm{i}} / g\right) 【 2 h^{\prime \prime}\left(\mathrm{d} x_{\mathrm{i}} / \mathrm{d} b\right)^{2}+h^{\prime}\left(\mathrm{d}^{2} \mathrm{x}_{\mathrm{i}} / \mathrm{d} b^{2}\right) 】  \tag{32}\\
& (i=1,2)
\end{align*}
$$

Inspection of（32）indicates that the question of determining the sign of the second derivative $\mathrm{d}^{2} S_{\mathrm{i}} / \mathrm{d} b^{2}$ is rather involved．While the first term of the right－hand side of （32）is positive since $\mathrm{d}_{x_{i}} / \mathrm{d} b$ is negative（see（29），the second－term is ambiguous in sign since the signs of $h$＂and $\mathrm{d}^{2} \mathrm{x}_{\mathrm{i}} / \mathrm{d} b^{2}$ are indeterminate．By inserting（29）and（39） into（32）and performing some calculations，we find the following：

$$
\begin{equation*}
\mathrm{d}^{2} S_{\mathrm{i}} / \mathrm{d} b^{2}=\left(g^{\prime} x_{\mathrm{i}}-g\right) F_{\mathrm{i}} / g^{2} D_{\mathrm{i}}{ }^{3}, \tag{33}
\end{equation*}
$$

where $\quad F_{\mathrm{i}}$ denotes the quantity defined as follows：

$$
\begin{align*}
F_{\mathrm{i}}= & D_{\mathrm{i}}\left(g^{\prime} x_{\mathrm{i}}-g\right) 【 D_{\mathrm{i}}+h^{\prime}\left(g^{\prime} x_{\mathrm{i}}-g\right)-2 g h^{\prime \prime} x_{\mathrm{i}} 】 \\
& -g^{\prime} h^{\prime} x_{\mathrm{i}} 【 2 g^{\prime \prime} x_{\mathrm{i}} D_{\mathrm{i}}-\left(g^{\prime} x_{\mathrm{i}}-g\right) E_{\mathrm{i}} 】 . \quad(i=1,2) \tag{33a}
\end{align*}
$$

We can thus establish the following proposition：

## Proposition 6

$\mathrm{d}^{2} S_{\mathrm{i}} / \mathrm{d} b^{2} \quad そ \quad 0 \quad \Leftrightarrow \quad F_{\mathrm{i}} \quad ३ 0 . \quad(i=1,2)$

This proposition shows the effect of acquiring demand information on each LMF"s expected profit per worker. As expected, it is generally ambiguous unless further restrictions are placed on the demand and production functions. With some conditions on $h$ and $g$, the second-order derivative $\mathrm{d}^{2} \boldsymbol{S}_{\mathrm{i}} / \mathrm{d} \boldsymbol{b}^{2}$ may be negative, implying that ignorance may be bliss. Since we believe that this is an intriguing result, we will conduct a more detailed analysis in the next section.

Let us turn to the case of fixed cost uncertainty. By differentiating the first-order condition (25) with respect to $k$, we now have the following results:

$$
\begin{array}{ll}
\mathrm{d} x_{\mathrm{i}} / \mathrm{d} k=g^{\prime} / D_{\mathrm{i}} ; & (i=1,2) \\
\mathrm{d}^{2} \mathrm{X}_{\mathrm{i}} / \mathrm{d} k^{2}=g^{\prime}\left(2 g^{\prime \prime} D_{\mathrm{i}}-g^{\prime} E_{\mathrm{i}}\right) / D_{\mathrm{i}^{3}}, \tag{35}
\end{array}
$$

By making use of (35), it is fairy easy to establish the following proposition:

## Proposition 7

$\mathrm{d}^{2} \mathrm{X}_{\mathrm{i}} / \mathrm{d} k^{2} \quad \gtreqless 0 \quad \Leftrightarrow \quad 2 g^{\prime \prime} D_{\mathrm{i}}-g^{\prime} E_{\mathrm{i}} \quad<\quad 0 . \quad(i=1,2)$

