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From General Equilibrium Theory to the Economics of Uncertainty : A Personal Perspective

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From General Equilibrium Theory to the Economics of Uncertainty : A Personal Perspective

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Abstract This note is concerned with the question of why and how my research interest has been changed from general equilibrium theory to the economics of uncertainty. though it is fundamentally a personal perspective, it is expected to have historical implications as well, thus serving as a good guide toward the new horizon of integrated social science. When I was started my research life, the people was involved in the "Cold War" between the capitalist bloc and the socialist bloc. In 1968, to escape from the Japanese university disturbance, I applied for the graduate program at the University of Rochester, with Professor Lionel W. McKenzie being a towering figure. While I took care of the math econ sequence at the University of Pittsburgh, I began to have a feeling of doubt about the practical applicability of general equilibrium theory a la McKenzie. Partly being motivated by a suggestion from Professor Oscar Morgenstern, who came to Pittsburgh for an academic lecture, I began to shift my research area from pure and abstract theories to more practical and applied subjects including the economics of uncertainty. In 1989, all of a sudden, the seemingly invincible Berlin Wall turned down, being followed by the collapse of the mighty Soviet Union. In the dreadful year of 2008, the world economies was involved in the most serious crisis since the Great Depression on the 1930s. At present, we are living in the "New Age of Uncertainty," hoping for the coming of the second Keynes and/or the second Knight. Thomas Piketty's new book on economic inequality would possibly lead to the promotion of an integrated social science in the new century.

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1 The "Cold War" on the academic front： a strange start

I was born in the pre-World War Ⅱ period. Since so many people have been born after the war, I sometimes feel as if I belonged to rare species. During those long years, the people saw a series of extraordinary things such as atomic bombing in Hiroshima and Nagasaki (1945), the rise of People's Republic of China (1949), the Cuba crisis between the capitalist bloc and the socialist bloc (1962), the Russian spaceship Sputnik over the earth (1964), the first man on the moon by American space project (1969), and the frequent occurrence of violent student movements (1968-1969). Those events which happened between 1940 and 1970 were more or less the products of the so-called Cold War between the two powerful blocs. [1]

I would like to add that another sort of cold war took place on the academic front as well. In this connection, it is recalled that Seishi Kaya, the chairman of the Japan Science Council, once remarked in a newspaper:

> On reflection it is really ridiculous that mankind cannot live on this globe peacefully with each other when they possess the knowledge and knowhow even of making a round-up trip to the moon. The most important thing from now on seems to be to join our efforts in making the time nearer when we can all visit the moon as friendly tourist, instead of being involved in the clash between communism and capitalism. (Kaya, 1957)

Kaya was then a famous natural scientist, also serving as the President of Tokyo University. Reading Kaya's article, Shigeto Tsuru, a noted economist with a Ph.D. degree from Harvard University, did not agree with Kaya and made the following counterargument:

> [T]he distinction between capitalism and socialism as a social system is not due to emotional antagonism of politicians or to doctrinaire rigidity of academic people. Dr. Kaya's wish for a harmonious world is everybody's wish; but he should be aware that there does exist here a scientific problem of differentiating different social systems by an objective criterion and that the difference between them cannot be wished away. (Tsuru, 1961, pp. 2-3)

The difference of opinions between Kaya and Tsuru was quite clear-cut and seemed to be almost irreconcilable. Speaking of myself, I was then an eager econ-major student, thus being inclined to side with Tsuru rather than Kaya. [2]

In my student days in the 1960s, there existed two popular yet opposing textbooks in economics. They were nicknamed the "red text" and the "blue text." In hindsight, this was a strange start for my student life. The red text was well-represented by *Economic Textbook* published by the U.S.S.R. Academy Economics Institute (1958), the most authoritative agency of the Soviet socialist bloc. The issue of "socialism versus capitalism" constituted the central theme of the red text. The coauthors of the text took pride in reaching the following conclusion:

> We have thus far discussed the whole processes of economic development of a society. As a result, we have reached the most important conclusion that from a historical viewpoint, capitalism is destined to collapse whereas socialism is marching for its final victory over capitalism. There should be no other way! We are so confident of such historical inevitability.

