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# Information Exchanges among Firms and Their Welfare Implications (Part Ⅲ) : Private Risks and Oligopoly Models

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# Information Exchanges among Firms and Their Welfare Implications (Part Ⅲ) : Private Risks and Oligopoly Models

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**Abstract** The long series of papers on the information exchanges among firms and their welfare implications contain three parts, namely Part I, Part II and Part III.

In the previous papers, we already discussed Parts I and II. Part I was concerned with the basic dual relations between the Cournot and Bertrand models. Part II dealt with the world of risk and uncertainty, focusing on the Cournot duopoly model with a common demand risk as a starting point. It then explored other types of duopoly models with a common risk.

The purpose of this paper is to discuss more complicated problems such as private risks and oligopoly models. When there exist more than two firms in an industry, the problem of the information exchange among firms becomes more complicated yet more intriguing. It will be seen that as the number of "producers as insiders" rises, the possibility of "consumers as outsiders" gaining their welfare is likely to increase. This is certainly the result which may agree with common sense. Some policy implications of our analysis will also be investigated.

**Keywords** Information exchange・oligopoly・welfare implications・the benefit of consumers as outsiders

## 1. The Case of Private Risks: An Introduction

The long series of survey papers consist of three parts. Part I has mainly discussed the dual relationship between the Cournot and Bertrand duopoly models in the absence of risks. This constitutes the starting point from which all of later discussions on oligopoly and information will effectively emerge.

On the basis of Part I, Part II has focused on various types of duopoly models facing a common risk of demand or cost. In other words, it has exclusively concerned with the situation of a common disturbance in the sense the two firms face the sole common disturbance to their demand/cost functions. Such an environment call be called a "common value problem" in the auction literature. It is worthy of attention, however, that there is another equally important environment named a "private values problem" in the same literature.

This paper represents Part III of the series. It will deal with the situation of idiosyncratic disturbances: there are now two different sources of risks, with each source being associated with one firm. It will be seen that in the case of a common risk, the welfare implications of information sharing are quite sensitive to many factors. Among those factors, the following four are very important: (i) the type of competition (Cournot or Bertrand), (ii) the nature of risk (demand or cost), (iii) the degree and direction of physical and stochastic interdependence among demand or cost parameters, and (iv) the number of participating firms. It will also be argued that our investigation of information pooling sheds new light both on the desirability of trade associations and on the merits or demerits of industrial policies.

The outline of this paper is as follows. The first section will continuously discuss a variety of duopoly models with private risks. The second section will explore more general types of oligopoly models, and carefully investigate the welfare implications of information exchanges among firms. Some concluding remarks will be made in the final section.

### 1.1 The Cournot Duopoly with Private Demand Risks

Let us start our inquiry with the Cournot duopoly model in which each firm faces its own demand risk. As in the case of a common risk, we suppose that there are two Cournot firms——firm 1 and firm 2. We particularly assume that the two demand parameters $\alpha_1$ and $\alpha_2$ are random parameters whose joint distribution $F(\alpha_1, \alpha_2)$ is a common knowledge to both firms. Although this is apparently a simple assumption, we believe that it is the necessary first step we have to take for our theoretical investigation. [1)]

Concerning the joint distribution $F(\alpha_1, \alpha_2)$, it is usually assumed that its regression equations are linear. The bivariate normal distribution represents a distinguished member of such a family. The linearity of regression equation makes our calculations fairly manageable, otherwise we would be entangled in a mathematical jungle, perhaps with no exit in sight.

It is quite convenient to express the information structure of our model in terms of the symbol $\eta_{ij}$ ($i = 1,2$; $j = 1,2$) such that

$\eta_{ij} = 1$ if firm $i$ knows the realized value of $\alpha_j$,

$\eta_{ij} = 0$ otherwise.

Let $\eta = [\eta_{11}\eta_{12}, \eta_{21}\eta_{22}]$. Since each $\eta_{ik}$ takes on either 1 or 0, there are totally $2^4 = 16$ information structures: namely, [00,00]; [10,00], [01,00], [00,10], [00,01]; [11,00], [10,10], [10,01], [01,10], [01,01], [00,11]; [11,10], [11,01], [10,11], [01,11]; and [11,11]. Among these sixteen cases, we will focus on the following three symmetric cases in this paper. [2)]

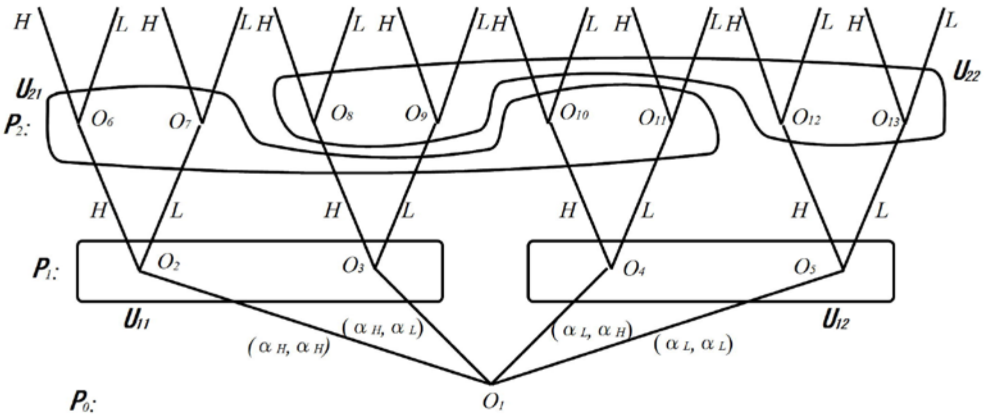
(i) $\eta^O$ = [00,00] : Neither its own demand $\alpha_1$ nor the rival's $\alpha_2$ is known to firm 1, and similarly for firm 2. In short, both firms are ignorant of $\alpha_1$ and $\alpha_2$.

(ii) $\eta^P$ = [10,01] : Firm 1 knows $\alpha_1$, but not $\alpha_2$, whereas firm 2 knows $\alpha_2$ but not $\alpha_1$. In other words, each firm acquires information about its own demand, but not the rival's.

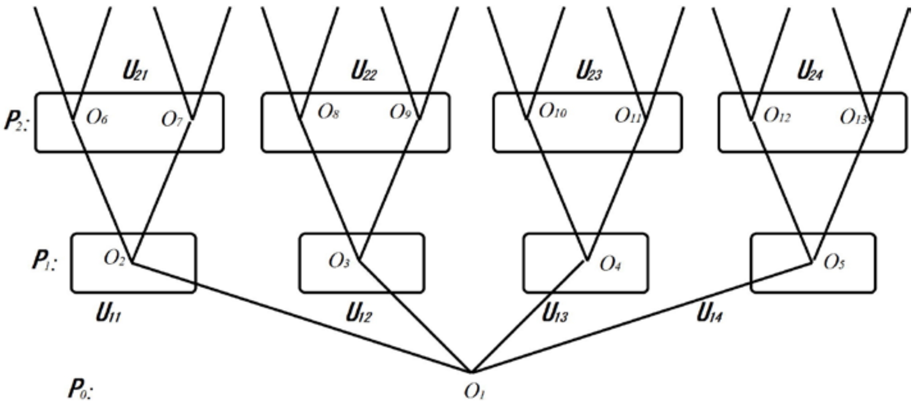
(iii) $\eta^S$ = [11,11]: Both firms 1 and 2 have information about $\alpha_1$ and $\alpha_2$. Namely, they share private information between them.

The first case $\eta^O$ stands for the case of no information and will be served as a reference point. The second case $\eta^P$ shows the case of private information, and the third case $\eta^S$ the case of shared information. Game-theoretic representations will be very instructive in understanding the difference between the last two cases.

In Fig. 1, the two forms of extensive games are depicted as (A) and (B). Although there exist essentially two players on the main stage, $P_1$ and $P_2$, we suppose that $P_0$, the "Nature," is also there behind the scene. We presume that $P_0$ acts like a person and chooses only two alternatives ($\alpha_H$ or $\alpha_L$) for each demand intercept, with a certain combination of parameters. As a result, there are the four strategies taken by $P_0$ : namely, $(\alpha_H, \alpha_H)$, $(\alpha_H, \alpha_L)$, $(\alpha_L, \alpha_H)$ and $(\alpha_L, \alpha_L)$. Let us further assume




(A)　The case of private information:　$\eta^P$




(B)　The case of shared information:　$\eta^S$

**Fig. 1　The Cournot duopoly：　extensive-game presentations**

that the two firms $P_1$ and $P_2$ must select either a high (H) or low (L) level of output.

Let us take a careful look at Panel (A). This represents the case of private information, $\eta^P$. in which $P_1$ knows $\alpha_1$, but not $\alpha_2$, whereas $P_2$ knows $\alpha_2$, but not $\alpha_1$. Therefore, the information structure $P_1$ consists of the two information sets: $U_{11} = \{O_2, O_3\}$ and $U_{12} = \{O_4, O_5\}$. It is noted that $O_2$ and $O_3$ belong to the same set whereas $O_2$ and $O_4$ belong to different sets. In a similar fashion, we can see that there $P_2$ has the two information sets: $U_{21} = \{O_6, O_7, O_{10}, O_{11}\}$ and $U_{22} = \{O_8, O_9, O_{12}, O_{13}\}$. It is noted that for instance, $O_6$ and $O_7$ belong to the same set whereas $O_6$ and $O_8$ belong to different sets.