As it is seen in this proposition, the expected output of each LMF may respond positively or negatively to the acquisition of fixed cost information: indeed, it depends on the form of the demand and production functions. This result is in sharp contrast to the PMF situation in which $\mathrm{x}_{i}$ is linear in $k$, and hence the expected output of each PMF remains unaffected by the acquisition of fixed cost information (see Proposition 4). If we consider the simple case where the demand function is linear and the production function is quadratic, then we may show that the production activity of each LMF, on average, increases by the information acquisition. 11)

Finally, let us explore the welfare impact of fixed cost information on the LMF. In the same manner as above, we can measure the impact by the following first-order and second-order derivatives:

$$
\begin{align*}
\mathrm{d} S_{\mathrm{i}} / \mathrm{d} k & =-(1 / g)\left[1+h^{\prime} x_{\mathrm{i}}\left(\mathrm{~d} x_{\mathrm{i}} / \mathrm{d} k\right)\right] \\
& =-\left(h^{\prime} g^{\prime} x_{\mathrm{i}}+D_{\mathrm{i}}\right) / g D_{\mathrm{i}} \tag{36}
\end{align*}
$$

$$
\begin{align*}
\mathrm{d}^{2} S_{\mathrm{i}} / \mathrm{d} k^{2} & =\left(g^{\prime} / g^{2}\right)\left(\mathrm{d} x_{\mathrm{i}} / \mathrm{d} k\right)\left[1+h^{\prime} x_{\mathrm{i}}\left(\mathrm{~d} x_{\mathrm{i}} / \mathrm{d} k\right)\right] \\
& -(1 / g) 【\left(2 h^{\prime \prime} x_{\mathrm{i}}+h^{\prime}\right)\left(\mathrm{d} x_{\mathrm{i}} / \mathrm{d} k\right)^{2}+h^{\prime} x_{\mathrm{i}}\left(\mathrm{~d}^{2} x_{\mathrm{i}} / \mathrm{d} k^{2}\right) 】 \\
& =\left(g^{\prime} / g^{2}\right)\left(G_{\mathrm{i}} / D_{\mathrm{i}}^{3}\right), \quad(i=1,2) \tag{37}
\end{align*}
$$

where $G_{\mathrm{i}}$ is the quantity newly defined as follows：

$$
\begin{align*}
G_{\mathrm{i}}= & g^{\prime} D_{\mathrm{i}}\left(D_{\mathrm{i}}+h^{\prime} g^{\prime} x_{\mathrm{i}}\right) \\
& -g 【 D_{\mathrm{i}} g^{\prime}\left(2 h^{\prime \prime} x_{\mathrm{i}}+h^{\prime}\right)+h^{\prime} x_{\mathrm{i}}\left(2 g^{\prime} D_{\mathrm{i}}-g^{\prime} E_{\mathrm{i}}\right) 】 \tag{37a}
\end{align*}
$$

We can thereby establish the following proposition：

## Proposition 8

$$
\mathrm{d}^{2} \boldsymbol{S}_{\mathrm{i}} / \mathrm{d} k^{2} \quad \gtreqless 0 \quad \Leftrightarrow \quad G_{\mathrm{i}} \gtrless 0 \quad . \quad(i=1,2)
$$

In order to see how fixed cost information affects the welfare of each LMF，it is necessary to examine the convexity or concavity of $S_{\mathrm{i}}$ with respect to $k$（see Proposition 1 above）．According to Proposition 8，we cannot say whether the second－order derivative $\mathrm{d}^{2} \boldsymbol{S}_{\mathrm{i}} / \mathrm{d} k^{2}$ is positive or negative unless we specify the exact form for the demand and production functions．Consequently，the effect of acquiring fixed cost information on each LMF＇s expected profit per worker may go in either direction．It will be shown in the next section，however，that when $h$ is linear and $g$ is quadratic，the welfare of each LMF increases by the acquisition of fixed cost information，which would agree with common sense．12）

## 2 A Simple Case of Linear Demand and Quadratic Production

Whereas the informational properties of the LM economy are more or less similar to those of its PM twin in some circumstances，the former may be entirely different from the latter in others．In order to make the distinction between these two regimes much clearer，in this section we will carry out a very detailed analysis of the simple yet important case in which the demand function is linear and each firm＇s production function is quadratic．

For simplicity, let us make the following assumption:0

## Assumption (DL-PQ)

```
(1) (Demand: Linear) \(\quad h(X)=x_{1}+x_{2}\).
(2) (Production: Quadratic)
\(g\left(x_{i}\right)=x_{X_{i}}{ }^{2} \quad(i=1,2)\)
```

