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**Implementing Arrow-Debreu Equilibria in Security Markets
with Infinite Dimensional Martingale Generator**

Koji Kusuda

September 2004

**Center for Risk Research
Faculty of Economics
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Implementing Arrow-Debreu Equilibria in Security Markets with Infinite Dimensional Martingale Generator

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Abstract. This paper presents that Arrow-Debreu equilibria in continuous-time market economy with jump-diffusion information generated by infinite dimensional martingale generator can be implemented in *approximately complete security markets* (Björk *et al.* [10]) in which every bond with any maturity date is traded and any contingent claim is approximately replicated with any given precision. A generalized security market equilibrium called *approximate security market equilibrium* is introduced in which each agent is allowed to choose a consumption plan approximately financed with any prescribed precision. It is shown that an Arrow-Debreu equilibrium in the economy can be identified with an approximate security market equilibrium in the approximately complete markets.

Keywords and Phrases: Approximately complete markets, Approximate security market equilibrium, Arrow-Debreu equilibrium, Infinite dimensional martingale generator, Jump-diffusion.

JEL Classification Numbers: C62, D51, G10.

This paper is a revision and expansion of each part of my working paper (Kusuda [33]) and Chapter 3 in my Ph.D. dissertation (Kusuda [35]) at the Department of Economics, University of Minnesota. I would like to thank my adviser Professor Jan Werner for his invaluable encouragement and advice. I am grateful for comments of participants of presentations at University of Minnesota, Japanese Economic Association Fall 2002 Conference, Institute of Economic Research, Hitotsubashi University, Institute for Advanced Studies, Vienna, and Shiga University.

1. INTRODUCTION

There is a strong evidence¹ that the dynamics of most financial processes such as equity prices, interest rates, and exchange rates are better described by jump-diffusion processes than pure diffusion processes which are assumed in the standard models such as the Black-Scholes model (Black and Scholes [12]) of stock price. Many studies² have also shown that the presence of jumps could have significant impact on asset pricing³ and portfolio choice⁴. Against the background of such studies, jump-diffusion security market models have been intensively studied in Finance and Financial Economics, and in particular, in the context of CAPM⁵, option pricing⁶, and portfolio choice⁷. In most of jump-diffusion security market models, the jump magnitude is specified as a random variable with a continuous distribution at each jump time. In this case, the *dimensionality of martingale generator*⁸ in the markets, which can be interpreted as “the number of sources of uncertainty,” is uncountably infinite, and no finite set of traded securities can complete the markets. While many equilibrium analyses have been conducted in security market economy with finite dimensional martingale generator,⁹ in security market economy with infinite dimensional martingale generator, no equilibrium analysis has been conducted to date as far as the author knows.

The purpose of this paper and its companion paper (Kusuda [37]) is to develop a general equilibrium (GE, hereafter) analysis in a class of GE models of security market economy with infinite dimensional martingale generator, in which Consumption-based CAPMs (Capital Asset Pricing Models) with jump risk and jump-diffusion option pricing models can be constructed. In this paper, the notion of a generalized security market equilibrium called *approximate security market* (ASM, hereafter) *equilibrium* is introduced, and then it is proven that an ASM equilibrium can be identified with an Arrow-Debreu equilibrium. In the corresponding Arrow-Debreu economy, the companion paper (Kusuda [37]) presents (1) a sufficient condition for the existence of Arrow-Debreu equilibria in the case of stochastic differential utilities, which are continuous-time version of Epstein-Zin utilities (Epstein and Zin [22]), and (2) sufficient conditions for the existence, uniqueness, and local uniqueness (or determinacy) of Arrow-Debreu equilibria in the case of time additive utilities. In subsequent papers, Consumption-based CAPMs with jump risk and a broad class of jump-diffusion option pricing models are presented in the

¹Akgiray and Booth [3], Andersen, Benzoni, and Lund [4], Bakshi, Cao, and Chen [6], Bates [7] [8] [9], Das [16], Eraker, Johannes, and Polson [23], Jorion [29], Pan [45], *etc.*

²Bakshi, Cao, and Chen [6], Bates [9], Duffie, Pan, and Singleton [19], Pan [45], Rietz [48], *etc.*

³Rietz [48] has claimed that jump risk premia could be high enough to explain high equity premia pointed out by Mehra and Prescott [42]. Pan [45] has shown that jump risk premia is high enough to explain volatility “smirks” implied by market quoted prices of options.

⁴Adachi [1], Daglish [14], Liu, Longstaff, and Pan [38], *etc.*

⁵Ahn and Thompson [2], Back [5], Kusuda [34], Madan [39], *etc.*

⁶Bakshi, Cao, and Chen [6], Bates [7] [8] [9], Björk *et al.* [10] [11], Duffie, Pan, and Singleton [19], Fujiwara and Miyahara [24], Merton [43], Naik and Lee [44], *etc.*

⁷Adachi [1], Daglish [14], Liu, Longstaff, and Pan [38], *etc.*

⁸For example, if the *filtration* in security markets, which can be interpreted as the “information,” is generated by a d -dimensional Wiener process, then a martingale generator is the Wiener process and its dimensionality is d .

⁹In security market economy in which the filtration is generated by a finite dimensional Wiener process, Duffie [17], Duffie and Zame [20], and Huang [26] show sufficient conditions for the existence of equilibria, and Karatzas, Lakner, Lehoczky, and Shreve [30], and Karatzas, Lehoczky, and Shreve [31] present sufficient conditions for the existence and uniqueness of equilibria. Dana and Pontier [15], and Duffie [17] show sufficient conditions for the existence of equilibria in security market economy in which the filtration is more general than the one generated by finite dimensional Wiener process. However, the martingale generator in their markets is still assumed to be finite dimensional.

class of GE models with infinite dimensional martingale generator (Kusuda [34]), and further interest rate derivative pricing models are proposed in the class of jump-diffusion option pricing models (Kusuda [32] [36]).

A summary of this paper is as follows. A continuous-time security market economy with an infinite dimensional martingale generator, which consists of a jump process given by a *marked point process* (see Appendix A) and a Wiener process, is considered. The markets are then incomplete as long as the number of traded securities is finite. It is very difficult to show the existence of GE in incomplete markets. Therefore, it is assumed that every zero-coupon bond with any maturity time is traded, in other words, a continuum of bonds are traded. In such markets, it is shown that under some regularity condition, the markets are *approximately complete* (Björk, Di Masi, Kabanov, and Runggaldier [10], Björk, Kabanov, and Runggaldier [11]). In usual complete markets, any contingent claim is replicated by an admissible self-financing portfolio. In approximately complete markets, any contingent claim is approximately replicated with any given precision by an admissible self-financing portfolio of bonds. The notion of a generalized security market equilibrium called ASM (Approximate Security Market) equilibrium is then introduced. In ASM equilibrium, each agent is allowed to choose any consumption plan that can be approximately financed with prescribed precision by a budgetary admissible portfolio. Next, a method of implementing Arrow-Debreu equilibria in security market economy is presented. In the existing method (Dana and Pontier [15], Duffie [17], Duffie and Zame [20], and Huang [26]) of implementation, it is assumed that the nominal price of every zero-coupon bond is always one. On the contrary, in this paper, the family of nominal bond prices is not specified in this way. Rather, a class of families of nominal bond prices is introduced at which the markets are arbitrage-free and approximately complete. In this method, any family of nominal bond prices in this class can be chosen. It is shown that this method of implementation can be interpreted as a generalization of the existing one. Finally, it is proven that for every family of nominal bond prices in the class, an ASM equilibrium can be identified with an Arrow-Debreu equilibrium. Main mathematical techniques used in this paper are jump-diffusion information versions of Ito's Formula, Girsanov's Theorem, and Martingale Representation Theorem.

The remainder of this paper is organized as follows. In Section 2, a specification of a security market economy with jump-diffusion information is provided. In Section 3, a review of arbitrage-free approximately complete security markets is introduced following Björk *et al.* [10] [11]. In Section 4, the notion of ASM equilibrium is introduced, and the method of implementing Arrow-Debreu Equilibria in our security markets is presented. In Section 5, it is proven that an ASM equilibrium can be identified with an Arrow-Debreu equilibrium. In Section 6, the proof of the author's main proposition is given.

2. SECURITY MARKET ECONOMY WITH JUMP-DIFFUSION INFORMATION

In this section, a specification of security market economy with jump-diffusion information is provided.