Now suppose that each firm agrees to exchange its private information with each other. Then we enter into the world of shared information, $\eta^S$, as is seen in Panel (B). Since $P_1$ may now distinguish any one point from the remaining three points, its information structure comprises the four information sets: $U_{11} = \{O_2\}$, $U_{12} = \{O_3\}$, $U_{13} = \{O_4\}$ and $U_{14} = \{O_5\}$. In an analogous way, we are able to see that $U_{21} = \{O_6, O_7\}$, $U_{22} = \{O_8, O_9\}$, $U_{23} = \{O_{10}, O_{11}\}$ and $U_{24} = \{O_{12}, O_{13}\}$. Comparison of Panels (A) and (B) enables us to easily confirm that the game under $\eta^S$ is a refinement of the game under $\eta^P$.

Given one of the information structures, each firm is assumed to play Nash, so that it has no incentive to deviate from an equilibrium whenever it is reached. More formally, the pair $(x_1^O, x_2^O)$ of output strategies is said to be an equilibrium under no information $\eta^O = [00,00]$ if the following conditions are met:

$$x_i^O = \text{Arg Max}_{x_i} \; E[\, \Pi_i (x_i, x_j^O, \alpha_i)\,]. \qquad (i, j = 1,2;\ i \neq j)$$

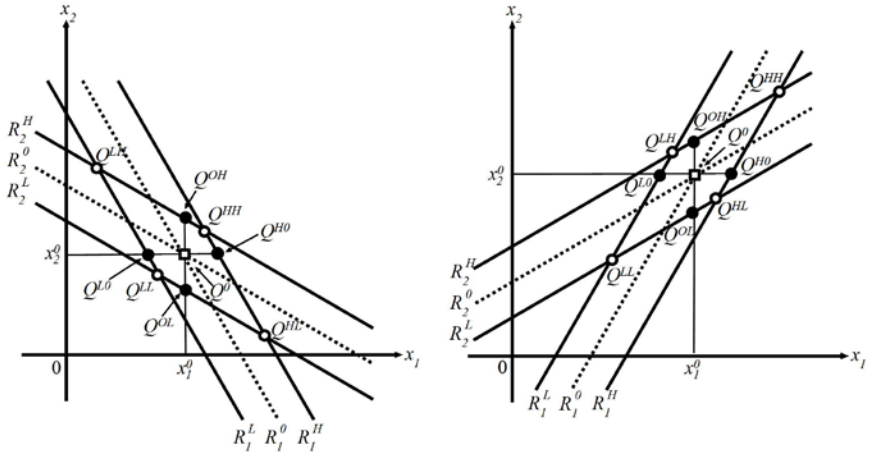
With no information about $\alpha_1$ and $\alpha_2$ available, each firm's optimal strategy must be a routine action in the sense that it does not take account of specific values of $\alpha_1$ and $\alpha_2$.

Now suppose that firm $i$ knows its own demand $\alpha_i$. Then its optimal strategy is no longer a routine action, but an action contingent on the true value of $\alpha$. Therefore, given $\eta^P = [10,01]$, we call the pair $(x_1^P(\alpha_1), x_2^P(\alpha_2))$ of output strategies an equilibrium pair under $\eta^P$ if for each $\alpha_i$, we find

$$x_i^P(\alpha_i) = \text{Arg Max}_{x_i} \; E[\, \Pi_i (x_i, x_j^P(\alpha_j), \alpha_i) \mid \alpha_i)\,], \qquad (i, j = 1,2;\ i \neq j)$$

where the expectation is taken over $\alpha_i$.

If both firms agree to exchange its private information with each other through a trade association or the like, we come to the situation of shared information $\eta^S = [11,11]$. The pair $(x_1^S(\alpha_1, \alpha_2), x_2^S(\alpha_1, \alpha_2))$ of output strategies is called an equilibrium pair under $\eta^S$ if for each $(\alpha_1, \alpha_2)$, we have




(A) $\theta > 0$ and $\rho = 0$ (B) $\theta < 0$ and $\rho = 0$

**Fig. 2 The Cournot duopoly equilibriums under $\eta^P$ and $\eta^S$**

$$x_i^S(\alpha_1, \alpha_2) \;=\; \text{Arg Max}_{x_i}\ \Pi_i(x_i, x_j^S(\alpha_1, \alpha_2),\ \alpha_i). \qquad (i, j = 1,2;\ i \neq j)$$

In this case, each firm's optimal strategy ought to be a contingent action with its contingency depending on both $\alpha_1$ and $\alpha_2$.

In order to intuitively understand the Cournot duopoly equilibriums under $\eta^O$, $\eta^P$ and $\eta^S$, visional illustrations by figures would be very helpful. Let us take a close look at Fig. 2. It is assumed that each demand intercept ($\alpha_i$) must be one of the two equally likely values: High ($H$) or Low ($L$). For an illustrative purpose, we also suppose that the two stochastic variables, $\alpha_1$ and $\alpha_2$, are uncorrelated, namely $\rho = 0$. In Fig. 2, the two parallel lines $R_i^H$ and $R_i^L$ respectively represent firm $i$'s reaction function when its private demand is high and low ($i = 1,2$); and a dotted middle line $R_i^O$ stands for the average of these two reaction functions. [3)]

When neither $\alpha_1$ and $\alpha_2$ is known to both firms, the center $Q^O$, which is indicated by a tiny empty square, stands for an equilibrium, with $(x_1^O, x_2^O)$ being the pair of equilibrium output strategies. If each firm becomes informed of its own demand but not its opponent's, an equilibrium is shown by a set of the four solid points, $Q^{HO}$, $Q^{LO}$, $Q^{OH}$ and $Q^{OL}$, with $(x_1^{HO}, x_1^{LO}; x_2^{OH}, x_2^{OL})$ being the vector of equilibrium output strategies. Let both firms agree to exchange its private information with each other. Then an equilibrium is denoted by a set of the four hollow points , $Q^{HH}$, $Q^{HL}$, $Q^{LH}$ *and* $Q^{LL}$ ; and the vector of equilibrium output strategies is given by $(x_1^{HH}, x_1^{HL}, x_1^{LH}, x_1^{LL}; x_2^{HH}, x_2^{HL}, x_2^{LH}, x_2^{LL})$. Panels (A) and (B) respectively demonstrate the cases of substitutes and complements. Unquestionably, it would be quite interesting for us to graphically see how the information pooling makes a set of four equilibrium points spread out like those panels. However, a graph is no more than a graph, and cannot perfectly be replaced by exact computation. Seeing may be believing, but should be supported by doing !

We are now in a position to do such exact computation. Provided one of the information structures, we are able to first find the equilibrium pair of output strategies, and to proceed to compute each firm's expected profit, expected producer surplus, expected consumer surplus, and expected total surplus. Since the computation is analogous to the one we did for a common risk in Part I, it may be omitted here. We are only content to record the following useful set of welfare equations: [4]

$$\Delta E\Pi_i = -\beta \Delta Var(x_i) - \beta\theta \Delta Cov(x_1, x_2) + \Delta Cov(\alpha_i, x_i), \quad (i = 1,2) \tag{1}$$

$$\Delta EPS = -\beta \Sigma_i \Delta Var(x_i) - 2\beta\theta \Delta Cov(x_1, x_2) + \Sigma_i \Delta Cov(\alpha_i, x_i), \tag{2}$$

$$\Delta ECS = (\beta/2) \Sigma_i \Delta Var(x_i) + \beta\theta \Delta Cov(x_1, x_2), \tag{3}$$

$$\Delta EPS = -(\beta/2) \Sigma_i \Delta Var(x_i) - \beta\theta \Delta Cov(x_1, x_2) + \Sigma_i \Delta Cov(\alpha_i, x_i). \tag{4}$$

Let us compare the two systems, namely the above system (1) − (4) for private demand risks, and the previous system (8)−(11) for a common demand risk. Then we are able to find that these two systems are very similar, the only difference being that there are now firm-specific parameters $\alpha_i$ $(i = 1,2)$ instead of a industry-wide parameter $\alpha$. As in the previous situations, there exist the two channels through which the information sharing between the two firms affects the equilibrium values of welfare quantities: the variation and efficiency channels.

A good summary of the welfare effects of information pooling for Cournot duopoly with private demand risks is provided by Table 1. The following shorthand notations are employed here:

**Table 1 The Cournot duopoly with private demand risks ($\alpha_1$, $\alpha_2$)**

| The Welfare Impact | Own Variation | | Cross Variation | Own Efficiency | | Cross Efficiancy | | Total |
|---|---|---|---|---|---|---|---|---|
| | OV1 | OV2 | CV | OE1 | OE2 | CE1 | CE2 | |
| | + | + | − | + | + | 0 | 0 | |
| $\Delta E\Pi_1$ | − | 0 | + | + | 0 | 0 | 0 | + |
| $\Delta E\Pi_2$ | 0 | − | + | 0 | + | 0 | 0 | + |
| $\Delta EPS$ | − | − | + | + | + | 0 | 0 | + |
| $\Delta ECS$ | + | + | − | 0 | 0 | 0 | 0 | − |
| $\Delta ETS$ | − | − | + | + | + | 0 | 0 | + |

Remark. OV1 = $\Delta Var(x_1)$, OV2 = $\Delta Var(x_2)$; CV = $\theta \Delta Cov(x_1, x_2)$;
OE1= $\Delta Cov(\alpha_1, x_1)$, OE2 = $\Delta Cov(\alpha_2, x_2)$;
CE1 = $\Delta Cov(\alpha_1, x_2)$, CE2 = $\Delta Cov(\alpha_2, x_1)$.