First, we will deal with the PM economy. Clealy, firm $i$ s profit function is written as $\Pi_{\mathrm{i}}=(b-h(\mathrm{X}))_{x_{\mathrm{i}}}-w g\left(x_{\mathrm{i}}\right)-k=\left(b-x_{\mathrm{i}}-x_{\mathrm{j}}\right) X_{\mathrm{i}}-{ }_{W} X_{\mathrm{i}}{ }^{2}-k$. So, if firm $i$ maximizes $\Pi_{i}$ under the Cournot-Nash assumption on its opponent's output, the following first-order condition must be met:

$$
\begin{equation*}
\left.\partial \Pi_{\mathrm{i}} / \partial_{x_{\mathrm{i}}} \equiv b-2(1+w) x_{\mathrm{i}}-x_{\mathrm{j}}=0 \quad, \quad(i, j=1,2 ;) i \neq j\right) \tag{38}
\end{equation*}
$$

Since $\quad \partial^{2} \Pi_{\mathrm{i}} / \partial x_{\mathrm{i}}{ }^{2} \equiv-2(1+w)<0$, the second-order condition is always satisfied. In the light of (7.38), we can derive each PMF 's reaction function as follows:

$$
\text { [PMF] } \quad \boldsymbol{R}_{\mathrm{i}}: \quad x_{\mathrm{i}}=【 1 / 2(1+w) 】\left(b-x_{\mathrm{j}}\right) . \quad(i \neq j)
$$

The two reaction curves, $R_{1}$ and $R_{2}$, are shown in Fig. 4. As is expected, both curves are downward-sloping straight lines. Point $Q^{*}$ represents a unique CournotNash equilibrium which is apparently (globally) stable. The equilibrium output of each PMF is then provided as follows:

$$
\begin{equation*}
x_{\mathrm{i}} *=b /(3+2 w) . \quad\left(x_{\mathrm{i}}=1,2\right) \tag{39}
\end{equation*}
$$

And, by virtue of (38), firm $i$ 's equilibrium profit can be rewritten as follows:

$$
\begin{equation*}
\Pi_{\mathrm{i}}{ }^{*}=(1+W)\left(x_{i} *\right)^{2}-k . \tag{40}
\end{equation*}
$$

We must determine the signs of the second-order derivatives with respect to the parameters in question in order to explore the impact of information acquisition by PMFs on equilibrium values. If we differentiate (39) and (40) with respect to $b$ twice, then we find the following:

$$
\mathrm{d} x_{\mathrm{i}} * / \mathrm{d} b^{2}=0 ;
$$

$$
\mathrm{d} \Pi_{\mathrm{i}} * / \mathrm{d} b^{2}=2(1+W) /(3+2 W)^{2}>0
$$

In this simple case, $x_{\mathrm{i}} *$ is a linear function of $b$. As a result, information acquisition about $b$ has no impact on each PMF 's expected output, but does increase its expected profit.


Fig. 4 Profit-Maximizing (PM) Duopoly: A Simple Case

Similarly, we can have the following:

$$
\begin{aligned}
& \mathrm{d}^{2} x_{\mathrm{i}} * / \mathrm{d} k^{2}=0 \\
& \mathrm{~d}^{2} \Pi_{\mathrm{i}} * / \mathrm{d} k^{2}=0
\end{aligned}
$$

Since both $x_{i} *$ and $\Pi_{\mathrm{i}} *$ are linear functions of $k$, it follows that information about $k$ does not affect the expected outputs and profits of PMF s in any way.