In contrast to the powerful red text, the blue text, presumably being regarded as a strong rival, seemed to be rather modest and even hesitant. To take an instance, Paul A. Samuelson (1955, 7th edition 1967) wrote the world-popular textbook *Economics*, a typical blue text defending the American capitalist bloc. Samuelson modestly argued:

> America leads Russia, but will the gap narrow? (Samuelson, 7th edition 1967, p. 791)

Comparing those two texts, I was clearly more impressed by the powerful red text than the moderate blue text. After some hesitation, however, I decided to go to the USA so that I could continue my graduate study without unduly political and psychological interruptions. I just wanted to get out of the Japanese university disturbance in the 1960s, thus daring to jump into the core of the capitalist economy. [3]

2 The mathematical beauty of general equilibrium theory: Lionel W. McKenzie as Professor Fixed Point

In the 1960s, Japanese universities were so noisy and turbulent that they became no longer good places for study and research. So, I intended to apply for admission of graduate schools at American universities. Very fortunately, in 1968, I was admitted to the Graduate School of Economics, the University of Rochester.

There was a very prominent professor at the Rochester economics faculty, who played a leading role for establishing the outstanding graduate program in economics. The name of that famous professor was Lionel W. McKenzie. He was one of those

pioneers who together with Kenneth Arrow and Gerard Debreu succeeded in introducing advanced mathematics such as differential topology into economics. One of his favorable mathematical tools was the fixed point theorem which was first established by great mathematicians including L.E.J. Brouwer and Shizuo Kakutani. So it would be quite natural to see that McKenzie was nicknamed by many graduate students "Professor Fixed Point." The way in which McKenzie taught general equilibrium theory at Rochester was legendary. Every time, he distributed to a selected group of graduate students his handwritten manuscripts, which contained a lot of mathematical symbols and complicated equations. As could easily be guessed, the symbols and equations were rather loosely written and sometimes almost incomprehensible. There was something dignified in his teaching; making use of large blackboards in a classroom, he always wrote so many definitions and axioms before proceeding to prove a series of mathematical propositions. All the students seemed to be just quiet, taking great pains in making their lecture notes faithfully. [4]

Professor McKenzie seemed to be very fond of mathematics; indeed, he was a passionate researcher in the application of differential topology to economic science. It is true that he almost always controlled his class authoritatively. No person should be perfect, however. There was an occasion when he was a bit nervous in mathematical derivations and pondered for some time while grasping a piece of white chalk in this right hand. Whenever his right hand got up and approached to his face, the color of his lips gradually changed from reddish to whitish: he forgot the inescapable reality that the white chalk happened to touched his lips. Occasionally, he spoke the names of several Japanese economists in heavy (or rather correct) English accents, "Morry-see-ma" (meaning Michio Morishima), "Woo-za-were" (Hirofumi Uzawa), "Knee-kai-dow" (Fukukane Nikaido), and "Nay-gee-see" (Takeshi Negishi). Yes, they must be Japanese names, but sounded to me like American names.

I still remember the occasion when McKenzie did not feel well and unfortunately got struck in a mathematical jungle. This incident happened exactly when he was about to finish the proof of general market equilibrium solution. He knew that the mathematical tool needed was no less than the effective use of the fixed point theorem, Then he stopped walking and began to fold his arms, holding a piece of white chalk with his right hand. After five minutes or so, his cheek suddenly got more brighter than ever before and nodded his head to himself, "I've got it!" And after completing the existence proof successfully, he convincingly yet rather quietly muttered with the following sigh: "Oh, it's so beautiful !" I should add that was only faintly heard to me: I was lucky enough to set on the front row.

Unquestionably, McKenzie's lecture on general equilibrium theory was mathematically so beautiful that it greatly impressed all the students in his class. Honestly speaking, however, his inclination toward mathematical beauty seemed too much to us. In academic activities, if we seek "beauty" in addition to "logic" and "ethics," we should be demanding too much. Then we could possibly be caught in inescapable pitfalls in the sense that we vainly searched for a sort of "utopian capitalism" or rather "elegant nihilism." A will-o'-the-wisp would be burning and sneer at us forever! [5]

3 The practical charm of the economics of uncertainty: a kind advice from Oscar Morgenstern

After receiving a Ph.D. degree in economics from the University of Rochester, I was lucky enough to get a teaching job at the University of Pittsburgh. At Pittsburgh, I was asked to take care of the mathematical economics sequence for both undergraduate and graduate students. General equilibrium theory constituted the core of "math econ," an abbreviation of mathematical economics, which used to be regarded with reverence as a sort of god by many economists. [6]

One day, a female student from Turkey asked me a very pointed question like this:

> Dr. Sakai, I am so impressed by the way you taught us general equilibrium theory. We have enjoyed by seeing the power and beauty of Kakutani's fixed point theorem, a mathematical tool to show the existence of competitive equilibrium. As you may know well, sir, I am not from well-advanced countries like the U.S. and Japan, but alas, from a less advanced country. I wonder whether and to what extent your general equilibrium lecture is applicable to the betterment of my home country. Please let me know.