OV1 = $\Delta Var(x_1)$ = an increment in the variance of $x_1$,
OV2 = $\Delta Var(x_2)$ = an increment in the variance of $x_2$,
CV = $\theta \Delta Cov(x_1, x_2)$ = the product of the substitution coefficient $\theta$ and an increment in the covariance of $x_1$ and $x_2$,
OE1= $\Delta Cov(\alpha_1, x_1)$ = an increment in the covariance of $\alpha_1$ and $x_1$,
OE2 = $\Delta Cov(\alpha_2, x_2)$ = an increment in the covariance of $\alpha_2$ and $x_2$,
CE1 = $\Delta Cov(\alpha_1, x_2)$ = an increment in the covariance of $\alpha_1$ and $x_2$, ,
CE2 = $\Delta Cov(\alpha_2, x_1)$ = an increment in the covariance of $\alpha_2$ and $x_1$,

Let us compare the two tables: Table 1 of Part III for *private* demand risks, and

Table 3 of Part II for a *common* demand risk. Then we immediately see that a mosaic-type diagram enchased with plus, minus and zero signs becomes much simpler in the sense that only one sign is attached to each block regardless of the value of $\theta$. The reason for it is that the transmission of information between the two firms is now "a two-way street" instead of "a one-way street," and thus both firms may be treated very symmetrically. Surely, symmetry makes everything simple and beautiful!

By taking a close look at Table 1, we are able to obtain the following welfare results:

(i) The exchange of private demand information between the two Cournot firms makes each firm's production activity more responsive to a change in demand, so that it increases the variance of each output (the own variation effect). This yields a fall in expected producer surplus as well as a rise in expected consumer surplus.

(ii) The information sharing has a tendency to reinforce the degree of (negative or positive) interaction between the strategies of the two firms (the cross variation effect). Since the reaction curves of firms are negatively (or positively) sloped whenever goods are substitutes (or complements) as is clearly seen in Fig. 1 , the information pooling always increases the product of $\theta$ and $(-Cov(x_1, x_2))$. The greater the strategic interaction between both firms, the more advantageous is the position of "producers as insiders" and the more disadvantageous the position of "consumers as outsiders."

(iii) The information pooling contributes to the efficiency allocation of resources (the efficiency effect). In fact, it increases the value of $Cov(\alpha_i, x_i)$, meaning that the firm facing greater (or smaller) demand is likely to have a larger market share. A better correspondence between demands and outputs means an additional gain in the welfare of producers. It is noted here that consumers are not *directly* affected by such reallocation, even if it could be *indirectly* influenced through corresponding changes in outputs.

(iv) The last column indicates the total welfare impact which combines the own and cross variation effects and the efficiency effects. The information sharing between the firms increases the producer welfare as well as the overall welfare, but decreases the consumer welfare. These implications would clearly agree with common sense.

### 1-2 Other Duopoly Models with Private Risks

Let us continue to assume that firms act as Cournot competitors and thus employ quantities as their strategic variables. Then as we noted Part II, whether private risks are about demands or costs does not matter at all. If we discuss the situation under which a stochastic vector under question is the vector ($\kappa_1$, $\kappa_2$) of cost parameters rather than vector ($\alpha_1$, $\alpha_2$) of demand parameters, we are able to draw a table analogous to Table 1, only the difference being that we must now compute the value of ($-\Delta\ Cov(\kappa_i, x_i)$) instead of that of $\Delta Cov(\alpha_i, x_i)$. Therefore, the welfare results obtained for the case of private *demands* can be applied to the present case of private *costs* with appropriate modifications.

Now, let us turn our attention to the situation under which firms play as Bertrand competitors and thus use prices as their strategic variables. In such a case of Bertrand duopoly, the question of whether private risks are about demands or costs becomes very important, may significantly affect the concluding part of welfare implications of information sharing in oligopoly.

Let us assume that each of Bertrand competitors faces its own demand risk. Specifically, we assume that the demand parameters $\alpha_1$ and $\alpha_2$ are random variables whose joint distribution is a bivariate normal distribution. We are concerned with comparing non-sharing information and sharing information equilibriums on an ex ante basis. Table 2 gives us a summary of such comparison. It is noted there that the following shorthand notations are conveniently used: [5]

OV1 = $\Delta\ Var(p_1)$ = an increment in the variance of $p_1$ ,

OV2 = $\Delta\ Var(p_2)$ = an increment in the variance of $p_2$ ,

CV = $\theta\ \Delta\ Cov(p_1, p_2)$ = the product of the substitution coefficient $\theta$ and an increment in the covariance of $p_1$ and $p_2$ ,

OE1 = $\Delta\ Cov(a_1, p_1)$ = an increment in the covariance of $a_1$ and $p_1$,

OE2 = $\Delta\ Cov(a_2, p_2)$ = an increment in the covariance of $a_2$ and $p_2$,

CE1 = $\Delta\ Cov(a_1, p_2)$ = an increment in the covariance of $a_1$ and $p_2$ ,

CE2 = $\Delta$ Cov ($a_2$, $p_1$) = an increment in the covariance of $a_2$ and $p_1$ .

A careful look at Table 2 enables us obtain the following results concerning the welfare impact of information pooling through variation and efficiency channels: [6]

(i) If the Bertrand firms to agree to exchange private demand information with each other, then each firm's price level becomes more responsive to a change in its private demand (the own variation effect). As a result, expected producer surplus and expected total surplus fall while expected consumer rises.

(ii) The information pooling has an effect of reinforcing the strategic interaction between the two firms (the cross variation effect). The greater such an interaction, the stronger will be the position of producers, and thus the weaker will be the position of consumers.

**Table 2 The Bertrand Duopoly with private demand risks ( $\alpha_1$ , $\alpha_2$ )**

| The Welfare Impact | Own Variation | | Cross Variation | Own Efficiency | | Cross Efficiancy | | Total |
|---|---|---|---|---|---|---|---|---|
| | OV1 | OV2 | CV | OE1 | OE2 | CE1 | CE2 | |
| | + | + | + | + | + | 0 | 0 | |
| $\Delta E\Pi_1$ | − | 0 | + | + | 0 | 0 | 0 | + |
| $\Delta E\Pi_2$ | 0 | − | + | 0 | + | 0 | 0 | + |
| $\Delta EPS$ | − | − | + | + | + | 0 | 0 | + |
| $\Delta ECS$ | + | + | − | − | − | 0 | 0 | − |
| $\Delta ETS$ | − | − | + | 0 | 0 | 0 | 0 | + |

Remark. OV1 = $\Delta\, Var(p_1)$ , OV2 = $\Delta\, Var(p_2)$ ; CV = $\theta\ \Delta\, Cov(p_1, p_2)$ ;
OE1 = $\Delta\, Cov(a_1, p_1)$, OE2 = $\Delta\, Cov(a_2, p_2)$ ;
CE1 = $\Delta\, Cov(a_1, p_2)$, CE2 = $\Delta\, Cov(a_2, p_1)$ .

(iii) A better correspondence between demands and prices is now possible by the information exchange between the two Bertrand firms (the own efficiency effect). This is not only beneficial to producers, but is now *definitely harmful* to consumers; which is a new feature of the Bertrand model with private demand risks. This may be contrasted with the previous Cournot world in which the own efficiency effect is not working for or against the interest of consumers. .

(iv) In order to investigate the welfare implications of the information sharing, we must take into consideration those three effects mentioned above. In so doing, we can take advantage of the dual relationship between the Bertrand equilibrium with substitutes (or complements) and the Cournot equilibrium with complements (or substitutes). Such a duality can be confirmed by comparing the sign pattern of $\Delta E\Pi_1$,

$\Delta E\Pi_2$ and $\Delta EPS$ in Table 2 and the corresponding sign pattern in Table 1: in fact, these two patterns are the same.

(v) In a sharp contrast to the Cournot case, however, there emerges a new sign pattern for the welfare impact on *ECS* and *ETS* through the own efficiency channel. It is noted that a gain in *EPS* and a loss in *ECS* via this route are just counterbalanced, so that *ETS* remains unaffected.

(vi) In spite of the appearance of the own efficiency effect on the part of consumers, it is remarkable to see that the total welfare impact of the private demand information sharing between the Bertrand firms is the same as the one between the Cournot firms. As in the Cournot case, the information pooling increases the welfare of producers and the total welfare, but it decreases the welfare of consumers.

Finally, let us discuss the situation under which the Bertrand firms are subject to private cost risks. Among the four cases of private risks, this constitutes the most delicate case in order to derive the welfare results. If we carry out our task of computation, we will be able to obtain Table 3 which summarizes the final results. The following shorthand notations are employed here: [7)]

OV1 = $\Delta\, Var(p_1)$ = an increment in the variance of $p_1$,
OV2 = $\Delta\, Var(p_2)$ = an increment in the variance of $p_2$,
CV = $\theta\, \Delta\, Cov(p_1, p_2)$ = the product of the substitution coefficient $\theta$ and
an increment in the covariance of $p_1$ and $p_2$
OE1 = $\Delta\, Cov(\kappa_1, p_1)$ = an increment in the covariance of $\kappa_1$ and $p_1$,
OE2 = $\Delta\, Cov(\kappa_2, p_2)$ = an increment in the covariance of $\kappa_2$ and $p_2$,
CE1 = $\Delta\, \mathrm{Cov}(\kappa_1, p_2)$ = an increment in the covariance of $\kappa_1$ and $p_2$,
CE2 = $\Delta\, \mathrm{Cov}(\kappa_2, p_1)$ = an increment in the covariance of $\kappa_2$ and $p_1$.

Let us have a very careful look at Table 3. Then we are able to have the following welfare results for this case:

(i) As in the previous cases, the pooling of private cost information between the Bertrand firms tends to increase the variance of each firm's price (the own variation effect) and to strengthen the degree of interaction between the two prices (the cross variation effect).