A continuous-time frictionless security market economy with time span $[0, T^\dagger]$ (abbreviated by \mathbf{T} , hereafter) for a fixed horizon time $T^\dagger > 0$ is considered. The agents' common subjective probability and information structure is modeled by a complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ where $\mathbb{F} = (\mathcal{F}_t)_{t \in \mathbf{T}}$ is the natural filtration generated by a d -dimensional Wiener process W and a *marked point process* $\nu(dt \times dz)$ (see Appendix A) on a Lusin space $(\mathbb{Z}, \mathcal{Z})$ (in usual applications,

$\mathbb{Z} = \mathbb{R}^n$, or \mathbb{N}^n , or a finite set) with the P -intensity kernel $\lambda_t(dz)$.¹⁰ If the mark space \mathbb{Z} is infinite, then the dimensionality of martingale generator is infinite because a martingale generator in this economy is $(W, (\nu(dt \times \{z\}) - \lambda_t(\{z\}))_{z \in \mathbb{Z}})$. The author's main concern is to consider the case in which \mathbb{Z} is infinite, although \mathbb{Z} is unspecified.

There is a single perishable consumption commodity. The commodity space is a Banach space $\mathbf{L}^\infty = \mathbf{L}^\infty(\Omega \times \mathbf{T}, \mathcal{P}, \mu)$ where \mathcal{P} is the predictable σ -algebra on $\Omega \times \mathbf{T}$, μ is the product measure of the probability measure P and the Lebesgue measure on \mathbf{T} . There are I agents. Each agent $i \in \{1, 2, \dots, I\}$ (abbreviated by \mathbf{I} , hereafter) is represented by (U^i, \bar{c}^i) , where U^i is a strictly increasing and continuous utility on the positive cone \mathbf{L}_+^∞ of the consumption process and $\bar{c}^i \in \mathbf{L}_+^\infty$ is an endowment process, which is assumed to be nonzero. The economy mentioned above is described by a collection

$$\mathbf{E} = ((\Omega, \mathcal{F}, \mathbb{F}, P), (U^i, \bar{c}^i)_{i \in \mathbf{I}}).$$

There are markets for the consumption commodity and securities at every date $t \in \mathbf{T}$. The traded securities are nominal-risk-free security (NOT the risk-free security) called the *money market account* and a continuum of zero-coupon bonds whose maturity times are $(0, T^\dagger]$, each of which pays one unit of cash (NOT one unit of the commodity) at its maturity time. Let p , B , and $(B^T)_{T \in (0, T^\dagger]}$ denote the consumption commodity price process, nominal money market account price process and nominal bond price processes, respectively. The collection $(B, (B^T)_{T \in (0, T^\dagger]})$ of security prices is abbreviated by \mathbf{B} , and called the *family of bond prices*.

3. APPROXIMATELY COMPLETE MARKETS

In this section, the approximately complete markets given by Björk *et al.* [10] [11] is reviewed, and a class of families of bond prices is introduced such that for every family of bond prices in this class, an ASM equilibrium can be identified with an Arrow-Debreu equilibrium.

Let $n \in \mathbb{N}$. Let \mathcal{L}^n denote the space of real-valued \mathcal{P} -measurable process X satisfying the integrability condition $\int_0^{T^\dagger} |X_s|^n ds < \infty$ P -a.e. Also, let $\mathcal{L}^n(\lambda_t(dz) \times dt)$ denote the space of real-valued $\mathcal{P} \otimes \mathcal{Z}$ -measurable process H satisfying the integrability condition $\int_0^{T^\dagger} \int_{\mathbb{Z}} |H_s(z)|^n \lambda_s(dz) ds < \infty$ P -a.e. Moreover, let \mathbf{C}^n denote the space of n -times continuously differentiable functions.

We say that a family $\mathbf{B} = (B, (B^T)_{T \in (0, T^\dagger]})$ of bond prices is *regular* if and only if the following conditions hold:

- (1) For every $T \in (0, T^\dagger]$, the dynamics of nominal bond price process B^T satisfies the following stochastic differential-difference equation

$$\frac{dB_t^T}{B_{t-}^T} = r_t^T dt + v_t^T \cdot dW_t + \int_{\mathbb{Z}} m_t^T(z) \{ \nu(dt \times dz) - \lambda_t(dz) dt \} \quad \forall t \in [0, T)$$

with $B_T^T = 1$ and $B_t^T = 0$ for every $t \in (T, T^\dagger]$ for some $r^T \in \mathcal{L}^1$, $v^T \in \prod_{j=1}^d \mathcal{L}^2$, and $m^T \in \mathcal{L}^1(\lambda_t(dz) \times dt)$. Moreover, it follows that:

- (i) For every $(\omega, t) \in \Omega \times \mathbf{T}$, $r_t(\omega), v_i(\omega) \in \mathbf{C}^1(\mathbf{T})$, and for every $(\omega, t, z) \in \Omega \times \mathbf{T} \times \mathbb{Z}$, $m_t(\omega, z) \in \mathbf{C}^1(\mathbf{T})$.

¹⁰This information structure is based on Björk, Kabanov, and Runggaldier [11]. More general information structures are considered in Björk, Di Masi, Kabanov, and Runggaldier [10] and in Jarrow and Madan [28].

- (ii) For every $T \in (0, T^\dagger]$, B^T is regular enough to allow for the differentiation under the integral sign and the interchange of integration order.¹¹
 - (iii) For every $t \in \mathbf{T}$, bond price curves B_t^{\cdot} are bounded P -a.e.
 - (iv) The family of jump magnitude functions $m_i(\cdot)$ is uniformly bounded μ -a.e.
- (2) The dynamics of nominal money market account price process B satisfies the following stochastic differential equation

$$\frac{dB_t}{B_t} = r_t^B dt \quad \forall t \in [0, T^\dagger)$$

with $B_0 = 1$ where r_t^B is given by $r_t^B = -\frac{\partial \ln B_t^T}{\partial T} \Big|_{T=t}$, and $r^B \geq 0$ μ -a.e.

Each agent is allowed to hold a portfolio consisting of the money market account and all of bonds at one time. For that purpose, the portfolio component of bonds is defined by a signed finite Borel measure on $[t, T^\dagger]$ for every event $\omega \in \Omega$ and time $t \in \mathbf{T}$.

Definition 1. A *portfolio* (also called “trading strategy”) is a stochastic process $\vartheta = (\vartheta^0, \vartheta^1(\cdot))$ that satisfies:

- (1) The component ϑ^0 is a real-valued \mathcal{P} -measurable process.
- (2) The component ϑ^1 is such that:
 - (i) For every $(\omega, t) \in \Omega \times \mathbf{T}$, the set function $\vartheta_t^1(\omega, \cdot)$ is a signed finite Borel measure on $[t, T^\dagger]$.
 - (ii) For every Borel set A , the process $\vartheta^1(A)$ is \mathcal{P} -measurable.

Let the family \mathbf{B} of bond prices be regular. A portfolio ϑ is said to be *feasible at \mathbf{B}* if and only if the following integrability conditions are satisfied:

$$B_t r_t^B \vartheta_t^0 \in \mathcal{L}^1, \quad \int_t^{T^\dagger} |B_t^T r_t^T| |\vartheta_t^1(dT)| \in \mathcal{L}^1,$$

$$\int_t^{T^\dagger} |B_t^T v_t^T| |\vartheta_t^1(dT)| \in \mathcal{L}^2, \quad \int_t^{T^\dagger} |B_t^T m_t^T(z)| |\vartheta_t^1(dT)| \in \mathcal{L}^1(\lambda_t(dz) \times dt).$$

Let $\Theta(\mathbf{B})$ denote the class of feasible portfolios at \mathbf{B} . The *value process* $V^{\mathbf{B}}(\vartheta)$ of a *feasible portfolio* $\vartheta \in \Theta(\mathbf{B})$ at \mathbf{B} is given by

$$V_t^{\mathbf{B}}(\vartheta) = B_t \vartheta_t^0 + \int_t^{T^\dagger} B_t^T \vartheta_t^1(dT) \quad \forall t \in \mathbf{T}.$$

A feasible portfolio $\vartheta \in \Theta(\mathbf{B})$ is said to be *self-financing at \mathbf{B}* if and only if the following equation holds:

$$V_t^{\mathbf{B}}(\vartheta) = V_0^{\mathbf{B}}(\vartheta) + \int_0^t \vartheta_s^0 dB_s + \int_0^t \int_s^{T^\dagger} \vartheta_s^1(dT) dB_s^T \quad \forall t \in \mathbf{T}.$$

Also, a self-financing portfolio $\vartheta \in \Theta(\mathbf{B})$ is said to be an *arbitrage portfolio at \mathbf{B}* if and only if either of the following condition holds:

- $V_0^{\mathbf{B}}(\vartheta) \leq 0$, and $V_{T^\dagger}^{\mathbf{B}}(\vartheta) > 0$, i.e. $V_{T^\dagger}^{\mathbf{B}}(\vartheta) \geq 0$ P -a.e. and $P(\{V_{T^\dagger}^{\mathbf{B}}(\vartheta) > 0\}) > 0$.
- $V_0^{\mathbf{B}}(\vartheta) < 0$, and $V_{T^\dagger}^{\mathbf{B}}(\vartheta) \geq 0$ P -a.e.