This question gave me a shock, sounding like a thunder out of blue. The questioner was a bright and beautiful female student from an agricultural country, who had been very attentive to my graduate course. It seemed that she wisely hit the mark. I myself was then a professor lecturing her. So, I took pains in hiding my true feeling before her, barely making the following answer:

> Oh, this sound a very clever question. My home country, Japan, is not so advanced as the United States either. As a matter of fact, there exist dual structures in Japan, namely, advanced and backward sectors. So, I am not in a good position to strongly believe that the general equilibrium

stuff can straightforwardly be applied to my home country. Well, all the students, I would like to say this! Now let us study together to carefully examine the applicability of pure theory to more complicated economies as they really are.

After several years of my stay at Pittsburgh, I found myself casting a somewhat skeptical eye upon the authoritative theory of general equilibrium. I vividly remember my exciting meeting with Oscar Morgenstern, a famous pioneer of the theory of games, who happened to come to Pittsburgh in 1973 to give us a general lecture on the history of economic theories. Being among the enthusiastic audience to listen to his lecture, I took a courage to ask a rather pointed question to him:

Professor Morgenstern, I have been so impressed by your instructive lecture today. To tell the truth, however, I am not so happy about the present state of economic science. It seems that there exists a big gap between the abstract assumptions of theory and the harsh facts in reality. In order to fill in the gap, I would strongly feel the necessity to establish a new approach to socio-economic problems today. If you have some suggestions in this direction, please let me know.

Morgenstern at first appeared to be perplexed a bit. but soon composed himself. Then he kindly began to answer my question:

Oh, Mr. Sakai, you look still young and have a lot of potential. I can tell you that fortunately, there now appear a new wave of economic thinking, that is the economics of uncertainty promoted by a group of young economists like you. Why don't you join the group right away?

The keyword "uncertainty" sounded to me like a sort of revelation or "Sermon on the Mount." What is uncertainty all about? Since then, I shifted all my energy into an investigation of uncertainty although I had already read several articles by George Akerlof, Michael Spence, and Joseph Stiglitz. Surely, there were all my contemporaries, so that I hopefully could join the "promising club of uncertainty." [7]

In hindsight, when I was a graduate student at Rochester, my research was never alien to the economics of risk and uncertainty. For one thing, Professors Edward Zabel and James Friedman jointly opened the graduate course "theory of uncertainty and games," in which I myself was a very attentive student. For another, Richard Thaler, one of my friends at Rochester and much later a Nobel prize winner, enthusiastically advised me to take a variety of courses including "econometric history " and "labor economics." To my regret, however, I did not give careful attention to the importance of

applied fields, so that I was just content to focus on pure theory of general equilibrium.

Following Morgenstern's advice aforementioned, I decided to read again Akerlof's paper (1970) on uncertainty, and this time I found myself greatly shocked in a psychological sense.

> We have been discussing economic models in which "trust" is important. Informal unwritten guarantees are preconditions for trade and production. Where these guarantees are indefinite, business will suffer — as indicated by our Gresham's law. This aspect of uncertainty has been explored by game theorists, as in the Prisoner's Dilemma, but usually it has not been incorporated in the more traditional Arrow-Debreu approach to uncertainty. But the difficulty of distinguishing good quality from bad in inherent in the business world; this may indeed explain many economic institutions and may in fact be one of the more important aspects of uncertainty.
>
> (Akerlof, 1970, pp. 15-16.)

As was stated by Akerlof, the issue of uncertainty was not incorporated in the traditional theory of general equilibrium. In fact, Professor McKenzie never seriously discussed the problem of uncertainty within the his framework of general equilibrium. At present, it would be safe to say that Akerlof's paper of 1970 served very well as a good springboard by which I could shift my research area from general equilibrium theory to the economics of uncertainty.

4 The second age of uncertainty: the return of old masters

History sometimes does something cruel. As the real history told us, a sequence of so many incredible things and events happened beyond all expectations. No persons on the earth should be perfect and almighty. According to Taleb (2007), the world is full of the "black swans," or the random events that are nearly impossible to predict in advance.