(ii) The own variation effect contributes negatively to the welfare of producers and the whole society, and positively to the welfare of consumers. It is interesting to

Table 3 The Bertrand duopoly with private cost risks ($\kappa_1$ , $\kappa_2$)

| The Welfare Impact | Own Variation | | Cross Variation | Own Efficiency | | Cross Efficiency | | Total |
|---|---|---|---|---|---|---|---|---|
| | OV1 | OV2 | CV | OE1 | OE2 | CE1 | CE2 | |
| | + | + | + | + | + | + | + | |
| $\Delta E\Pi_1$ | − | 0 | + | + | 0 | − | 0 | ±(*) |
| $\Delta E\Pi_2$ | 0 | − | + | 0 | + | 0 | − | ±(*) |
| $\Delta EPS$ | − | − | + | + | + | − | − | ±(*) |
| $\Delta ECS$ | + | + | − | 0 | 0 | 0 | 0 | − |
| $\Delta ETS$ | − | − | + | + | + | − | − | − |

Remark. (*) $\Delta E\Pi_i$ (i=1,2), $\Delta EPS \gtreqless 0 \Leftrightarrow \theta\rho \gtreqless \dfrac{4-3\theta^2}{2(2-\theta^2)}$

OV1 = $\Delta Var(p_1)$, OV2 = $\Delta Var(p_2)$; CV = $\theta \Delta Cov(p_1, p_2)$; OE1 = $\Delta Cov(\kappa_1, p_1)$,

OE2 = $\Delta Cov(\kappa_2, p_2)$; CE1 = $\Delta Cov(\kappa_1, p_2)$, CE2 = $\Delta Cov(\kappa_2, p_1)$

see that such cross variation effect has exactly opposite welfare implications from the own variation effect.

(iii) Switching our attention from variation channels to efficiency channels, the information exchange yields an improved correspondence between the cost and price of each firm (the own efficiency effect). Therefore, just as in the case of private demand risks, this has a beneficial effect of the welfare of producers and the whole society. However, contrary to the situation of private demand risks, it has no effect whatever on the welfare of consumers.

(iv) Remarkably, there is another kind of allocation repercussion across firms, which is represented by $Cov(\alpha_i, x_j)$ $(i \neq j)$. It can be shown that if goods are substitutes (or complements) then the information pooling increases (or decreases ) the covariance between the cost of one firm and the price of the other. Such repercussions have a disturbing impact of resource allocation across firms, regardless of the technical substitution between goods. Presence of such cross efficiency effect distinguishes the welfare analysis of the Bertrand duopoly with private cost risks from all other duopoly cases with private risks. In short, the cross allocation is literally the crossing factor that disturbs our welfare analysis !

(v) The final column indicates the total welfare impact taking account of the four effects — the own and cross variation effects and the own and cross efficiency effects. The information sharing may benefit firms in some situations but it may hurt them in other situations, depending on the degree of substitutability, $\theta$, and the degree of correlation, $\rho$. The product ($\theta\rho$) of these two parameters measures the degree of combined interaction between the two firms, joining together the physical and stochastic factors. In general, the cross variation and efficiency effects operate in mutually opposing directions. If, and only if, the combined interaction is large enough (more exactly, $\theta\rho > (4 - 3\theta^2)/[2(2 - \theta^2)]$ ), the cross variation effect would dominate the cross efficiency effect, so that the exchange of private cost information would benefit the participating firms.

This is really a very important point. So let us discuss it in more rigorous ways, both mathematically and graphically. In fact, we are able to obtain the following equation: [8)]

$$\Delta EPS = \frac{2b\sigma^2\theta^2(1-\rho^2)}{(4-\theta^2)^2(2-\theta\rho)^2}\left[2\theta\rho(2-\theta^2)-(4-3\theta^2)\right], \tag{5}$$

First of all, it is noted that this equation tells us that $\Delta$ EPS vanishes whenever $\theta = 0$ or $\rho = \pm 1$.

Next, we easily find the following interesting relation:

$$\Delta EPS \gtreqless 0 \text{ according to whether } \theta\rho \gtreqless \frac{4-3\theta^2}{2(2-\theta^2)}, \tag{6}$$

whence $\Delta EPS$ is positive if the product ($\theta\rho$) is greater than the fractional quantity $(4 - 3\theta^2)/[2(2 - \theta^2)]$.

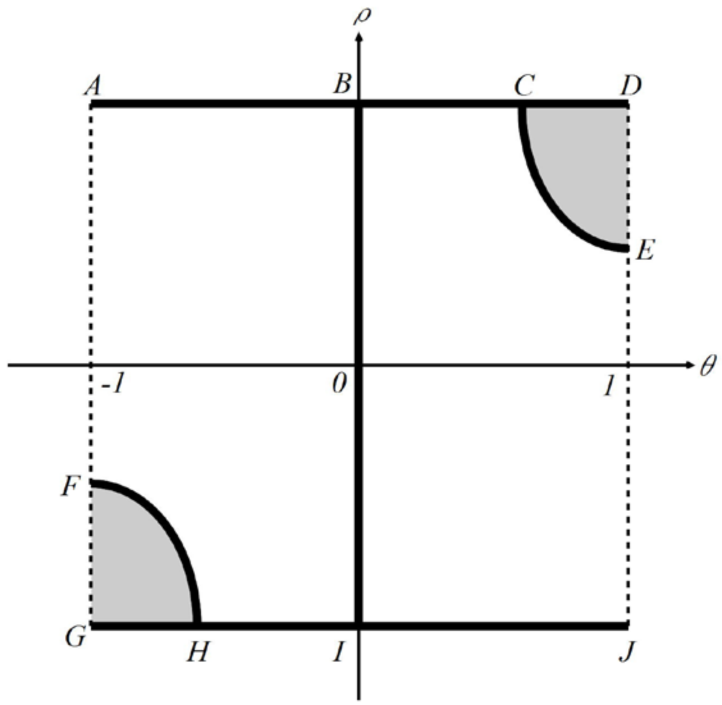



**Fig. 3 The ( $\theta$ , $\rho$ ) diagram for $\Delta EPS$ : the Bertrand duopoly with private cost risks ( $\kappa_1$ , $\kappa_2$ )**

Those results aforementioned may graphically be summarized in Fig. 3 , which shows how the sign of the quantity $\Delta EPS$ is sensitive to the combination of $\theta$ and $\rho$. In the interior of the shaded areas *CDE* and *FGH*, this quantity takes on positive values, so that the information sharing is beneficial to firms. It really vanishes on the curves *CE* and *FH* where the equation $\theta\rho = (4 - 3\theta^2)/[2(2 - \theta^2)]$ holds, and also on the horizontal line segments *AD* and *GJ*, and on the vertical one *BL*, meaning that the firms' gains due to the information pooling is then nil. Moreover, any point in the remaining blank area which is quite large represents the situation under which the information pooling is harmful to firms. More information may mean less benefit !

(vi) Concerning the consumer side, only the cross variation effect is operating against the consumer surplus, whereas there is no (own or cross) efficiency effect present. The result is that the information sharing is detrimental to consumers.

(v) In sharp contrast to all previous cases with private cases, the information pooling is not socially desirable. More significantly, except when the combined interaction is positive and strong, the pooling case must be Pareto inferior to the non-pooling case, This is presumably the worst possible situation we could imagine among all types of duopoly under private risks.

In conclusion, as we have often been told, everything has two sides — a bright side and a dark side. In almost all cases, the information sharing is good for producers, and may also be good for consumers if a side payment from producers to consumers is appropriately accompanied. There is an exception to this general rule, however. We bear in mind that as (v) above indicates, the information pooling may make every member of the society too sensitive to fluctuations, thus possibly making all of them worse off.

## 2. Oligopoly Models

In the above, we have carried out a detailed analysis of welfare implications of the information transmission between firms. We have found that those welfare implications are sensitive to strategic variables (outputs versus prices), the source of risk (demands versus costs), and the type of risks (an industry-wide common risk versus firm-specific private risks). What we are going to do in this section is to show that the implications are also very sensitive to the number of firms in an industry. In particular, we must be very careful of extending the consumer welfare analysis from the simple case of duopoly to the general case of oligopoly with more than two firms. This is because the possibility that the information sharing among firms benefits "consumers as outsiders" would arise and gradually grow as the number of "producers as insiders" increases. As can naturally be expected, such "insider-outsider story" or "spillover story" in oligopoly under risks may emerge and become more complicated in a more general framework.

While we aim to extend our welfare analysis to the general case of oligopoly, we limit our attention to the situation under which Cournot or Bertrand firms face *private cost risks*. In the light of the previous discussions on many types of duopoly, we believe that this case constitutes the most interesting one in the world of oligopoly, and that all other cases may be handled in a more or less analogous fashion.

## 2-1 The Basic Model

The generalization of a duopoly model to an oligopoly model is rather straightforward if each firm is treated symmetrically. On the production side, we have an oligopoly sector with $n$ firms, with firm $i$ producing a differentiated output $x_i$ ($i$ = 1,2,..., $n$), a competitive sector producing a numéraire good $x_o$. Let $p_i$ be the unit price of $x_i$ ($i$ = 1,2,..., $n$).

On the consumption side, we have a continuum of consumers of the same type such that the utility function of the representative consumer is of the following form:

$$U = x_o + \alpha \Sigma_i x_i - (1/2) \beta ( \Sigma_i x_i^2 + \theta \Sigma_i \Sigma_{j \neq i} x_i x_j ), \quad (7)$$

where both $\alpha$ and $\beta$ are positive. Without loss of generality, we assume that $\beta$ is unity.