Now, let us turn to the role of information in the LM economy. Note that under Assumption (DL-PQ), we have $S_{\mathrm{i}} \equiv \prod_{\mathrm{i}} / I_{\mathrm{i}}=\left[\left(b-x_{\mathrm{i}}-x_{\mathrm{j}}\right)_{X_{\mathrm{i}}-W X_{\mathrm{i}}}{ }^{2}-k\right] / x_{\mathrm{i}}{ }^{2}$ by means of (24). So, if firm $i$ maximizes $S_{\mathrm{i}}$ under the Cournot-Nash assumption on
its rival's output, the following first-order conditions should be satisfied:

$$
\begin{equation*}
\left.\partial S_{\mathrm{i}} / \partial x_{\mathrm{i}} \equiv\left[x_{\mathrm{i}}\left(x_{\mathrm{j}}-b\right)+2 k\right] / x_{\mathrm{i}}{ }^{3}=0 . \quad(i, j=1,2 ;) i \neq j\right) \tag{41}
\end{equation*}
$$

Making use of (41), firm $i$ 's equilibrium profit per worker is expressed by the following equation:

$$
\begin{equation*}
S_{\mathrm{i}} *=k /\left(x_{\mathrm{i}} *\right)^{2}-1-W . \tag{42}
\end{equation*}
$$

The second-order condition for profit per worker requires the following:

$$
\begin{equation*}
\partial^{2} \boldsymbol{S}_{\mathrm{i}} / \partial x_{\mathrm{i}}{ }^{2} \equiv 2\left[x_{\mathrm{i}}\left(b-x_{\mathrm{i}}\right)-3 k\right] / x_{\mathrm{i}}{ }^{4}<0 . \tag{42a}
\end{equation*}
$$

Because $\quad x_{\mathrm{i}}\left(b-x_{\mathrm{i}}\right)-3 k=-k<0$ by means of (41), the second-order condition is satisfied. From (41), each LMF 's reaction function is given as follows:
[LMF] $\quad \boldsymbol{R}_{\mathrm{i}}: \quad x_{\mathrm{i}}=2 k /\left(b-x_{\mathrm{i}}\right) . \quad(i \neq j)$

The two reaction curves for the LM economy are shown in Fig. 5. For convenience, assume that $b^{2}$ is greater than $8 k$. Then, as is clear from the figure, there exist two


Fig. 5 Labor-Managed (LM) Duopoly: A Simple Linear Case
equilibrium points, $Q^{*}$ and $Q^{* *}$. The corresponding equilibrium outputs are then given as follows :

$$
\begin{array}{ll}
\boldsymbol{Q}^{*}: & x_{\mathrm{i}}{ }^{*}=\left(b-\left(b^{2}-8 k\right)^{1 / 2}\right) / 2 \\
\boldsymbol{Q}^{* *}: & x_{\mathrm{i}}{ }^{*}=\left(b+\left(b^{2}-8 k\right)^{1 / 2}\right) / 2 \tag{44}
\end{array}
$$

Let us compare this Fig. 5 with the last Fig. 4. Then, we see that even in the simple case of linear demand and quadratic production, each LMF 's reaction curve is no longer a straight line but a rectangular hyperbola. Thus, it is a more difficult job to investigate the working of the LM economy than that of its PM twin.

In Fig. 5 , Point $Q^{*}$ is stable but Point $Q^{* *}$ is not so. Therefore, for our comparative static analysis to be valid, we have only to focus on the stable point, namely Point $Q^{*}$. As a result, $x_{i}{ }^{*}$ stands for the only relevant equilibrium output of firm $i$.

We are ready to examine the informational implications for the LM economy. By repeatedly differentiating (43) and (42) with respect to $b$, we obtain the following second derivatives:

$$
\mathrm{d}^{2} x_{\mathrm{i}} * / \mathrm{d} b^{2}=2 x_{\mathrm{i}} *\left(b-x_{\mathrm{i}} *\right) /\left(b-2 x_{\mathrm{i}} *\right)^{3}=4 k /\left(b^{2}-8 k\right)^{3 / 2}>0 ;
$$

$$
\begin{align*}
& \mathrm{d}^{2} S_{\mathrm{i}} * / \mathrm{d} b^{2}=\left(b-x_{\mathrm{i}} *\right)\left(b-4 x_{\mathrm{i}} *\right) / x_{\mathrm{i}} *\left(b-2 x_{\mathrm{i}} *\right)^{3} \\
& \quad=【 \mathrm{~b}+\left(b^{2-8 k}\right)^{1 / 2} 】 【 2\left(b^{2-8 k}\right)^{1 / 2}-b 】 / 【 \mathrm{~b}-\left(b^{2}-8 k\right)^{1 / 2} 】\left(b^{2-8 k}\right)^{3 / 2} \\
& \quad \gtrless \quad 0 . \tag{45}
\end{align*}
$$