¹¹For the marked point process integrals, we can apply the ordinary Fubini Theorem, and for the interchange of integration with respect to dW and dt , we can apply the Stochastic Fubini Theorem (see Protter [46]).

For a real-valued \mathcal{P} -measurable process X , the *discounted process of X at \mathbf{B}* is denoted by \tilde{X} . Thus, $\tilde{X} = \frac{X}{\tilde{\mathbf{B}}}$. The collection $(\tilde{B}, (\tilde{B}^T)_{T \in \mathbf{T}})$ of security prices is abbreviated by $\tilde{\mathbf{B}}$.

To eliminate unrealistic portfolios such as those based on a *doubling strategy* (see Chapter 6 in Duffie [18]), the class of feasible portfolios is restricted to the class of *admissible portfolios*, which is equivalent to the class of *credit-constrained portfolios* introduced by Dybvig and Huang [21].

Definition 2. Let \mathbf{B} be regular. A feasible portfolio $\vartheta \in \Theta(\mathbf{B})$ at \mathbf{B} is *admissible at \mathbf{B}* if and only if the discounted value process $\tilde{V}^{\mathbf{B}}(\vartheta)$ is bounded below P -a.e.

Let $\mathcal{Q}(\tilde{\mathbf{B}})$ denote the class of admissible portfolios at \mathbf{B} . Definitions of *arbitrage-free markets* and the *risk-neutral measure* (also called “spot martingale measure”).

Definition 3. Let \mathbf{B} be regular.

- (1) Markets are *arbitrage-free at \mathbf{B}* if and only if there exists no admissible arbitrage portfolio at \mathbf{B} .
- (2) A probability measure $\tilde{P}^{\mathbf{B}}$ on (Ω, \mathcal{F}) is a *risk-neutral measure at \mathbf{B}* if and only if $\tilde{P}^{\mathbf{B}}$ is equivalent to P and the discounted family $\tilde{\mathbf{B}}$ of bond prices is a martingale under $\tilde{P}^{\mathbf{B}}$.

It is well known that the existence of risk-neutral measures implies that markets are arbitrage-free.

Lemma 1. *Let \mathbf{B} be regular. If there exists a risk-neutral measure at \mathbf{B} , then markets are arbitrage-free at \mathbf{B} .*

Proof. See the proofs of Theorem 6.F and Corollary 6.F in Duffie [18]. \square

Suppose that the family of bond prices \mathbf{B} is regular. The following lemma then shows a necessary and sufficient condition on \mathbf{B} for the existence of risk-neutral measures.

Lemma 2. *Let \mathbf{B} be regular. Then it follows that:*

- (1) *There exists a risk-neutral measure $\tilde{P}^{\mathbf{B}}$ at \mathbf{B} if and only if there exists a martingale process $\Lambda^{\mathbf{B}}$ such that*

$$\frac{d\Lambda_t^{\mathbf{B}}}{\Lambda_{t-}^{\mathbf{B}}} = -v_t^{\mathbf{B}} \cdot dW_t - \int_{\mathbb{Z}} m_t^{\mathbf{B}}(z) \{ \nu(dt \times dz) - \lambda_t(dz) dt \} \quad \forall t \in [0, T^\dagger)$$

with $\Lambda_0^{\mathbf{B}} = 1$ where $(v^{\mathbf{B}}, m^{\mathbf{B}}) \in (\prod_{j=1}^d \mathcal{L}^2) \times \mathcal{L}^1(\lambda_t(dz) \times dt)$ satisfies the following equation

$$r_t^T = r_t^B + v_t^{\mathbf{B}} \cdot v_t^T + \int_{\mathbb{Z}} m_t^{\mathbf{B}}(z) m_t^T(z) \lambda_t(dz) \quad \forall t \in [0, T^\dagger). \quad (3.1)$$

- (2) *If there exists a martingale process $\Lambda^{\mathbf{B}}$ satisfying the above conditions, then it follows that:*

- (i) *The probability measure $\tilde{P}^{\mathbf{B}}$ given by the Radon-Nikodym derivative*

$$d\tilde{P}^{\mathbf{B}} = \Lambda_{T^\dagger}^{\mathbf{B}} dP \quad (3.2)$$

is a risk-neutral measure at \mathbf{B} .

- (ii) *The process $\tilde{W}^{\mathbf{B}}$ given by*

$$\tilde{W}_t^{\mathbf{B}} = W_t + \int_0^t v_s^{\mathbf{B}} ds \quad \forall t \in \mathbf{T} \quad (3.3)$$

is a $\tilde{P}^{\mathbf{B}}$ -Wiener process.

- (iii) The marked point process $\nu(dt \times dz)$ has the $\tilde{P}^{\mathbf{B}}$ -intensity kernel $\tilde{\lambda}_t^{\mathbf{B}}(dz)$ such that

$$\tilde{\lambda}_t^{\mathbf{B}}(dz) = (1 - m_t^{\mathbf{B}}(z)) \lambda_t(dz) \quad \forall (t, z) \in \mathbf{T} \times \mathbb{Z}. \quad (3.4)$$

Proof. These results follow from Ito's Formula (see Appendix B) and Girsanov's Theorem (see Appendix C). \square

Let \mathcal{B} denote the class of families of regular bond prices satisfying conditions in Lemma 2. The process $A^{\mathbf{B}}$ is called the *density process of $\tilde{P}^{\mathbf{B}}$ relative to P* . The processes $v_t^{\mathbf{B}}$ and $m_t^{\mathbf{B}}(z)\lambda_t(dz)$ are called the *market price of (nominal) diffusive risk* and the *market price of (nominal) jump risk*, respectively.

Suppose that the mark space \mathbb{Z} is finite, and let $\mathbf{B} \in \mathcal{B}$. Then it should be noted that solutions $(v^{\mathbf{B}}, m^{\mathbf{B}}) \in (\prod_{j=1}^d \mathcal{L}^2) \times \mathcal{L}^1(\lambda_t(dz) \times dt)$ of the equation (3.1) are not necessarily unique, which implies that risk-neutral measures at \mathbf{B} are not necessarily unique. If risk-neutral measures at \mathbf{B} are not unique, then markets are incomplete, which makes it difficult to show the existence of GE. Therefore, some regularity condition on \mathbf{B} is imposed such that the solutions of (3.1) are unique at \mathbf{B} . In the case of security markets with pure diffusion information, the uniqueness of risk-neutral measures is equivalent to the completeness of markets. Unfortunately, this is no longer true for security markets with jump-diffusion information; that is, in the case of security markets with jump-diffusion information, the uniqueness of risk-neutral measures does not imply the completeness of markets. Therefore, Björk *et al.* [10] [11] introduced the notion of *approximately complete* defined in the following.

Definition 4. Let $\mathbf{B} \in \mathcal{B}$.