If the utility function $U$ is to be concave, the following matrix must be positive definite:

$$\Theta = \begin{bmatrix} 1 & \theta & \theta & \dots & \theta \\ \theta & 1 & \theta & \cdots & \theta \\ \vdots & & & \ddots & \vdots \\ \theta & \theta & \theta & \cdots & 1 \end{bmatrix}$$

This implies that the value of $\theta$ must lie between (-1)/($n$-1)and 1 . For instance, $-1 < \theta < 1$ for $n = 2$; $-1/2 < \theta < 1$ for $n = 3$; $-1/3 < \theta < 1$ for $n = 4$; and so on. [9)]

We assume that the consumer maximizes $U$ subject to the budget constraint. The inverse demand functions are then provided by the set of linear equations:

$$p_i = \alpha - x_i - \theta \Sigma_{j \neq i} x_j \qquad (i = 1, ..., n), \quad (8)$$

provided that prices are positive. It is noted that any two goods are substitutes, independent, or complements according to whether $\theta$ is greater, equal to, or less than zero.

If we solve for $x_i$ in (8), we may obtain the direct demand functions as

$$x_i = a - b[1 + (n - 2)\theta]p_i + b\theta\,\Sigma_{j\neq i}\,p_j \quad , \tag{9}$$

provided that outputs are positive. It is easy to see that $a = \alpha/[1 + (n - 1)\theta]$ and $b = 1/(1-\theta)[1 + (n - 1)\theta]$.

As in the previous case of duopoly, we assume that the technology exhibits constant returns to scale, whence firm $i$ has constant unit cost $\kappa_i$ ($i = 1,..., n$). In order to make our computation manageable, let us assume that ($\kappa_1,..., \kappa_n$) is a stochastic vector, whose joint distribution follows the normal distribution with the mean vector ($\mu,..., \mu$) and the covariance matrix $\Sigma$, in which we find

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho & \rho & \dots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \vdots & & & \ddots & \vdots \\ \rho & \rho & \rho & \cdots & 1 \end{bmatrix}$$

Because the matrix $\Sigma$ is positive definite, the value of $\rho$ must lie between $(-1)/(n - 1)$ and 1. It should be noted that the taste matrix $\Theta$ and the covariance matrix $\Sigma$ are both symmetric and take exactly the same form.

It should be pointed out here that the specific form of a normal distribution is not essential for analysis. What we need to have for the sake of computational convenience is the property of linearity of regression equations. Indeed, the normal case meets such requirement and the regression equations can be written in the following way:

$$E(\kappa_j \mid \kappa_i) = \rho(\kappa_i - \mu) + \mu \qquad (i, j = 1,..., n\,;\, i \neq j). \tag{10}$$

Profits of firm $i$ are given by

$$\Pi_i = (p_i - \kappa_i)\,x_i \quad . \qquad (i = 1,..., n\ ) \tag{11}$$

Then the producer surplus is the sum of those profits over $i$: namely, $PS = \Sigma_i \Pi_i$. If the utility function is provided by (7) above, it is not hard to see that the consumer surplus is simply measured by

$$CS = U - x^o - \Sigma_i\, p_i\, x_i = (1/2)\ \Sigma_i\ (\alpha_i - p_i)\ x_i \tag{12}$$

Concerning the information structure of our oligopoly model, we are content to focus our attention to the following two cases as we did for the previous case of duopoly with private risks:

(i) the case of private information in which each firm acquires information about its own cost, but not its rival's;

(ii) the case of shared information where each firm gets information about both its own cost and its rival's cost.

In the following, we wish to systematically compare the private and shared information equilibriums on an *ex ante* basis. We are then able to explore how and to what extent the information exchange agreements among the firms made before costs are realized will affect the welfare of producers, consumers, and the whole society.

## 2-2 The Cournot Oligopoly

To begin with, let us suppose that firms are Cournot-type competitors, with outputs being their strategic variables. Following the same method of computation as we did for the previous case of duopoly, we are able to derive various equilibrium values for the present oligopoly cases of private and shared information. [10)]

For the sake of convenience, let us introduce the following notations:

$$OWN\ VARI = -\beta \Sigma_i Var(x_i), \tag{13}$$

$$CROSS\ VARI = -\beta\theta \Sigma_i \Sigma_{i \neq j} Cov(x_i, x_j), \tag{14}$$

$$OWN\ EFFI = -\Sigma_i Cov(\kappa_i, x_i). \tag{15}$$

Then by making use of (13)-(15), it is a bit lengthy yet straightforward task to obtain the following set of welfare equations: [11)]

$$\Delta EPS = \Delta(OWN\ VARI) + \Delta(CROSS\ VARI) + \Delta(OWN\ EFFI), \tag{16}$$

$$\Delta ECS = -(1/2)\Delta(OWN\ VARI) - (1/2)\Delta(CROSS\ VARI), \tag{17}$$

$$\Delta ETS = (1/2)\Delta(OWN\ VARI) + (1/2)\Delta(CROSS\ VARI) + \Delta(OWN\ EFFI). \tag{18}$$

It would be natural to respectively refer to the terms (*OWN VARI*), (*CROSS VARI*) and (*OWN EFFI*) as the *own variation* term, the *cross variation* term and the *own efficiency* term. The first term (*OWN VARI*) consists of the variance of $x_i$, whereas the

Table 4 The Cournot oligopoly with private cost risks ($\kappa_1, \ldots, \kappa_n$)

| The Welfare Impact | Variation | | Efficiency | | Total |
|---|---|---|---|---|---|
| | OWN | CROSS | OWN | CROSS | |
| $\Delta EPS$ | − | + | + | 0 | + |
| $\Delta ECS$ | + | − | 0 | 0 | ? $\begin{cases} -(n<9) \\ \pm(n\geq 10) \end{cases}$ |
| $\Delta ETS$ | − | + | + | 0 | + |

second term (*CROSS VARI*) comprises the covariance of $x_i$ and $x_j$ $(i \neq j)$ and the degree of technical substitution between them. These two are related to the variation side of firms' strategic variables. In contrast, the third term (*OWN EFFI*) is associated with the covariance between the cost and output of each firm, shedding light on the efficiency side of firms in an industry.

Table 4 summarizes the welfare impact of the information exchange between firms via variation and efficiency channels. This table can be regarded as a generalization of Table 1 to the present case of Cournot oligopoly. This is because Table 1 is applicable not only to the Cournot *duopoly* case with private *demand* risks, but also to the one with private *cost* risks.

Interestingly enough, we are able to draw several welfare implications of the information sharing among Cournot firms from Table 4.

(i) First of all, let us look at this table vertically from top to down. Then we can immediately see in which direction the welfare of producers, consumers and the whole society is influenced through each given channel. Next. let us look at the table horizontally from left to right. Then we may understand how the welfare of producers, consumers or the whole society must change through variation and efficiency channels.

By taking a look at Table 4 either vertically or horizontally, we can see that there is a general tendency that a minus sign is possibly followed by a plus sign which is in turn possibly followed by a minus sign ... Such a mixed sequence of minus and plus signs makes our welfare analysis considerably complicated yet extremely interesting.

(ii) The last column teaches us the total welfare impact of the information sharing among firms, taking account of many opposing effects working on the variation and efficiency sides. First, the information pooling tends to increase the welfare of producers, regardless of the number of firms in an industry. Second, it has a tendency to improve the overall welfare as well, meaning that information is good for the society. These results are the same as those obtained for the simple case of duopoly.

(iii) When we turn to the welfare impact for consumers, the situation changes drastically and becomes much more intricate in the sense that the impact is quite sensitive to the number of firms. The quantity $\Delta EPS$ may move in either direction, depending on the number of producers.

Since there is no efficiency effect term present on the consumer side, the direction of change is determined by the relative strength of the own and cross variation effects. If there are a few firms (more exactly, less than nine firms for our model), then the information sharing increases the variance of each firm's output so much that the own variation effect overpowers the cross variation effect. It is noted that this result is obtained regardless of the degree of substitutability, $\theta$, and the direction of correlation, $\rho$. If, however, there are many firms (at least as many as ten firms for our model), then the power of the cross effect is weakened and thus the position of consumers as outsiders is relatively strengthened. As a result, the information pooling among firms may even be beneficial to consumers as well. [12)]

Among those three welfare results (i), (ii) and (iii), the third result is of the greatest importance. This is because it shows the possibility that the information situation is Pareto superior to the non-pooling situation whenever the number of participating firms is large enough. In order to understand this point more precisely, let us draw Fig. 4. This figure indicates very specifically how the sign of the quantity $\Delta ECS$ is sensitive to the values of $\theta$ and $\rho$ when $n$ takes on four values: namely, $n = 1, 10, 20$, and 50.

In the interior of the shaded area in Fig. 4, the quantity $\Delta ECS$ takes on positive values, meaning that the information sharing among firms benefits consumers in terms of expected consumer surplus.