It is seen that $x_{i} *$ is a convex function of $b$ ．Thus，for the simple case of linear demand and quadratic production，we can unequivocally determine the impact of information acquisition about $b$ on $E_{x_{i}}$ ．Obtaining demand information increases the expected output of each LMF．This result may be contrasted with the PMF case of linear demand and quadratic production，where as stated above，the expected output remains unaffected by such information acquisition．

In the present simple case of PMF，we have shown that $\Pi_{i}$＊is a convex function of $b$ ，implying that the welfare of each PMF increases by the acquisition of demand function．It is seen in（45），however，$S_{\mathrm{i}} *$ is neither convex nor concave in $b$ ，whence the impact of the demand information acquisition on the welfare of each LMF is ambiguous．In fact，the second－order derivative $\mathrm{d}^{2} S_{\mathrm{i}} * / \mathrm{d} b^{2}$ depends on the sign of the quantity 【2 $\left(b^{2-8 k}\right)^{1 / 2}-b 】$ ，which is positive or negative according to whether $b$ is greater than or less than the quantity $(32 k / 3)^{1 / 2}$ ．

Fig． 6 demonstrates the relationship between $b$ and $S_{\mathrm{i}} *$ ．Apparently，Curve $S_{\mathrm{i}} *$ is inverse－S shaped with M being a reflection point．For instance，let us consider the following discrete uniform distribution of $b$ ，that is，$\phi(b)=1 / 2$ for $b=$ $(32 \mathrm{k} / 3)^{1 / 2}+m, \quad(32 \mathrm{k} / 3)^{1 / 2}+2 \mathrm{~m} ; \quad$ and $\phi(b)=0$ otherwise．（Note that $m$ is a given positive integer．）


Fig． 6 The Relationship between $h$ and $S_{\mathrm{i}}$＊

Then，the information transmission makes each LMF better off（ or worse off） whenever $m$ is positive（or negative）．Even working with the case of linear demand and quadratic production，we have thus been led to a＂perverse result，＂characteristic of the LM economy．Once again，we have learned that the value of information may be negative and ignorance may be bliss ！

Finally，let us analyze the effects of obtaining fixed cost information on the equilibrium values．In view of（43）and（45），it is not difficult to derive the following second－order derivatives：

$$
\begin{aligned}
& \mathrm{d}^{2} x_{\mathrm{i}} * / \mathrm{d} k^{2}=8 /\left(b-2 x_{\mathrm{i}} *\right)^{3}=8 /\left(b^{2}-8 k\right)^{3 / 2}>0 ; \\
& \mathrm{d}^{2} S_{\mathrm{i}} * / \mathrm{d} k^{2}=4 b\left(b-3 x_{\mathrm{i}} *\right) /\left(x_{\mathrm{i}} *\right)^{3}\left(b-2 x_{\mathrm{i}} *\right)^{3} \\
& \quad=16 b 【 3\left(b^{2}-8 k\right)^{1 / 2}-b 】 / 【 b-\left(b^{2}-8 k\right)^{1 / 2} \rrbracket^{3} 【 b^{2}-8 k \rrbracket^{3 / 2}>0 .
\end{aligned}
$$

It is noted by（41）and（45）that at equilibrium，$b-3 x_{\mathrm{i}} *=2\left[k-\left(x_{\mathrm{i}} *\right) 2\right] / x_{\mathrm{i}} *$
$=x_{\mathrm{i}} *\left(W^{2}+S_{\mathrm{i}} *\right)>0$. Therefore, the second-order derivative $\mathrm{d}^{2} S_{\mathrm{i}} * / \mathrm{d} k^{2}$ must be positive as shown above.

Since both $x_{\mathrm{i}} *$ and $S_{\mathrm{i}} *$ are convex functions of $k$, it follows from Proposition above that the acquisition of fixed cost information increases the expected output and expected profit per worker of each LMF. These results are in marked contrast to the PMF world where the information acquisition has no influence on the expected output and expected profit $f$ each PMF.