- (1) For every $T \in (0, T^\dagger]$, a *contingent T -claim* is a \mathcal{F}_T -measurable random variable X_T such that $\tilde{X}_T = \frac{X_T}{B_t} \in \mathbf{L}_+^\infty(\Omega, \mathcal{F}_T)$ where $\mathbf{L}^\infty(\Omega, \mathcal{F}_T)$ is the space of almost surely bounded \mathcal{F}_T -measurable random variables.
- (2) A contingent T -claim X_T is *replicable at \mathbf{B}* if and only if there exists an admissible self-financing portfolio $\vartheta \in \underline{\mathcal{Q}}(\tilde{\mathbf{B}})$ such that the discounted value process is bounded, and satisfies $V_T^{\mathbf{B}}(\vartheta) = X_T$.
- (3) Markets are *complete at \mathbf{B}* if and only if every T -contingent claim X_T is replicable for every $T \in (0, T^\dagger]$.
- (4) Markets are *approximately complete at \mathbf{B}* if and only if for any $T \in (0, T^\dagger]$ and any T -contingent claim X_T , there exists a sequence of replicable claims $(X_{Tn})_{n \in \mathbb{N}}$ converging to X_T in $\mathbf{L}^2(\Omega, \mathcal{F}_T, \tilde{P}^{\mathbf{B}})$ where $\tilde{P}^{\mathbf{B}}$ is a risk-neutral measure at \mathbf{B} .

Let $\mathbf{B} \in \mathcal{B}$. Björk, Di Masi, Kabanov, and Runggaldier [10] prove that risk-neutral measures are unique at \mathbf{B} if and only if markets are approximately complete at \mathbf{B} . They also show that if \mathbb{Z} is finite, then risk-neutral measures at \mathbf{B} are unique if and only if markets are complete at \mathbf{B} .

Proposition 1. Let $\mathbf{B} \in \mathcal{B}$.

- (1) Each of the following conditions is necessary and sufficient for \mathbf{B} to have a unique risk-neutral measure.
 - (i) Markets are approximately complete at \mathbf{B} .
 - (ii) For every $(\omega, t) \in \Omega \times \mathbf{T}$, the equation

$$\tilde{\mathcal{O}}_t^*(\omega) \vartheta_t^1(\omega) = \begin{pmatrix} v_t(\omega) \\ m_t(\omega, \cdot) \end{pmatrix} \quad (3.5)$$

can be solved on a dense subset of $\mathbb{R}^d \times \mathbf{L}^2(\mathbb{Z}, \mathcal{Z}, \tilde{\lambda}_t^{\mathbf{B}}(dz))$ where $\tilde{\mathcal{O}}_t^*(\omega) : \mathbf{M}(\mathbf{T}) \rightarrow \mathbb{R}^d \times \mathbf{L}^2(\mathbb{Z}, \mathcal{Z}, \tilde{\lambda}_t^{\mathbf{B}}(\omega, dz))$ is defined by

$$\tilde{\mathcal{O}}_t^*(\omega) : \vartheta_t^1(\omega) \mapsto \left(\begin{array}{c} \int_t^{T^\dagger} \tilde{B}_{t-}^{T^\dagger}(\omega) v_t^T(\omega) \vartheta_t^1(\omega, dT) \\ \int_t^{T^\dagger} \tilde{B}_{t-}^{T^\dagger}(\omega) m_t^T(\omega, \cdot) \vartheta_t^1(\omega, dT) \end{array} \right)$$

where $\mathbf{M}(\mathbf{T})$ denotes the space of measures on \mathbf{T} with finite total variation, which is equivalent to the dual space of $\mathbf{C}^0(\mathbf{T})$.

- (2) Suppose that the mark set \mathbb{Z} is finite. Then each of the following conditions is necessary and sufficient for \mathbf{B} to have a unique risk-neutral measure.
- (i) Markets are complete at \mathbf{B} .
 - (ii) For every $(\omega, t) \in \Omega \times \mathbf{T}$, the equation (3.5) can be solved on $\mathbb{R}^d \times \mathbf{L}^2(\mathbb{Z}, \mathcal{Z}, \tilde{\lambda}_t^{\mathbf{B}}(\omega, dz))$.

Proof. See the proof of Proposition 6.10 in Björk, Di Masi, Kabanov, and Runggaldier [10]. \square

We introduce a class of families of bond prices such that for every family of bond prices in this class, an ASM equilibrium can be identified with an Arrow-Debreu equilibrium.

Definition 5. A family of bond prices $\mathbf{B} \in \mathcal{B}$ is *implementable* if and only if the following two conditions hold:

- (1) Risk-neutral measures at \mathbf{B} are unique.
- (2) The discounted density process $\tilde{\lambda}^{\mathbf{B}}$ of $\tilde{P}^{\mathbf{B}}$ relative to P is bounded above and bounded away from zero μ -a.e.

Let $\tilde{\mathcal{B}}$ denote the class of families of implementable bond prices.

4. ASM EQUILIBRIUM AND ARROW-DEBREU EQUILIBRIUM

In this section, the notion of ASM (Approximate Security Market) equilibrium is introduced, and then the method of implementing Arrow-Debreu equilibria in security market economy is presented.

4.1. ASM (Approximate Security Market) Equilibrium. Before introducing the notion of ASM equilibrium, the following *security market equilibrium* first introduced by Radner [47] is considered.

Definition 6. A collection $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B}) \in \prod_{i \in \mathbf{I}} \mathbf{L}_+^\infty \times \mathbf{L}_+^\infty \times \tilde{\mathcal{B}}$ constitutes a *security market equilibrium for \mathbf{E}* if and only if the following conditions hold:

- (1) For every $i \in \mathbf{I}$, \hat{c}^i solves the problem

$$\max_{c^i \in \mathcal{C}^i(p, \mathbf{B})} U^i(c^i)$$

where

$$\mathcal{C}^i(p, \mathbf{B}) = \left\{ c^i \in \mathbf{L}_+^\infty : \exists \vartheta^i \in \underline{\Theta}(\tilde{\mathbf{B}}) \quad \text{s.t.} \right.$$

$$\left. \begin{array}{l} V_t^{\mathbf{B}}(\vartheta^i) = \int_0^t \vartheta_s^{i0} dB_s + \int_0^t \int_s^{T^\dagger} \vartheta_s^{i1}(dT) dB_s^T + \int_0^t p_s(\bar{c}_s^i - c_s^i) ds \quad \forall t \in \mathbf{T}, \\ V_{T^\dagger}^{\mathbf{B}}(\vartheta^i) = 0 \end{array} \right\}.$$

- (2) The commodity market is cleared as $\sum_{i \in \mathbf{I}} \hat{c}^i = \sum_{i \in \mathbf{I}} \bar{c}^i$.

Remark 1. Note that if $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ constitutes a security market equilibrium for \mathbf{E} , then the security market clearing condition is satisfied in the sense that there exists a $(\hat{\vartheta}^i)_{i \in \mathbf{I}} \in \prod_{i \in \mathbf{I}} \mathcal{Q}(\tilde{\mathbf{B}})$ with $\sum_{i \in \mathbf{I}} \hat{\vartheta}^i = 0$ such that $\hat{\vartheta}^i$ supports \hat{c}^i , *i.e.*

$$\begin{aligned} V_t^{\mathbf{B}}(\hat{\vartheta}^i) &= \int_0^t \hat{\vartheta}_s^{i0} dB_s + \int_0^t \int_s^{T^\dagger} \hat{\vartheta}_s^{i1}(dT) dB_s^T + \int_0^t p_s(\hat{c}_s^i - \tilde{c}_s^i) ds \quad \forall t \in \mathbf{T}, \\ V_{T^\dagger}^{\mathbf{B}}(\hat{\vartheta}^i) &= 0 \end{aligned}$$

for every $i \in \mathbf{I}$. This immediately follows from the commodity market clearing condition and the linearity of value process. Hence, the security market clearing condition has been removed out of the definition of security market equilibrium.

Let $(p, \mathbf{B}) \in \mathbf{L}_+^\infty \times \tilde{\mathbf{B}}$. As shown in Proposition 1, the markets are then approximately complete in which a contingent claim may not be exactly replicated. Thus, an agent's maximization problem may not be well defined since the optimal consumption plan may not be exactly financed by any portfolio in the budget constraint set $\mathcal{C}^i(p, \mathbf{B})$. We now introduce the notion of ASM equilibrium in which each agent is allowed to choose any consumption plan that is approximately financed with any prescribed precision by a budgetary admissible portfolio.