Now let us a close look on the thick lines in which $\theta = 0$ or $\rho = 1, (-1)/(n-1)$, and the thick curves $HnMnKn$ ($n = 10, 20, 50$). The latter curves correspond to those in which the pair $(\theta, \rho)$ satisfies the following equation:

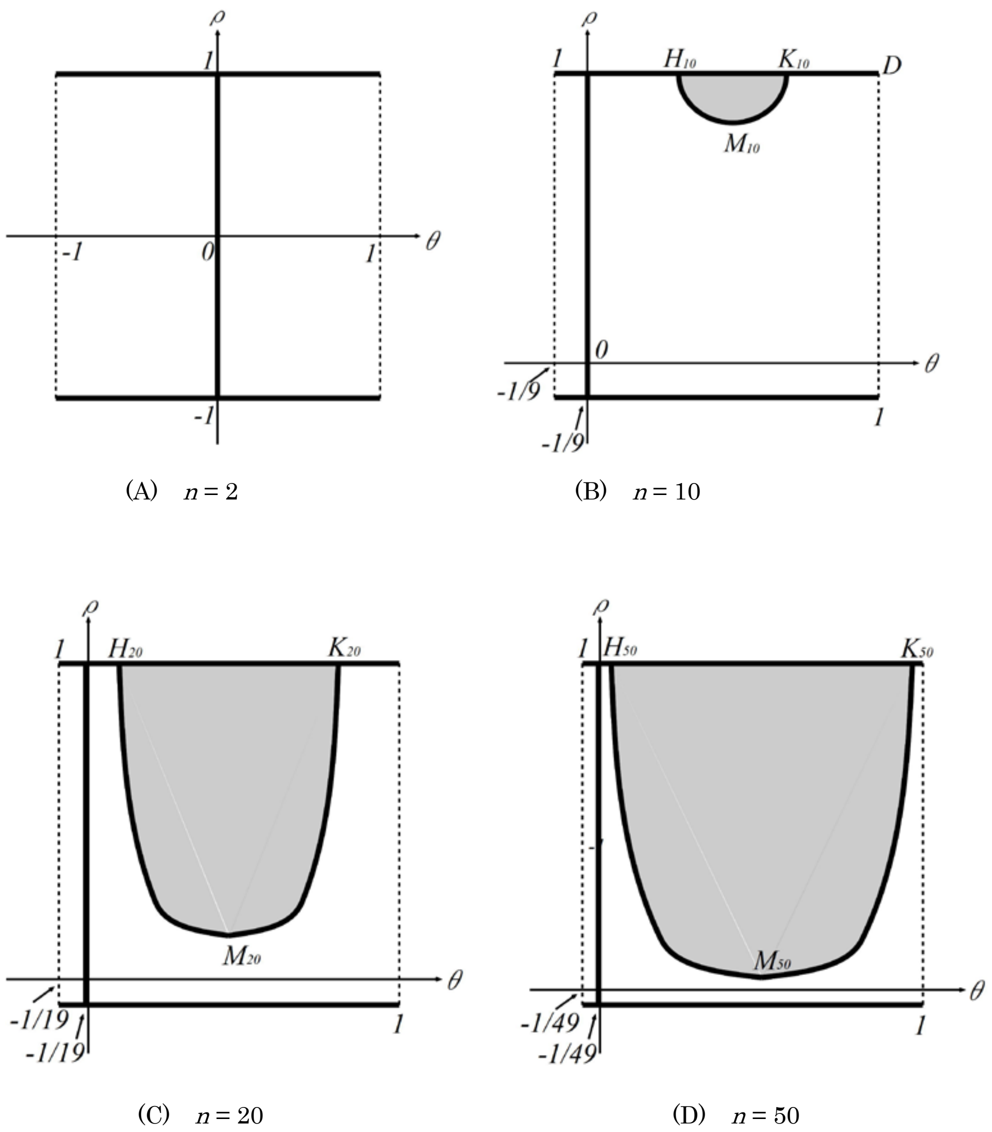



**Fig. 4 The effect of information sharing on consumers: the Cournot oligopoly with private cost risks ( $\kappa_1, \ldots, \kappa_n$ )**

$$\rho = [4 + (n-1)\theta^2] \,/\, \theta^2(n-1)[(n-2) - (n-1)\theta]. \qquad (19)$$

Interestingly enough, on the solid curves $H_nM_nK_n$, the quantity $\Delta ECS$ vanishes, so that consumers' gains due to the information pooling are just nil.

Any point in the remaining blank area represents the situation under which the information exchange has a harmful effect on consumers. It is noted here that the coordinates of the points $H_n$, $M_n$ and $K_n$ are approximately given as follows:

$H_{10}$ = (0.334, 1), $M_{10}$ = (0.5, 0.794), $K_{10}$ = (0.666, 1) ;
$H_{20}$ = (0.120, 1), $M_{20}$ = (0.5, 0.217), $K_{20}$ = (0.880, 1) ;
$H_{50}$ = (0.043, 1), $M_{50}$ = (0.5, 0.057), $K_{50}$ = (0.957, 1) .

In the simple case of duopoly (i.e., $n$ = 2), there is no shaded area present, whence the information sharing among producers is harmful to consumers as was already discussed above. The tongue-like shaded area appears in the upper middle of the ($\theta$, $\rho$) square only after $n$ =10, and grows very rapidly as $n$ increases. It should be noted here that any tongue-like area in Fig. 4 should not be a symmetric figure although it might appear nearly so. This is because the equation (19) above does not take on a simple quadratic form but rather a more complicated quotient form where the numerator is quadratic but the denominator cubic. [13)]

As we may see in Fig 4, when there are many firms in an industry, the situation under which the information sharing benefits consumers takes place if goods are moderately substitutable and costs are positively correlated. Moreover, when a sufficiently large number of firms exist, a great part of the ($\theta$, $\rho$) square is swallowed by the shaded tongue, meaning that consumers may almost always enjoy the " benefit of a third party" from the information exchange among producers. In other words, a sort of "spill over effect" or "dripping down effect" may be working behind, thus benefiting the outsiders here! [14)]

## 2-3 The Bertrand Oligopoly

We are now in a position to discuss the situation under which firms are Bertrand competitors which employ price levels as their strategic variables. For the purpose of presentation, let us bring in the following notations: [15)]

$$OWN\ VARI = -\ b\,(1+(n\text{-}2)\,\theta)\ \ \Sigma_i\ Var(p_i), \tag{20}$$
$$CROSS\ VARI = b\,\theta\ \Sigma_i \Sigma_{i\neq j}\ Cov(p_i, p_j)\ , \tag{21}$$
$$OWN\ EFFI = b\,(1+(n\text{-}2)\,\theta)\ \ \Sigma_i\ Cov(\kappa_i, p_i)\ , \tag{22}$$
$$CROSS\ EFFI = -\ b\,\theta\ \Sigma_i \Sigma_{i\neq j}\ Cov(\kappa_i, p_j)\ . \tag{23}$$

Then we are able to derive the following set of welfare equations:

$$\Delta EPS = \Delta(OWN\ VARI) + \Delta(CROSS\ VARI) + \Delta(OWN\ EFFI) + \Delta(CROSS\ EFFI) \quad , \tag{24}$$

$$\Delta ECS = -(1/2)\Delta(OWN\ VARI) - (1/2)\Delta(CROSS\ VARI), \tag{25}$$

$$\Delta ETS = (1/2)\Delta(OWN\ VARI) + (1/2)\Delta(CROSS\ VARI) + \Delta(OWN\ EFFI) + \Delta(CROSS\ EFFI) \,. \tag{26}$$

If we compare Eqs. (20)-(23) with Eqs. (13)-(15), we can see that there is now a cross efficiency term (*CROSS EFFI*) associating $\kappa_i$ with $p_j$ $(i \neq j)$. The presence of a new cross term is expected to make our welfare analysis of the Bertrand oligopoly clearly different from the one of the Cournot oligopoly. In fact, as is seen in Table 5, the own and cross efficiency effects are working in opposite directions in the determination of $\Delta EPS$. If Bertrand firms agree to exchange their private information with each other, we can expect to have an allocation benefit arising from a better correspondence between the cost and price of each firm because the firm with a higher (or lower) cost is likely to have a smaller (or larger) market share. In the case of Bertrand competition, however, there is another kind of allocation repercussion across firms. If goods are substitutes (or complements), then the information pooling increases (or decreases) the covariance between the cost of one firm and the price of any other firm. Such a repercussion has a disturbing impact on resource allocation across firms, regardless of technical substitution between goods.

The following welfare implications of the information pooling among the Bertrand competitors may be drawn from Table 5:

(i) We can look at Table 5 either horizontally or vertically. In either way, there exist no sequences of simple sign pattern such as plus-sign only or minus-sign only: indeed, both plus and minus signs appear in every sequence. As in the case of Cournot competition, there are both variation and efficiency channels through which the information pooling among firms affects the welfare of any member of the society. The variation channel consists of two sub-channels——own and cross sub-channels. Moreover, unlike the Cournot situation, the efficiency channel is now decomposed into own and cross sub-channels as well. It is quite interesting to see that, those two sub-channels on the efficiency side are working in opposing directions. Like we are walking at crossroads, we must be very careful of any kind of crossing effects in the academic world!

Table 5 The Bertrand oligopoly with private cost risks ($\kappa_1, \ldots, \kappa_n$)

| The Welfare Impact | Variation | | Efficiency | | Total |
|---|---|---|---|---|---|
| | OWN | CROSS | OWN | CROSS | |
| $\Delta EPS$ | − | + | + | − | ? possibly positive for all $n$ |
| $\Delta ECS$ | + | − | 0 | 0 | ? $\begin{cases} -\ (n<9) \\ \pm\ (n\geq 10) \end{cases}$ |
| $\Delta ETS$ | − | + | + | − | − |

(ii) The total impact taking care of all possible channels is shown in the last column. First of all, the information sharing has a general tendency of decreasing expected total surplus. Therefore, in sharp contrast to the Cournot situation, more information means less benefit. This is particularly so because the presence of the cross efficiency term has a strong effect of pulling down the level of welfare. As the title of a famous movie teaches us, "the man who knows too much" might be trapped in a dangerous situation!