Against the wildest expectations of almost everyone, the seemingly invincible Berlin Wall turned down all of a sudden in 1989, being followed by the total collapse of the mighty Soviet Union into so many independent countries in 1991.

> The Wall has collapsed completely! It's just gone! You'll see that we can now come and go so freely in Berlin!

This was the lively conversation I could hear here and there when I was invited as

a guest speaker at the Berlin international conference in 1993. Strangely enough, it reminded me of my young days when the issue of "socialism versus capitalism" was a lively discussion topic among fellow students. At the same time, I wondered if the collapse of one social system would sooner or later lead to the decline and even breakdown of its rival. As the saying goes, power will collapse, and absolute power will collapse absolutely!

In the dreadful year of 2008, the world economy was involved in the most serious crisis since the Great Depression of the 1930s. Share prices plunged throughout the world; in particular, the USA lost 33.8 % of its value in that single year. The seemingly strong Lehman brothers collapsed, being followed by the total breakdown of Yamaichi security company in Japan. Even some economists liked to call the financial crisis of 2008 the Great Recession, named after the famous Great Depression.

The Queen Elizabeth of the United Kingdom used to have a large portion of her huge wealth in the form of stocks and securities. All of a sudden, the Great Recession took place hitting the wealth of the British monarchy very harshly: The personal of the Queen's fortune was then estimated to have fallen £25 million in the 2008 credit crunch. This was really an intolerable shock to the Queen! In 2008, at the opening of a new building at the London School of Economics, the Queen uncharacteristically asked the following brief yet sharp question to the audience, including Professor Luis Garicano, then the director of research at the LSE:

> If these things were so large, how come everyone missed it?

Professor Gricano apparently got off-guard, thus defending himself by telling the Queen:

> In every stage, someone was relying on somebody else and everyone thought they were doing the right thing.

This episode seemed to be summed up as the simple sentence, "Sorry Maa'm, we just didn't see it coming." More academically, I would rather like to say that we are now living in the "Second Age of Uncertainty," named after the catchy word of the "(First) Age of Uncertainty" by John K. Galbraith (1977). In this connection, Posner (2009) eloquently remarked in a rather long sentence:

> I also emphasize some points that have received relatively little coverage in other accounts:

> the depression's political dimensions, the disappointing performance of the economics profession in regard to anticipating and providing guidance to responding to the depression, how ideology can distort economic policy, the inherent limitations of depression economics, how the self-interested decisions of rational businessmen and consumers can give rise to a depression (so there is no need to look for psychological explanations9, and how the failure of officials and economists to anticipate the financial crisis and prevent its ripening into a depression echoes the failure of other officials and other professionals to anticipate and prevent other catastrophic events, like the Pearl Harvor or 9/11 attacks or the devastation of New Orlean by Hurricane Katrina. (Posner 2009, Preface, pp. xiv - xv)

Perhaps, I would not need to add to Posner's eloquent remark. I just want to confirm here the disappointing performance of the economics profession as an anticipating guidance to the Great Recession of 2008. Surely, it may echoes the Queen Elizabeth's pointed question on the credit crunch of 2008: "why did nobody notice it?"

Many people might simply believe that the collapse of the Soviet Union in 1991 means the fundamental failure of socialism. The crisis of 2008, however, teaches us that the capitalist victory is not so clear. A failure, if not the failure, of sapitalism should also be a very fashionable topic again in the academia. In short, we are now living in the New Age of Uncertainty in which the rivalry between capitalism and socialism, or more exactly, the one between Market or Control, is still alive in many ways. We look forward to the return of the old masters including J.M. Keynes and F.H. Knight. Of course, we should not simply imitate the old teachings, nor merely apply them to new realities. Simple imitation and rude applications do not work here! We must do something else; we must rather go beyond those old masters in a variety ways toward the new horizon of integrated social science. [8]

5 Econs versus humans： no clear goals in sight

In the light of the history of economic theory, it seems that there has existed the academic struggle between "Econs" and "Humans." According to Richard H. Thaler (2015), a respected representative of our Rochester graduates, many standard models tend to use a fictional creature called home economics, or simply Econs. Econs are regarded as "rational fools" a la Amartia Sen (1987) in the sense that they rationally choose goods by optimizing their utilities and have always rational expectations about market equilibriums. Although the traditional expected utility model is an extention of the standard rational model to the world of risk and uncertainty, its fundamental

structure remains the same as before.