Definition 7. A collection $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B}) \in \prod_{i \in \mathbf{I}} \mathbf{L}_+^\infty \times \mathbf{L}_+^\infty \times \tilde{\mathbf{B}}$ constitutes an ASM (*Approximate Security Market*) equilibrium for \mathbf{E} if and only if the following conditions hold:

- (1) For every $i \in \mathbf{I}$, \hat{c}^i solves the problem

$$\max_{c^i \in \mathcal{C}^i(p, \mathbf{B})} U^i(c^i)$$

where

$$\bar{\mathcal{C}}^i(p, \mathbf{B}) = \left\{ c^i \in \mathbf{L}_+^\infty : \exists (\vartheta_n^i)_{n \in \mathbb{N}} \in \prod_{n \in \mathbb{N}} \mathcal{Q}(\tilde{\mathbf{B}}) \text{ s.t.} \right.$$

$$\left. \begin{aligned} V_t^{\mathbf{B}}(\vartheta_n^i) &= \int_0^t \vartheta_{ns}^{i0} dB_s + \int_0^t \int_s^{T^\dagger} \vartheta_{ns}^{i1}(dT) dB_s^T + \int_0^t p_s(\bar{c}_s^i - c_s^i) ds \quad \forall (n, t) \in \mathbb{N} \times \mathbf{T}, \\ \lim_{n \rightarrow \infty} V_{T^\dagger}^{\mathbf{B}}(\vartheta_n^i) &= 0 \end{aligned} \right\}.$$

- (2) The commodity market is cleared as $\sum_{i \in \mathbf{I}} \hat{c}^i = \sum_{i \in \mathbf{I}} \bar{c}^i$.

4.2. Implementation of Arrow-Debreu Equilibria. A collection $((\hat{c}^i)_{i \in \mathbf{I}}, \pi) \in \prod_{i \in \mathbf{I}} \mathbf{L}_+^\infty \times \mathbf{L}_+^\infty$ is said to constitute an *Arrow-Debreu equilibrium* for \mathbf{E} if and only if the following conditions hold:

- (1) For every $i \in \mathbf{I}$, \hat{c}^i solves the problem

$$\max_{c^i \in \mathcal{C}^i(\pi)} U^i(c^i)$$

where $\mathcal{C}^i(\pi) = \{c^i \in \mathbf{L}_+^\infty : \int_0^{T^\dagger} c_s^i ds = \int_0^{T^\dagger} \bar{c}_s^i ds\}$.

- (2) The commodity market is cleared as $\sum_{i \in \mathbf{I}} \hat{c}^i = \sum_{i \in \mathbf{I}} \bar{c}^i$.

To implement Arrow-Debreu equilibria in security market economy, Dana and Pontier [15], Duffie [17], Duffie and Zame [20], and Huang [26] assume that *Dynamic Spanning Condition* (see Section 10.D in Duffie [18]) is satisfied, which ensures the market completeness, and that every nominal security price process $S(D)$ with a nominal cumulative dividend process D satisfies

$$S_t(D) = E[D_{T^\dagger} - D_t | \mathcal{F}_t] \quad \forall t \in \mathbf{T}. \quad (4.1)$$

Then they set the commodity price $p = \pi$ for every Arrow-Debreu equilibrium $((\hat{c}^i)_{i \in \mathbf{I}}, \pi)$. It is easy to see that the equation (4.1) implies that every nominal

zero-coupon bond price is always one, or equivalently that the nominal-risk-free rate is always zero. This assumption is too restricted and unrealistic.

On the contrary, in the author's method of implementation, any implementable family $\mathbf{B} \in \bar{\mathcal{B}}$ of bond prices can be assumed. The implementation procedure is as follows. Choose an appropriate implementable family $\mathbf{B} \in \bar{\mathcal{B}}$ of bond prices. For every Arrow-Debreu equilibrium $((\hat{c}^i)_{i \in \mathbf{I}}, \pi)$, set the commodity price $p = (\tilde{\Lambda}^{\mathbf{B}})^{-1} \pi$ where $\tilde{\Lambda}^{\mathbf{B}}$ is the discounted density process of $\tilde{P}^{\mathbf{B}}$ relative to P . Then it will be shown in the next section that $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ is an ASM equilibrium for \mathbf{E} .

Assume that $B_t^T = 1$ for every $T \in (0, T^\dagger]$ and $t \in [0, T]$. This assumption implies the equation (4.1) and $p = \pi$ since $\tilde{\Lambda}^{\mathbf{B}} = 1$. Then the author's method of implementation is reduced to that of Dana and Pontier [15], Duffie [17], Duffie and Zame [20], and Huang [26], although in this case, another set of securities are required to make markets approximately complete since the bond markets are not approximately complete. In this sense, the author's method of implementation is a generalization of their one.

Remark 2. Such exogenously given family of bond prices in the author's method is interpreted as the agents' common subjective probability for the yield of nominal interest rates. The central bank controls the yield of nominal interest rates in order to control the commodity price. One explanation for justifying the agents' common subjective probability for the yield of nominal interest rates may be that it is formed by such policy of the central bank.

5. EQUIVALENCE OF ASM AND ARROW-DEBREU EQUILIBRIA

In this Section, it is proven that for every implementable family of bond prices, an ASM equilibrium can be identified with an Arrow-Debreu equilibrium. In concrete, it is proven that for every implementable family of bond prices $\mathbf{B} \in \bar{\mathcal{B}}$, an ASM equilibrium $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ for \mathbf{E} is identified with an Arrow-Debreu equilibrium $((\hat{c}^i)_{i \in \mathbf{I}}, \pi)$ for \mathbf{E} under the relation $\tilde{\Lambda}^{\mathbf{B}} p = \pi$. It is also shown that if the mark space is finite, then for every implementable family of bond prices, ASM equilibrium is reduced to be security market equilibrium.

We derive our main result, Proposition 2, using Proposition 1 and Martingale Representation Theorem. This proposition shows that: (1) For every implementable family of bond prices, a budget constraint set in ASM equilibrium can be identified with a budget constraint set in Arrow-Debreu equilibrium. (2) If the mark space is finite, then for every implementable family of bond prices and commodity price, the budget constraint set in ASM equilibrium is equivalent to the budget constraint set in security market equilibrium.

Proposition 2. *Let $\mathbf{B} \in \bar{\mathcal{B}}$ and $i \in \mathbf{I}$. It follows that:*

- (1) (i) *Let $c^i \in \mathcal{C}^i(\pi)$ where $\pi \in \mathbf{L}_{++}^\infty$. Define $p = (\tilde{\Lambda}^{\mathbf{B}})^{-1} \pi$. Then $p \in \mathbf{L}_{++}^\infty$, and $c^i \in \bar{\mathcal{C}}^i(p, \mathbf{B})$.*
- (ii) *Conversely, let $c^i \in \bar{\mathcal{C}}^i(p, \mathbf{B})$ where $p \in \mathbf{L}_{++}^2$. Define $\pi = \tilde{\Lambda}^{\mathbf{B}} p$. Then $\pi \in \mathbf{L}_{++}^\infty$, and $c^i \in \mathcal{C}^i(\pi)$.*
- (2) *Suppose that the mark space \mathbb{Z} is finite. Then for every $p \in \mathbf{L}_{++}^\infty$, $\mathcal{C}^i(p, \mathbf{B}) = \bar{\mathcal{C}}^i(p, \mathbf{B})$.*

Proof. See Section 6. □

Using Proposition 2, we finally obtain Theorem 1, which shows that: (1) For every implementable family of bond prices, an ASM equilibrium can be identified with an Arrow-Debreu equilibrium. (2) If the mark space is finite, then for every implementable family of bond prices, ASM equilibrium is reduced to be security market equilibrium.

Theorem 1. Let $\mathbf{B} \in \bar{\mathcal{B}}$. It follows that:

- (1) (i) Let $((\hat{c}^i)_{i \in \mathbf{I}}, \pi)$ be an Arrow-Debreu equilibrium for \mathbf{E} . Define $p = (\tilde{\Lambda}^{\mathbf{B}})^{-1}\pi$. Then $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ is an ASM equilibrium for \mathbf{E} .
 - (ii) Conversely, let $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ be an ASM equilibrium for \mathbf{E} . Define $\pi = \tilde{\Lambda}^{\mathbf{B}}p$. Then $((\hat{c}^i)_{i \in \mathbf{I}}, \pi)$ is an Arrow-Debreu equilibrium for \mathbf{E} .
- (2) Suppose that the mark space \mathbb{Z} is finite. Then $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ is an ASM equilibrium for \mathbf{E} if and only if $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ is a security market equilibrium for \mathbf{E} .