Table 1 summarizes the effects f obtaining demand and cost information on the equilibrium values for the PM and LM duopolies. While we limit our attention to the simple case of linear demand and quadratic production, we may easily understand that the information implications for the LM economy are different from those for the PM economy. In fact, it appears that information plays a greater role in the LM economy than in the PMeconomy. This is because the sign pattern for the former economy looks more complicated and more intriguing than for the latter economy.
:

Table 1 The Impact of Information Acquisition: A Simple Case of Linear Demand and Quadratic Production

| Uncertainty | PM duopoly |  | LM duopoly |  |
| :---: | :---: | :---: | :---: | :---: |
| Demand | $\mathrm{d}^{2} X_{\mathrm{i}}{ }^{*} / \mathrm{d} b^{2}$ | 0 | $\mathrm{d}^{2} \boldsymbol{X}_{\mathrm{i}} * / \mathrm{d} b^{2}$ | + |
|  | $\mathrm{d}^{2} \Pi_{\mathrm{i}} * / \mathrm{d} b^{2}$ | + | $\mathrm{d}^{2} S_{\mathrm{i}} * / \mathrm{d} b^{2}$ | $\pm$ a) |
| Fixed | $\mathrm{d}^{2} \mathrm{X}_{\mathrm{i}}{ }^{*} / \mathrm{d} k^{2}$ | 0 | $\mathrm{d}^{2} \boldsymbol{X}_{\mathrm{i}}{ }^{*} / \mathrm{d} k^{2}$ | + |
| Cost | $\mathrm{d}^{2} \Pi_{\mathrm{i}} * / \mathrm{d} k^{2}$ | + | $\mathrm{d}^{2} \boldsymbol{S}_{\mathrm{i}}{ }^{*} / \mathrm{d} k^{2}$ | + |

## 3 The Labor-Managed Economy Also Matters: Conclusions

In this paper, we have been concerned with the role of information played in PM and LM duopoly models. We are especially interested in seeing how and to what extent the LMF 's response to the acquisition of demand or cost information is different from the traditional PMF twin. To this end, we have first developed a unified approach to the role of information in a two-person game under uncertainty, and then applied the approach to the PM and LM duopolies.

Generally, it is a more demanding task to explore the informational implications for the LM duopolies than those for the PM duopolies. This is because in contrast to the equal treatment of labor and other factors of production in the conventional PM economy, a special status is accorded to labor as a vital factor of production. As can easily be understood, such non-symmetric treatment in the LM economy requires special care in the computations and interpretations of welfare results. In short, human beings play special status at the LM firm, thus being fundamentally different from other raw materials and machinery. This distinction should clearly be recognized in the discussion of the LM economy.

If we want to discuss the working and performance of the capitalist economy, it is correct to say that the PMF matters as one of the conventional forms. We want to assert, however, that the LMF also matters as an alternative form. .

Michael E. Porter (1947-) is a distinguished American academic who is well-known for his theories on economic science, business incentives, and social relations. In a very influential article in Harvard Business Review, Porter (2011) and his fellow worker Mark R. Kramer have once remarked:

The capitalist system is under siege. In recent years business increasingly has been criticized as a major cause of social, environmental, and economic problems. Companies are wisely thought to be prospering at the expense of their communities.
(Porter \& Kramer 2011, p. 11)

We think that Porter's recognition of the capitalist system as being "the system under siege" is of the utmost importance. We agree with Porter that a big business should be viewed as a major cause of social, environmental, and economic problems: Companies may be prospering at the expense of other players such as smaller businesses, non-profit organizations and the general public. According to Porter,
companies could bring business and the society back together if we redefine their purpose as "creating shared value " rather than "producing surplus value," meaning that what is good for business is also good for the society (also see Porter (2011) ) . This remind us of the traditional philosophy of Japanese merchants of "three-way advantages," implying that what is good for the seller is good for the buyer and also good for the society as a whole. In other words, the seller, buyer and the society should create shared value. In this respect, it is recalled that Ryutaro Komira, a influential Japanese economist, has once regarded the Japanese firm as a sort of LMF. Besides, Masahiko Aoki (1990), another distinguished economist, has worked hard toward an economic model of the Japanese firm within the framework of a "cooperative game theory." 13)

In conclusion, in reality, there exist a variety of capitalist firms, forming a wide spectrum containing the American-type PMF at one end and the Japanese-type LMF at the other end. We would like to stress that LMF also matters: it is worthy of serious investigation.