In contrast, Humans are just human beings, namely homo sapiens. Compared with fictional Econs, Humans do a lot of misbehaving, implying that economic models lead to a lot of bad predictions. Humans are supposed to have a lot of non-rational feelings such as envy, optimism, pessimism, sympathy, compassion, and the like. In the world where many Humans live, the traditional economic theory is far from satisfactory. We need to establish a more comprehensive model of human behavior including a variety of complicated psychologies. Besides, in contrast to Econs who are abstract and homogeneous entities independent of any historical and cultural constraints, Humans tend to behave as ordinary persons in the street, thus being always affected by their historical and cultural backgrounds. If homogeneity, uniformity and rationality are keywords to describe Econs, then heterogeneity, diversity and non-rationality constitute the basic characteristics of Humans.

It is recalled that Albert Einstein (1879-1955), the probably the greatest scientist of the twentieth century, once talked about his own philosophy of science:

> I have little patience with scientists who take a board of wood, look for its thinnest part, and drill a great number of holes where drilling is easy. (Einstein, quoted by Phillip Frank 1949)

In the real world, we are exposed to various temptations for easy-going lives. Ao, it would be a sad fact that we have a tendency to drill a great number of holes on a board of wood in which drilling is rather easy. However, we have to take a very broad view of the wood without looking at only small trees. Let me make it plain again. We are really Humans rather than Econs!

I strongly hope that this note on my personal perspective would serve as a helpful guide to a newly integrated theory of risk and uncertainty that should be built on the basis of J.M. Keynes and F.H. Knight. I am sure that we can learn new lessons from a set of old teachings. We live in the New Age of Uncertainty. Unfortunately, however, there are no clear goals are in sight. The simple return of the old masters would be no help. Probably, we need a new Keynes and/or a new Knight.

Recently, French economist Thomas Piketty (2013) has published a highly exciting book, first written in French and the immediately translated into English. It deals with the dynamics of wealth and income inequality covering a long span of the last 200 years. Piketty persuasively argues that we are now on the way back to the old-fashioned capitalism, in which the wealth and income inequality are widening again and thus social and economic instabilities are also increasing. Since its publication

there have been many pros and cons for the book. Paul Krugman (2014), a nobel prize winner in economic science, praised it very highly:

> It seems safe to say that Capital in the Twenty-First Century, the magnum opus of the French economist Thomas Piketty, will be the most important economics of the year — and maybe the decade. (Krugman 2014,)

We are not quite certain whether and to what extent Krugman's appraisal of Piketty is correct. If we think of the happenings of big unexpected events such as Lehman shock of 2008 and Fukushima nuclear disaster f 2011, however, we must eagerly hope for the coming of new economic science. Piketty's new and ambitious analysis will perhaps be one of the most important books for many years to come.

In short, we have to go on and beyond Keynes and Knight. We need for the second Piketty. Life is a challenge! As the saying goes, where there is a will, there is a way.

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Footnotes

1 The Cold War and its academic impact were extensively discussed in Chapter 1 of Sakai (2009).

2 Shigeto Tsuru (1912-2006) was a Harvard educated economist and later became the President of Hitotsubashi University, thus influencing in many ways in the post-World War Ⅱ period.

3 To the best of my recollection, in the 1950s and the 1960s, the red text was more popular than the blue text among Japanese students. In contrast, in the same period , the blue text was dominating and the red text non-existent among American students.

4 See Arrow & Debreu (1954), Debreu (1959), McKenzie (1954, 1955, 1969), Morishima (1964), Negishi (1960), Nikaido (1970), and Uzawa (1962).

5 For the fixed point theorems of Brouwer and Kakutani, see Brouwer (1910) and Kakutani (1941). Kakutani served as a research assistant to von Neumann, who together with Oscar Morgenstern wrote von Neumann-Morgenstern (1944), an epoch-making book on game theory. Also see Hirota (2004)..

6 In retrospect, McKenzie's lecture looked like a solemn ritual based on the Kant's philosophy of "truth, good and beauty." At Pittsburgh in the early 1970s, my lecture was also influenced by McKenzie, my mentor. For further details, see Sakai (1972) and Chapter 5 of Sakai (2019).

7 For my work on the economics of uncertainty, see Sakai (1982) and many others. Although I was writing its English version as well, I could not finish it.

8 The crash of 2008 and its historical implications were also discussed by Soros (2008), Krugman (2008) and others. See Keynes (1921), Keynes (1936), and Knight (1921). The similarity and difference between those Keynes and Knight were carefully discussed by Sakai (2016, 2019).