Proof. Let $\mathbf{B} \in \bar{\mathcal{B}}$. (2) directly follows from Proposition 2(ii), therefore (1) is proven. Note that $\tilde{\Lambda}^{\mathbf{B}}, (\tilde{\Lambda}^{\mathbf{B}})^{-1} \in \mathbf{L}_{++}^{\infty}$ for $\mathbf{B} \in \bar{\mathcal{B}}$.

Proof of (1)(i). Let $((\hat{c}^i)_{i \in \mathbf{I}}, \pi)$ be an Arrow-Debreu equilibrium for \mathbf{E} . First, $\pi \in \mathbf{L}_{++}^{\infty}$ because agents' utilities are strictly increasing. Define $p = (\tilde{\Lambda}^{\mathbf{B}})^{-1}\pi$. Then $p \in \mathbf{L}_{++}^{\infty}$ since $(\tilde{\Lambda}^{\mathbf{B}})^{-1} \in \mathbf{L}_{++}^{\infty}$. Next, by definition of Arrow-Debreu equilibrium, $(\hat{c}^i)_{i \in \mathbf{I}}$ satisfies the commodity market clearing condition in ASM equilibrium. Let $i \in \mathbf{I}$. It follows from Proposition 2(1)(i) that $\hat{c}^i \in \bar{\mathcal{C}}^i(p, \mathbf{B})$. Suppose that \hat{c}^i is not a utility maximizer in $\bar{\mathcal{C}}^i(p, \mathbf{B})$. Then Proposition 2(1)(ii) implies that \hat{c}^i is not a utility maximizer in $\mathcal{C}^i(\pi)$, which contradicts that $((\hat{c}^i)_{i \in \mathbf{I}}, \pi)$ is an Arrow-Debreu equilibrium for \mathbf{E} . Thus, \hat{c}^i is a utility maximizer in $\bar{\mathcal{C}}^i(p, \mathbf{B})$, and hence $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ is an ASM equilibrium for \mathbf{E} .

Proof of (1)(ii). Let $((\hat{c}^i)_{i \in \mathbf{I}}, p, \mathbf{B})$ be an ASM for \mathbf{E} . First, $p \in \mathbf{L}_{++}^{\infty}$ because agents' utility functions are strictly increasing. Define $\pi = \tilde{\Lambda}^{\mathbf{B}}p$. Then $\pi \in \mathbf{L}_{++}^{\infty}$ since $\tilde{\Lambda}^{\mathbf{B}} \in \mathbf{L}_{++}^{\infty}$. Let $i \in \mathbf{I}$. It suffices to show that \hat{c}^i is a utility maximizer in $\mathcal{C}^i(\pi)$. It follows from Proposition 2(1)(ii) that $\hat{c}^i \in \mathcal{C}^i(\pi)$. Suppose that \hat{c}^i is not a utility maximizer in $\mathcal{C}^i(\pi)$. Then Proposition 2(1)(i) implies that \hat{c}^i is not a utility maximizer in $\bar{\mathcal{C}}^i(p, \mathbf{B})$. This is a contradiction, and therefore \hat{c}^i is a utility maximizer in $\mathcal{C}^i(\pi)$. \square

6. PROOF OF PROPOSITION 2

Let $\mathbf{B} \in \bar{\mathcal{B}}$ and $i \in \mathbf{I}$. Note that $\tilde{\Lambda}^{\mathbf{B}}, (\tilde{\Lambda}^{\mathbf{B}})^{-1} \in \mathbf{L}_{++}^{\infty}$ for $\mathbf{B} \in \bar{\mathcal{B}}$. First, it follows from Bayes' rule and integration by parts that

$$\begin{aligned}
 \tilde{E}^{\mathbf{B}} \left[\int_0^{T^\dagger} \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \right] &= \frac{1}{\Lambda_0^{\mathbf{B}}} E \left[\Lambda_{T^\dagger}^{\mathbf{B}} \int_0^{T^\dagger} \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \right] \\
 &= E \left[\int_0^{T^\dagger} \Lambda_s^{\mathbf{B}} \tilde{p}_s(\bar{c}_s^i - c_s^i) ds + \int_0^{T^\dagger} \int_0^s \tilde{p}_{s'}(\bar{c}_{s'}^i - c_{s'}^i) ds' d\Lambda_s^{\mathbf{B}} + \int_0^{T^\dagger} d \left[\Lambda_s^{\mathbf{B}}, \int_0^s \tilde{p}_{s'}(\bar{c}_{s'}^i - c_{s'}^i) ds' \right] \right] \\
 &= E \left[\int_0^{T^\dagger} \pi_s(\bar{c}_s^i - c_s^i) ds \right].
 \end{aligned} \tag{6.1}$$

Define $\tilde{\mathcal{C}}^i(p, \mathbf{B})$ for every $p \in \mathbf{L}_{++}^{\infty}$ by

$$\begin{aligned}
 \tilde{\mathcal{C}}^i(p, \mathbf{B}) &= \left\{ c^i \in \mathbf{L}_+^{\infty} \mid \exists (\vartheta_n^i)_{n \in \mathbb{N}} \in \prod_{n \in \mathbb{N}} \underline{\mathcal{Q}}(\tilde{\mathbf{B}}) \quad \text{s.t.} \right. \\
 &\quad \tilde{V}_t^{\mathbf{B}}(\vartheta_n^i) = \int_0^t \int_s^{T^\dagger} \vartheta_{ns}^{i1}(dT) d\tilde{B}_s^T + \int_0^t \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \quad \forall (n, t) \in \mathbb{N} \times \mathbf{T} \\
 &\quad \left. \lim_{n \rightarrow \infty} \tilde{V}_{T^\dagger}^{\mathbf{B}}(\vartheta_n^i) = 0 \right\}.
 \end{aligned}$$

Step 1 – $\bar{\mathcal{C}}^i(p, \mathbf{B}) = \tilde{\mathcal{C}}^i(p, \mathbf{B})$ where $p \in \mathbf{L}_{++}^{\infty}$: See Appendix D.

Step 2 – Proof of (1)(ii): See Appendix D.

Step 3 – Proof of (1)(i): Let $c^i \in \mathcal{C}^i(\pi)$ where $\pi \in \mathbf{L}_{++}^\infty$. Then the budget constraint equation holds:

$$E \left[\int_0^{T^\dagger} \pi_s (\bar{c}_s^i - c_s^i) ds \right] = 0. \quad (6.2)$$

Define $p = (\tilde{\Lambda}^{\mathbf{B}})^{-1} \pi$. Then $p \in \mathbf{L}_{++}^\infty$ because $(\tilde{\Lambda}^{\mathbf{B}})^{-1} \in \mathbf{L}_{++}^\infty$. It follows from (6.1) and (6.2) that

$$\tilde{E}^{\mathbf{B}} \left[\int_0^{T^\dagger} \tilde{p}_s (\bar{c}_s^i - c_s^i) ds \right] = 0. \quad (6.3)$$

Since $\tilde{E}_t^{\mathbf{B}} \left[\int_0^{T^\dagger} \tilde{p}_s (c_s^i - \bar{c}_s^i) ds \right]$ is a $\tilde{P}^{\mathbf{B}}$ -martingale, by *Martingale Representation Theorem* (see Chapter IV Section 4 in Jacod and Shiryaev [27]) and (6.3), there exists a unique predictable representation $(v^i, m^i) \in (\prod_{j=1}^d \mathcal{L}^2) \times \mathcal{L}^1(\lambda_t(dz) \times dt)$ satisfying

$$\tilde{E}^{\mathbf{B}} \left[\int_0^{T^\dagger} \|v_s^i\|^2 ds \right] < \infty, \quad \tilde{E}^{\mathbf{B}} \left[\int_0^{T^\dagger} \int_{\mathbb{Z}} |m_s^i(z)|^2 \tilde{\lambda}_t^{\mathbf{B}}(dz) ds \right] < \infty,$$

and for every $t \in \mathbf{T}$,

$$\begin{aligned} & \tilde{E}_t^{\mathbf{B}} \left[\int_0^{T^\dagger} \tilde{p}_s (c_s^i - \bar{c}_s^i) ds \right] \\ &= \tilde{E}^{\mathbf{B}} \left[\int_0^{T^\dagger} \tilde{p}_s (c_s^i - \bar{c}_s^i) ds \right] + \int_0^t v_s^i \cdot d\tilde{W}_s^{\mathbf{B}} + \int_0^t \int_{\mathbb{Z}} m_s^i(z) \{ \nu(ds \times dz) - \tilde{\lambda}_s^{\mathbf{B}}(dz) ds \} \\ &= \int_0^t v_s^i \cdot d\tilde{W}_s^{\mathbf{B}} + \int_0^t \int_{\mathbb{Z}} m_s^i(z) \{ \nu(ds \times dz) - \tilde{\lambda}_s^{\mathbf{B}}(dz) ds \}. \end{aligned} \quad (6.4)$$