(iii) It is seen in (ii) that the "economic pie" gets really smaller by the information pooling. Here comes a more serious question. This is the question of how a smaller pie should be distributed between producers and consumers. The information sharing may make producers worse off or better off, and it may hurt or benefit consumers, depending upon the relative strength of the following three factors:

① the degree of technical substitution between any two goods, $\theta$;

② the value of stochastic correlation of any two cost, $\rho$ ;

③ the number of Bertrand firms, $n$ .

Let us take a careful look at Table 5 again. On the one hand, for *any* finite number of firms, the information pooling may benefit producers. On the other hand, there exists a critical value of the number of firms: below that value, information is harmful to consumers, but beyond it, information becomes beneficial.

(iv) There is no possibility at all that both *EPS* and *ECS* simultaneously rise through the information exchange. This is because *ETC* as the sum of *EPS* and *ECS* must decline.

In our opinion, the welfare results (1) - (iv) are all intriguing. We believe, however, that among those four, the result (iii) is the most remarkable one, and thus requires a more detailed investigation. The question at issue is how a change in *EPS* or *ECS* is related to the values of $n$ , $\theta$ and $\rho$ . Fig. 5 gives us an answer to the question when $n$ = 2, 10, 20, 50.

In Fig 5, there are two shaded areas in the ( $\theta$, $\rho$) diagram: That is, the shaded area located in the upper right corner and the one of the lower left corner. The interiors of those two areas indicate the set of combination of $\theta$ and $\rho$ for which the information sharing between producers has a positive effect on themselves.

Fig. 5 contains there different kinds of thick lines or curves. The first kind is the vertical thick line in which $\theta = 0$ , and the second kind, a pair of the horizontal thick lines where $\rho = 1$ and $\rho = (-1)/(n-1)$. The third kind corresponds to a pair of the thick curves, one in the upper right and another in the lower left, where the pair ( $\theta$, $\rho$) satisfies the following equation:

$$\rho = \frac{[1+(n-2)\theta]\{4(1-\theta)[1+(n-1)\theta]+(n-1)\theta^2\}}{(n-1)\theta\ \{(1-\theta)[2+(2n-3)\theta]+[1+(n-1)\theta][2+(n-3)\theta]\}} \tag{27}$$

Every point on those three thick lines or curves stands for the case in which the information pooling has no influence at all on producers. The remaining blank area shows the situation under which the information exchange has a negative effect on producers.

The coordinates of the point $P_n$, $Q_n$, $R_n$ and $S_n$ may approximately be shown in the following way:

$P_2$ = (0.7808, 1 ), $Q_2$ = ( 1, 0.5), $R_2$ = (− 0.7808, −1), $S_2$ = (−1, − 0.5);
$P_{10}$ =(0.1929, 1 ), $Q_{10}$ = ( 1, 0.1), $R_{10}$ = (− 0.1105, −1), $S_{10}$ = (−1, − 0.1);
$P_{20}$= (0.0983, 1 ), $Q_{20}$ = ( 1, 0.05), $R_{20}$ = (− 0.0525, −1), $S_{20}$ = (−1, − 0.05);

$P_{50}$= (0.0397,　1 ),　$Q_{20}$ = ( 1,　0.02),　$R_{20}$ = (− 0.02040, −1),　$S_{20}$ = (−1, − 0.02).

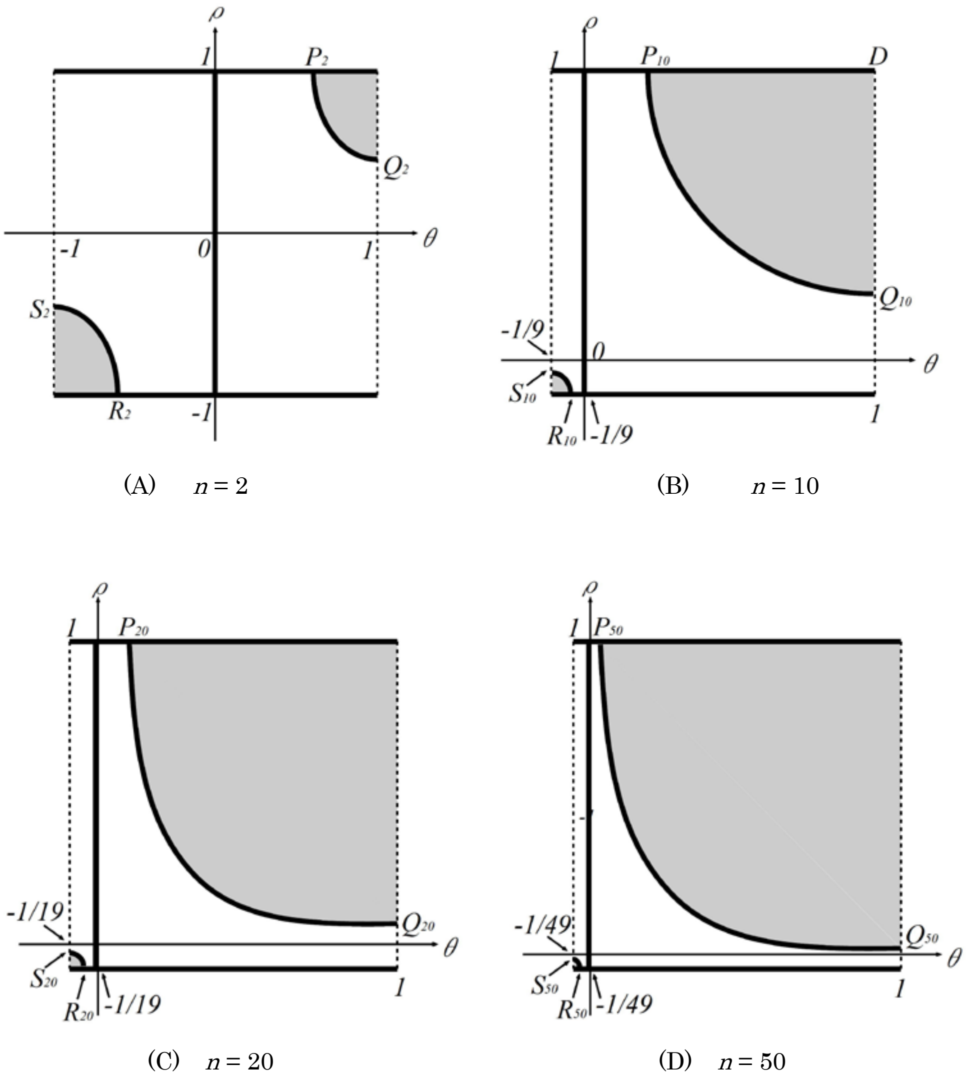



**Fig.5　The Effect of information sharing on producers:　the Bertrand oligopoly**

Remark. Since the case with $n = 2$　means duopoly, Chart (A) here must be identical to Fig. 3 above.

It is noted that a pair of shaded areas appear already when $n$ = 2, and that the shaded area in the positive quadrant gets larger and the one in the negative quadrant gets smaller as $n$ increases.　In other words, even when there are only two firms in an industry, the exchange of cost information between them may benefit firms either if goods are strong substitutes and costs are positively correlated or if goods are strong complements and costs are negatively correlated. When there are a larger number of firms, a great portion of the ($\theta$, $\rho$) square is swamped by the fan-like shaded areas. Since the total welfare never increases by the information pooling, this shows an increasing possibility of the conflict of interests between producers and consumers.

In general, if there is an information exchange among Bertrand firms, it is likely to put consumers in a less advantageous position. However, the possibility that it may even be beneficial to consumers cannot be excluded. Whether consumers suffer from outsiders or enjoy the benefit of a third party depends on the three factors again: Namely, $\theta$, $\rho$ and $n$. A brief summary regarding this point will be recorded down below without a detailed proof:

(i) If $\theta > 0$, or if $\rho \leqq 0$, then it can be shown that $\Delta ECS < 0$.

(ii) Besides, whenever $n$ is at most as great as nine, $\Delta ECS$ is also negative.

(iii) In our Bertrand model, only when $n$ is at least as great as ten, there appears a combination of $\theta$ and $\rho$ for which $\Delta ECS$ becomes positive. This is due to the fact that the own variation effect gets stronger and the cross variation effect gets weaker as the number of firms gets larger.

In order to take an example, let us consider the case of $\theta = (-1)/n$. Then it is not hard to obtain the following relationship:

$$\Delta ECS > 0 \quad \Leftrightarrow \quad \rho > \frac{2(n+15)}{(n-1)(n-3)} \tag{28}$$

Let us put $\rho^* = 2(n+15)/ (n-1)(n-3)$. Clearly, the amount of this $\rho^*$ represents a critical value on which consumers can enjoy the benefit of a third party. Specifically speaking, se can show the following results:

(i) For $n = 10$, $\rho^* = 50/63 \fallingdotseq 0.7937$ ;

(ii) For $n = 20$, $\rho^* = 70/323 \fallingdotseq 0.2167$ ;

(iii) For $n = 50$, $\rho^* = 130/2303 \fallingdotseq 0.05645$ .

These results clearly demonstrate the possibility that the information pooling among "producers as insiders" benefits "consumers as outsiders" becomes greater as the number of producers becomes greater. As social psychology teaches us, the "insider-outsider" story is both complicated and intriguing !

## 3 Concluding Remarks

It is true that this paper is mainly a theory-oriented work. We believe, however, the welfare results obtained so far are expected to have interesting policy implications regarding the effectiveness and limits of information-sharing agreements among firms.