A few final words. There are many directions in which we can extend our analysis of the role of information in the LM duopoly markets. First, in this paper, we have not fully examined the impact of information acquisition on the welfare of the consumer as a third party. Second, no attention was not paid to the possibility that information acquisition may favorably affect the group solidarity that presumably distinguishes the LM economy from the PM economy.

Third, we have ignored the problem of risk aversion and firm-specific risks. As shown by Sakai and Yoshizumi (1991a, 1991b), the presence of risk aversion has an effect of increasing the degree of concavity of the objective function (the whole profit or per capita profit function) of each firm, this enhancing the possibility that information is harmful rather than beneficial. Fourth, we could apply our analysis to the case of differentiated products and/or the Bertrand-type situation where the strategy of each LMF is not its quantity but its price. As the work of Okuguchi (1986) indicates, the comparison between Bertrand and Cournot equilibriums for the LM economy under product information is very important. Finally, in this paper, the number of firms in an industry is limited to only two. We believe that the generalization of our analysis to an oligopolistic market would be a challenging problem to tackle. Those and other related problems will be left for further research. So far so good. However, so many unsolved problems are waiting for us !

## References

Aoki M (1990) Toward an Economic Model of the Japanese Firm, Journal of Economic Literature, Vol. 28, No. 1, pp. 1-27.

Basar T, Ho Y (1974) Informational Properties of the Nash Solution of Two Stochastic Nonzero-Sum Games. Journal of Economic Theory, Vol. 7, April, pp. 370-384.
Bonin P, Putterman L (1987) Economics of Cooperation and Labor-Managed Economy, Harwood Academic Publishers, New York

Bulow J, et. al. (1985) Multimarket Oligopoly: Strategic Substitutes and Complements. Journal of Political Economy, Vol.93, June, pp. 488-511.
Friedman JW (1977) Oligopoly and the Theory of Games, North-Holland, Amsterdam.
Fukuda W (1980) The Theory of the Labor-Managed Firm under Uncertainty, Kobe Economic Review, Vol. 26, December, pp. 46-61.
Hey JD (1981) A Unified Theory of the Behavior of Profit-Maximizing, Labour-Managed and Joint-Stock Firms Operating under Uncertainty, Economic Journal, Vol. 91, June, pp. 364-374.

Hey JD, Suckling J (1980) On the Theory of the Competitive Labor-Managed Firm under Price Uncertainty: Comment, Journal of Comparative Economics, Vol. 4, September, pp. 338-342.

Imai K (1992) Competition among Different Systems in Capitalism, Chikura Publishing Company, Tokyo.
Komiya R (1988a) The Structural and Behavioral Characteristics of the Japanese Firm: Part I, Tokyo University Economic Review, Vo. 54, July, pp. 2-16.

Komiya R (1988a) The Structural and Behavioral Characteristics of the Japanese Firm: Part II, Tokyo University Economic Review, Vo. 54, October, pp. 54-66.
Miyamoto Y (1980) The Labor-Managed Firm and Oligopoly, Osaka City University Economic Review, Vol. 16, pp 17-31.

Miyamoto Y (1980) The Labor-Managed Firm's Reaction Function Reconsidered, Working Paper, Faculty of Economics, Osaka City University, pp 27-42.
Muzondo Y (1982) On the Theory of Competitive Labor-Managed Firm under Price Uncertainty, Journal of Comparative Economics, Vol. 3, April, pp.127-144.

Ogura E (1980) Ohmi Merchants: Their Origins and Developments (in Japanese), Nihon Keizai Oublishers, Tokyo.

Ogura E (1991) The Ideas and Thought of Ohmi Merchants (in Japanese), Sunrise Publishers, Hikone, Japan.

Oi WY (1961) The Desirability of Price Instability under Perfect Competition, Econometrics, Vo. 29, February, pp. 58-61.

Okuguchi K (1978) The Stability of Price-Adjusting Oligopoly with Conjectural Variations, Journal of Economics, Vol. 38, February, pp. 55-60.

Okuguchi K (1986) Labor-Managed Bertrand and Cournot Oligopolies, Journal of Economics, Vol. 46, June, pp. 115-122.

Ponssard JP (1979) The Strategic Role of Information on the Demand Function in an Oligopolistic Market, Management SCience, Vol. 25, March, pp. 243-250.