Since $\mathbf{B} \in \bar{\mathcal{B}}$, by Proposition 1(1) and the proof Proposition 6.9 in Björk, Di Masi, Kabanov, and Runggaldier [10], there exists a pair of sequences $(v_n^i, m_n^i)_{n \in \mathbb{N}} \in \prod_{n \in \mathbb{N}} \left(\prod_{j=1}^d \mathcal{L}^2 \right) \times \mathcal{L}^1(\lambda_t(dz) \times dt)$ such that:

- (1) For every $(\omega, t) \in \Omega \times \mathbf{T}$, $(v_{nt}^i(\omega), m_{nt}^i(\omega, \cdot))$ converges to $(v_t^i(\omega), m_t^i(\omega, \cdot))$ in $\mathbb{R}^d \times \mathbf{L}^2(\mathbb{Z}, \mathcal{Z}, \tilde{\lambda}_t^{\mathbf{B}}(\omega, dz))$ as $n \rightarrow \infty$.
- (2) For every $n \in \mathbb{N}$, there exists $\vartheta_n^{i1} \in \mathbf{M}(\mathbf{T})$ satisfying

$$\tilde{\mathcal{O}}_t^*(\omega) \vartheta_{nt}^{i1}(\omega) = \left(\int_t^{T^\dagger} \tilde{B}_{t-}^T(\omega) v_t^T(\omega) \vartheta_{nt}^{i1}(\omega, dT) \right) = \left(\begin{array}{c} v_{nt}^i(\omega) \\ m_{nt}^i(\omega, \cdot) \end{array} \right)$$

for every $(\omega, t) \in \Omega \times \mathbf{T}$, and

$$\int_0^t \int_s^{T^\dagger} \vartheta_{ns}^{i1}(dT) d\tilde{B}_s^T \in \mathbf{L}^\infty. \quad (6.5)$$

Thus, it follows from (6.4) that for every $t \in \mathbf{T}$,

$$\begin{aligned}
& \int_0^t \int_s^{T^\dagger} \vartheta_{ns}^{i1}(dT) d\tilde{B}_s^T \\
&= \int_0^t \int_s^{T^\dagger} \tilde{B}_s^T v_s^T \vartheta_{ns}^{i1}(dT) \cdot d\tilde{W}_s^{\mathbf{B}} + \int_0^t \int_s^{T^\dagger} \int_{\mathbb{Z}} \tilde{B}_s^T m_s^T(z) \vartheta_{ns}^{i1}(dT) \{ \nu(ds \times dz) - \tilde{\lambda}_s^{\mathbf{B}}(dz) ds \} \\
&= \int_0^t v_{ns}^i \cdot d\tilde{W}_s^{\mathbf{B}} + \int_0^t \int_{\mathbb{Z}} m_{ns}^i(z) \{ \nu(ds \times dz) - \tilde{\lambda}_s^{\mathbf{B}}(dz) ds \} \\
&\quad \rightarrow \int_0^t v_s^i \cdot d\tilde{W}_s^{\mathbf{B}} + \int_0^t \int_{\mathbb{Z}} m_s^i(z) \{ \nu(ds \times dz) - \tilde{\lambda}_s^{\mathbf{B}}(dz) ds \}
\end{aligned} \tag{6.6}$$

in $\mathbf{L}^2(\Omega, \mathcal{F}_t, \tilde{P}^{\mathbf{B}})$ as $n \rightarrow \infty$. Define $(\vartheta_n^{i0})_{n \in \mathbb{N}}$ by

$$\vartheta_{nt}^{i0} = - \int_t^{T^\dagger} \tilde{B}_t^T \vartheta_{nt}^{i1}(dT) + \int_0^t \int_s^{T^\dagger} \vartheta_{ns}^{i1}(dT) d\tilde{B}_s^T + \int_0^t \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \quad \forall (n, t) \in \mathbb{N} \times \mathbf{T}.$$

Substituting this into $\tilde{V}_t^{\mathbf{B}}(\vartheta_n^i) = \vartheta_{nt}^{i0} + \int_t^{T^\dagger} \tilde{B}_t^T \vartheta_{nt}^{i1}(dT)$ yields

$$\tilde{V}_t^{\mathbf{B}}(\vartheta_n^i) = \int_0^t \int_s^{T^\dagger} \vartheta_{ns}^{i1}(dT) d\tilde{B}_s^T + \int_0^t \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \quad \forall (n, t) \in \mathbb{N} \times \mathbf{T}. \tag{6.7}$$

Note that $\tilde{p} = \frac{p}{B} \in \mathbf{L}^\infty$ because $B \geq 1$ μ -a.e. It follows from (6.5) and $\tilde{p}, \bar{c}^i, c^i \in \mathbf{L}^\infty$ that $\tilde{V}^{\mathbf{B}}(\vartheta_n^i) \in \mathbf{L}^\infty$. Hence, (6.7) implies that $\vartheta_n^i \in \underline{\mathcal{Q}}(\tilde{\mathbf{B}})$. Finally, it follows from (6.4), (6.6), and (6.7) that

$$\begin{aligned}
\lim_{n \rightarrow \infty} \tilde{V}_{T^\dagger}^{\mathbf{B}}(\vartheta_n^i) &= \lim_{n \rightarrow \infty} \left\{ \int_0^{T^\dagger} \int_s^{T^\dagger} \vartheta_{ns}^{i1}(dT) d\tilde{B}_s^T + \int_0^{T^\dagger} \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \right\} \\
&= \lim_{n \rightarrow \infty} \left\{ \int_0^{T^\dagger} v_{ns}^i \cdot d\tilde{W}_s^{\mathbf{B}} + \int_0^{T^\dagger} \int_{\mathbb{Z}} m_{ns}^i(z) \{ \nu(ds \times dz) - \tilde{\lambda}_s^{\mathbf{B}}(dz) ds \} \right\} + \int_0^{T^\dagger} \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \\
&= \int_0^{T^\dagger} v_s^i \cdot d\tilde{W}_s^{\mathbf{B}} + \int_0^{T^\dagger} \int_{\mathbb{Z}} m_s^i(z) \{ \nu(ds \times dz) - \tilde{\lambda}_s^{\mathbf{B}}(dz) ds \} + \int_0^{T^\dagger} \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \\
&= \tilde{E}_{T^\dagger}^{\mathbf{B}} \left[\int_0^{T^\dagger} \tilde{p}_s(c_s^i - \bar{c}_s^i) ds \right] + \int_0^{T^\dagger} \tilde{p}_s(\bar{c}_s^i - c_s^i) ds = 0.
\end{aligned} \tag{6.8}$$

Equations (6.7) and (6.8) show $c^i \in \tilde{\mathcal{C}}^i(p, \mathbf{B})$, and therefore $c^i \in \bar{\mathcal{C}}^i(p, \mathbf{B})$.

APPENDIX A. MARKED POINT PROCESS

A double sequence $(s_n, Z_n)_{n \in \mathbb{N}}$ is considered, where s_n is the occurrence time of an n th jump and Z_n is a random variable taking its values on a measurable space $(\mathbb{Z}, \mathcal{Z})$ at time s_n . Define a random counting measure $\nu(dt \times dz)$ by

$$\nu([0, t] \times A) = \sum_{n \in \mathbb{N}} 1_{\{s_n \leq t, Z_n \in A\}} \quad \forall (t, A) \in [0, T^\dagger] \times \mathcal{Z}.$$

This counting measure $\nu(dt \times dz)$ is called the \mathbb{Z} -marked point process.

Let λ be such that

- (1) For every $(\omega, t) \in \Omega \times (0, T^\dagger]$, the set function $\lambda_t(\omega, \cdot)$ is a finite Borel measure on \mathbb{Z} .
- (2) For every $A \in \mathcal{Z}$, the process $\lambda(A)$ is \mathcal{P} -measurable and satisfies $\lambda(A) \in \mathcal{L}^1$.

The marked point process $\nu(dt \times dz)$ is said to have the P -intensity kernel $\lambda_t(dz)$ if and only if the following equation

$$E \left[\int_0^{T^\dagger} Y_s \nu(ds \times A) \right] = E \left[\int_0^{T^\dagger} Y_s \lambda_s(A) ds \right] \quad \forall A \in \mathcal{Z}$$

holds for any nonnegative \mathcal{P} -measurable process Y , then it is said that the marked point process $\nu(dt \times dz)$ has the P -intensity kernel $\lambda_t(dz)$.

Let $\nu(dt \times dz)$ be a \mathbb{Z} -marked point process with the P -intensity kernel $\lambda_t(dz)$. Let H be a $\mathcal{P} \otimes \mathcal{Z}$ -measurable function. It follows that:

- (1) If the following integrability condition

$$E \left[\int_0^{T^\dagger} \int_{\mathbb{Z}} |H_s(z)| \lambda_s(z) ds \right] < \infty$$

holds, then the process $\int_0^t \int_{\mathbb{Z}} H_s(z) \{ \nu(ds \times dz) - \lambda_s(dz) ds \}$ is a P -martingale.

- (2) If $H \in \mathcal{L}^1(\lambda_t(dz) \times dt)$, then the process $\int_0^t \int_{\mathbb{Z}} H_s(z) \{ \nu(ds \times dz) - \lambda_s(dz) ds \}$ is a local P -martingale.

Proof. See p. 235 in Brémaud [13]. □

APPENDIX B. ITO'S FORMULA

Let $X = (X^1, \dots, X^d)'$ be a d -dimensional semimartingale, and g be a real-valued \mathcal{C}^2 function on \mathbb{R}^d . Then $g(X)$ is a semimartingale of the form

$$\begin{aligned} g(X_t) = g(X_0) &+ \sum_{i=1}^d \int_0^t \frac{\partial}{\partial x_i} g(X_{s-}) dX_s^i + \frac{1}{2} \sum_{i=1}^d \sum_{j=1}^d \int_0^t \frac{\partial^2}{\partial x_i \partial x_j} g(X_{s-}) d\langle X^{ic}, X^{jc} \rangle \\ &+ \sum_{0 \leq s \leq t} \left\{ g(X_s) - g(X_{s-}) + \sum_{i=1}^d \frac{\partial}{\partial x_i} g(X_{s-}) \Delta X_s^i \right\} \end{aligned}$$

where X^{ic} is the continuous part of X^{ic} and $\langle X^{ic}, X^{jc} \rangle$ is the quadratic covariation of X^{ic} and X^{jc} .

APPENDIX C. GIRSANOV'S THEOREM

- (1) Let $v \in \prod_{j=1}^d \mathcal{L}^2$ and $m \in \mathcal{L}^1(\lambda_t(dz) \times dt)$. Define a process A by

$$\frac{dA_t}{A_{t-}} = -v_t \cdot dW_t - \int_{\mathbb{Z}} m_t(z) \{ \nu(dt \times dz) - \lambda_t(dz) dt \} \quad \forall t \in [0, T^\dagger)$$

with $A_0 = 1$, and suppose $E[A_{T^\dagger}] = 1$. Then there exists a probability measure \tilde{P} on $(\Omega, \mathcal{F}, \mathbb{F})$ given by the Radon-Nikodym derivative

$$d\tilde{P} = A_{T^\dagger} dP$$

such that:

- (i) The measure \tilde{P} is equivalent to P .
- (ii) The process given by

$$\tilde{W}_t = W_t + \int_0^t v_s ds \quad \forall t \in \mathbf{T}$$

is a \tilde{P} -Wiener process.

- (iii) The marked point process $\nu(dt \times dz)$ has the \tilde{P} -intensity kernel such that

$$\tilde{\lambda}_t(dz) = (1 - m_t(z)) \lambda_t(dz) \quad \forall (t, z) \in \mathbf{T} \times \mathbb{Z}.$$

- (2) Every probability measure equivalent to P has the structure above.

APPENDIX D. PROOFS OF STEPS 1 AND 2 IN PROOF OF PROPOSITION 2

Step 1 – $\tilde{C}^i(p, \mathbf{B}) = \tilde{C}^i(p, \mathbf{B})$ where $p \in \mathbf{L}_{++}^\infty$: First, let $c^i \in \tilde{C}^i(p, \mathbf{B})$. Then it follows that $\lim_{n \rightarrow \infty} \tilde{V}_{T^\dagger}^{\mathbf{B}}(\vartheta_n^i) = (B_t)^{-1} \lim_{n \rightarrow \infty} V_{T^\dagger}^{\mathbf{B}}(\vartheta_n^i) = 0$. Also, applying integration by parts yields for every $(n, t) \in \mathbb{N} \times \mathbf{T}$,

$$\begin{aligned}
\tilde{V}_t^{\mathbf{B}}(\vartheta_n^i) &= \tilde{V}_0^{\mathbf{B}}(\vartheta_n^i) + \int_0^t B_s^{-1} dV_s^{\mathbf{B}}(\vartheta_n^i) + \int_0^t V_s^{\mathbf{B}}(\vartheta_n^i) dB_s^{-1} + \int_0^t d[V_s^{\mathbf{B}}(\vartheta_n^i), B_s^{-1}] \\
&= \int_0^t B_s^{-1} \left\{ \vartheta_{n,s}^{i0} dB_s + \int_s^{T^\dagger} \vartheta_{n,s}^{i1}(dT) dB_s^T + p_s(\bar{c}_s^i - c_s^i) ds \right\} \\
&\quad + \int_0^t \left\{ \vartheta_{n,s}^{i0} B_s + \int_s^{T^\dagger} B_s^T \vartheta_{n,s}^{i1}(dT) \right\} dB_s^{-1} \\
&= \int_0^t \vartheta_{n,s}^{i0} \{ B_s^{-1} dB_s + B_s dB_s^{-1} \} \\
&\quad + \int_0^t \int_s^{T^\dagger} \vartheta_{n,s}^{i1}(dT) \{ B_s^{-1} dB_s^T + B_s^T dB_s^{-1} \} + \int_0^t \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \\
&= \int_0^t \vartheta_{n,s}^{i0} d\tilde{B}_s + \int_0^t \int_s^{T^\dagger} \vartheta_{n,s}^{i1}(dT) d\tilde{B}_s^T + \int_0^t \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \\
&= \int_0^t \int_s^{T^\dagger} \vartheta_{n,s}^{i1}(dT) d\tilde{B}_s^T + \int_0^t \tilde{p}_s(\bar{c}_s^i - c_s^i) ds
\end{aligned} \tag{D.1}$$

where $[X^1, X^2]$ is the optional quadratic covariation of X^1 and X^2 . Therefore, $c^i \in \tilde{C}^i(p, \mathbf{B})$. Next, let $c^i \in \tilde{C}^i(p, \mathbf{B})$. Then in the same way, $c^i \in \tilde{C}^i(p, \mathbf{B})$ is obtained.

Step 2 – Proof of (1)(ii): Let $c^i \in \tilde{C}^i(p, \mathbf{B})$ where $(p, \mathbf{B}) \in \mathbf{L}_{++}^\infty \times \bar{\mathcal{B}}$. First, $c^i \in \tilde{C}^i(p, \mathbf{B})$ follows from Step 1. Define $\pi = \tilde{A}^{\mathbf{B}} p$. Then $\pi \in \mathbf{L}_{++}^\infty$ since $\tilde{A}^{\mathbf{B}} \in \mathbf{L}_{++}^\infty$. Also it follows from $c^i \in \tilde{C}^i(p, \mathbf{B})$ that

$$E \left[\int_0^{T^\dagger} \tilde{p}_s(\bar{c}_s^i - c_s^i) ds \right] = \tilde{E}^{\mathbf{B}} \left[\lim_{n \rightarrow \infty} \tilde{V}_{T^\dagger}^{\mathbf{B}}(\vartheta_n^i) - \lim_{n \rightarrow \infty} \int_0^{T^\dagger} \int_s^{T^\dagger} \vartheta_{n,s}^{i1}(dT) d\tilde{B}_s^T \right] = 0. \tag{D.2}$$

Combining (D.2) with (6.1) yields $E[\int_0^{T^\dagger} \pi_s(\bar{c}_s^i - c_s^i) ds] = 0$, and therefore $c^i \in \mathcal{C}^i(\pi)$.

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