Porter ME, Kramer, MR (2011) Creating Shared Value, Harvard Business Review, Vol. 89, Issue 1/2, Jan/Feb pp. 2-77.

Porter M (2011) Rethinking Capitalism, Harvard Business School, An Interview. Blog. http//zum.io/2011/01/07/michael-porter-on-shared-value/

Rothenberg TI, Smith KR (1971) The Effect of Uncertainty on Resource Allocation, Quarterly Journal of Economics, Vol. 85, October, pp. 440-453,
Sakai Y (1990) Information Sharing in Oligopoly; Overview and Evaluation: Part I, Alternative Models with a Common Risk, Keio Economic Studies, Vol. 27, October, pp.17-41.

Sakai Y (1991) Information Sharing in Oligopoly; Overview and Evaluation: Part II, Private Risks and Oligopoly Models, Keio Economic Studies, Vol. 28, April, pp. 51-71.

Sakai Y, Yoshizumi A (1991a) The Impact of Risk Aversion on Information Transmission between Firms, Journal of Economics, Vol. 53, February, pp. 51-73.

Sakai Y, Yoshizumi A (1991b) Risk Aversion and Duopoly: Is Information Exchange Always Beneficial to Firms? Pure Mathematics and Applications, Vol. 2, September, pp. 129-145.

Samuelson PA (1946) The Foundations of Economic Analysis, MIT Press, U.S.A.
Ward B (1958) The Firm in Illyria: Market Syndicalism, American Economic Review, vol. 68, pp. 566-589.

## Footnotes

1) For the critical role of information in the Japanese economy, see Imai (1992).
2) For an overview and evaluation of information sharing in the PM economy, see Sakai (1990, 1991).
3) An excellent summary of the literature on the LM economy was given by Bonin and Putterman (1987).
4) For instance, Assumption (L) is met in the pioneering models of Basar and Ho (1974) and Ponssard (1979). Also see Sakai (1990, 1991).
5) For the stability analysis of a two-person game and its application to duopoly, see Friedman (1977) and Okuguchi (1978).
6) We may say that $y_{i}$ and $y_{i}$ are strategic substitutes (or strategic complements) if and only if the second cross derivative $\partial^{2} Z_{\mathrm{i}} / \partial y_{\mathrm{i}} \partial y_{\mathrm{j}}$ is negative (or positive) . See Bulow et al. (1985).
7) In general, the inverse demand function should be written as $p=H(X, b)$, where $b$ represents a shift parameter. To make our analysis manageable, we assume here that the function $H$ is separable such that $H(X, b)=b-h(X)$.
8) It is Oi (1961) , one of Sakai's respected teachers at Rochester, who first noticed the convexity of the profit function with respect to price under perfect competition, arguing that price instability makes the PMF better off. Later, Rothenberg and Smith (1971) extended Oi's analysis to cover a general competitive model where the feedback for other sector is also allowed for. In their analysis, the PMF is expected to instantaneously adjust its output decision ex post for a random demand; which, in effect, means the PMF takes a contingent action on an ex ante basis. In the present paper, by reformulating the ex post Oi-Rothenberg-Smith analysis as an ex ante one, we shed new light on the value of information in duopoly.
9) See Bonin and Putterman (1987).
10) It is noted that the reaction function may be positively or negatively sloped, depending on the form of the demand and production functions.
11) In passing, we note that $\mathrm{d} x_{i} / \mathrm{d} k$ is positive (see (7.34). Therefore, output and labor respond positively to increase in fixed cost. Interestingly, this is another "perverse behavior" of the LMF.
12) This result is sensitive to the assumption that the PMF is a risk-neutral player in the sense that it seeks maximal expected profit. If we instead assume that the PMF displays
risk aversion and hence maximizes its expected utility of profit, then the acquisition of cost information is expected to have a significant influence on the welfare of the PMF. The same reservation should be kept for the LMF world. In short, risk aversion really matters! For this point, see Sakai and Yoshizumi (1991a, 1991b).
13) The Ohmi merchants of Japan give us a good example of the traditional philosophy of "three way advantages," namely, advantages for the seller, the buyer and the society. It seems that the philosophy is still alive in Japan today. See Ogura (1980, 1991).