On the one hand, trade associations may be regarded as those nice examples of the institutions in which the information transmission between firms takes place and is properly organized. Any kind of information-sharing agreement is seen to be double-edged: it may strengthen the power of coalition among firms, whereas it enhance the efficiency of resource allocation across firms. In the light of those mutually opposing welfare effects working behind, antitrust authorities in the U.S. have not taken a clear-cut position on the agreements on information pooling. This is admittedly an ambiguous and even confusing fact. [16)]

On the other hand, there are many economists who think that, among a set of industrial policies undertaken by the Japanese authorities, those policies which explicitly or implicitly contribute to the improvement of flows of industrial information have been very successful measures. In short, there are some industrial policies which may be effective in Japan but may not be so in the U.S. This may in part reflect cultural and historical differences between the two countries. [17)]

It is strongly hoped that our theoretical investigation of the information transmission among firms sheds new light both on the effectiveness or limitations of trade associations and on the merits or demerits of industrial policies. It seems that we can derive the following set of policy implications from our theoretical analysis conducted above.

(i) The most important thing we must bear in mind is that the welfare implications of the information transmission among firms are sensitive to many factors. They are enumerated as follows: the type of competition (Cournot or Bertrand), the nature of risk (demand or cost), the character of information (a common value or private values), and the number of participating firms (two, three or any finite number). Even if every one of those factors is specified, the welfare results may as well depend on the degree of technical substitution between any two outputs and the value and direction of stochastic interdependence between any two demand or cost parameters.

(ii) It goes without saying that the policy implications are closely linked to the welfare results, given a certain criterion of social welfare. Even if we regard the expected sum of the producer and consumer surpluses as a good measure of social welfare, we should be very careful of which kind of oligopoly we are discussing, and of which sort of risk and information we are talking about. As can naturally be expected, different assumptions on oligopoly, risk and information are likely to lead to different policy implications.

(iii) In order to have a clear-cut conclusion on the merits or demerits of the information transmission agreements, it is first necessary to determine whether the risk each firm is confronted with is of a common industry-wide type or a firm-specific type. Suppose that every Cournot or Bertrand firm belonging to the same industry is subject to the same demand or cost risk. Then, as our welfare analyses aforementioned have shown, the information flow from one firm to others results in an increase in expected social surplus, with the exception of the case in which firms are Betrand competitors facing a common demand risk and goods are not strong substitutes. Besides, in all those favorable cases, if side payments are permitted between firms and goods are moderately substitutable or complementary, such information transmission is most likely to represent a Pareto improvement in the sense that it makes both producers and consumers better off.

Therefore, except the situation of Betrand oligopoly with a common demand risk, the government authority should pursue a policy which encourages the spreading of information among firms. If such a policy happens to harm consumers although it does increase total surplus, it appears that we are a sort of dilemma, since consumer protection is often regarded as antitrust policy makers as their main objective. It follows that public policies for information transmission should be supplemented with income distribution policies, so that some of the increased social surplus may be shifted to consumers, for instance, through taxes and subsidies.

(iv) The most troublesome case rests with the situation under which firms are Bertrand competitors facing a common demand risk. Unless goods are strong substitutes, the demand transmission among firms has a negative effect on social welfare. In such a case, the authority should be discouraged from engaging in the information transfer.

(v) Let us turn our attention to the more interesting case where each firm faces its own demand risk or cost risk. In the case of such private firm-specific risk, the number of participating firms plays an important role in deciding the effect of the information sharing among producers on the welfare consumers.

Apart from the Bertrand oligopoly with cost risk, any information pooling agreement yields an increase in expected producer surplus and in expected total surplus, whatever the degree of technical substitution and the value of stochastic correlation.

Regarding the effect on consumers, there appears a dividing line between "a few firms" and "many firms." When the number of firms is "small," the information pooling among producers is always harmful to consumers, showing the need of introduction of supplementary income redistribution policies. If, however, the number becomes "large," then the situation would change completely. Then unless goods are homogeneous (which is unlikely in today's business circle), the shared information case is most likely to be Pareto superior to the non-shared information case. This is no doubt the most fortunate case we could have when we ask the authority to interfere information flows in private sectors.

(vi) If firms are Bertrand competitors facing private cost risk, the more information means less social benefit in the sense that the information pooling makes the "economic pie" smaller. This is presumably the most unfortunate situation among possible combinations of oligopoly and risk. Although the authority is not recommended to help diffuse private cost risk across firms, it might do so under the pressure of business circle because the information sharing is likely to increase the share of producers in social surplus if the number of producers is sufficient large. To make the problem even more complicated, there are some other circumstances in which the information pooling among producers may increase the welfare of consumers if the number of firms is "large."

(vii) To sum up, policy implications of an information transmission agreement among firms depends on whether risk is of an industry-wide type or of a firm-specific type, whether information is about demand or cost, and on whether inter-firm competition is of the Cournot quantity type or the Bertrand price type. Moreover, those implications are also sensitive to the degree of technical substitution among goods, the value and direction of stochastic correlation among demand or cost parameters, and the number of participating firms.

The above considerations seem to lead to making a case-by-case analysis quite effective if we have to take much care of adopting a Pareto-improving policy. If, however, we allow for a certain kind of side payment among firms, the scheme of welfare-enhancing policy becomes much simpler. This is due to the fact that unless the oligopoly in question is Bertrand oligopoly with a common demand risk or private cost risks, any government policy of promoting the information flows among firms has an effect of increasing total welfare although it might decrease the welfare certain members of the society. Since the economic pie *per se* gets larger by the information transmission among firms, it is possible to make every member better off if an information-flow-promoting policy is supplemented by a series of income redistribution policies.

On the other hand, there are a limited number of cases in which the information transmission or sharing among firms does indeed hurt total welfare. Those unfortunate cases are only two: namely, the Bertrand oligopoly with a common demand risk and the same-type of oligopoly with private cost risks. Besides. there are more possible cases where the information pooling is harmful to consumers as outsiders if the number of producers as insiders is rather small. What we have learned from our detailed analysis so far is that these "bad cases" may clearly be identified and should be distinguished from many other "good cases." The government agencies should have sharp eyes to select only "good cases" and, if necessary, should supplement policies for information transfer with policies for income redistribution. Needless to say, how much effective these policies really are solely based on the social trust by the people for their democratic government. No good democracy, no good policies !

It should be noted that there remain some limitations in our welfare analysis and many other directions in which the analysis may be further extended.

First of all, we have been working with a simple oligopoly model with explicit functional forms assumed for the utility functions of consumers, the cost functions of producers, and the density functions of stochastic variables. It is our strong belief that simplification is the essence of science and may be justified if it straightforwardly leads us to the heart of the matter.

Second, we have intentionally ignored the problem of risk aversion on the part of producers and/or consumers along with the problem of information cost. Needless to say, we are quite aware of the fact that people in the street tend to avoid any risk , and that information *per se* is sometimes a very expensive good. [17)]

Third, the question of partial information sharing and possible garbling has not been discussed at all in this paper. We know, however, that any kind of partial commitment and any degree of cheating are conceivable in every aspect of people's behaviors. [18)]

These problems aforementioned remain unsolved and will be the target of future research. And finally, we have paid no attention to the leader-follower model of Stackelberg in this paper. Stackelberg competition could employ either quantities or prices as their strategic variables. Besides, in the Stackelberg framework, the risk in question may be of an industry-wide type or a firm-specific type, and the information in question may be about demand or cost parameters. Taking these factors into account, we would have so many Stackelberg models to work with. Then there would be a certain class of circumstances under which a less informed firm is willing to act as a follower, with a more informed firm playing the role of a leader. Such an analysis would threw new light on the long-standing problem of the first mover advantage versus the second mover advantage.

In conclusion, we believe that economists should share any kind of information with each other through oral discussions or written papers or even E-mails, with the strong faith that information is power in our academic circle. Let us recall of the wise maxim followed by J.H. Fabre (1823-1915), a legendary French entomologist: Laboremus !

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## Endnotes

1) As far as the Cournot model is concerned, whether the information in question is about demand or cost does not matter at all. The line of research with the Cournot duopoly under private risks was initiated by Okada (1982) and Sakai (1985) for a homogenous product case, and was extended by Gal-Or (1986), Fried (1984), Sakai (1987), and others to deal with a wider range of product differentiation.

2) All of those sixteen cases were comprehensively discussed by Sakai (1985) for the extreme case of perfect substitutes (viz., $\theta = 1$). While Fried (1984) was a fine piece of work doing the welfare analysis of information sharing within a similar framework, he picked up only nine cases out of those sixteen, and unfortunately failed to consider the welfare impact on consumers and the whole society. It is also noted that in his pioneering work, Okada (1982) limited his attention to barely four cases.

3) For a graphical presentation only, we make the assumption of no correlation here (namely, $\rho = 0$). It is noted, however, that the welfare analysis of this paper can cover the whole range of correlation from minus unity to plus unity (namely, $-1 \leqq \rho \leqq 1$). Since Gal-Or (1986) assumes that goods are substitutes and that stochastic parameters are non-correlated (namely, $\theta > 0$ and $\rho = 0$), her analysis corresponds very well to Panel (A) in Fig. 2.

4) See Sakai (1991a, 91b) for detailed derivations.

5) The Bertrand model with private demand risks was first studied by Sakai (1987). It is unfortunate, however, that the welfare analysis of information sharing was not complete in this earlier paper. It not only failed to investigate the welfare impact on consumers and the whole society, but also neglected the decomposition into variation and efficiency channels.

6) To save the space, we omit those detailed tables which indicate the welfare impact through variation and efficiency channels for the present and following cases. For more detailed explanations, see Sakai (1989).

7) The Betrand duopoly under private cost risks was studied by Gal-Or (1986) for the special case where goods are substitutes and costs are not correlated (i.e., $\theta > 0$ $\rho = 0$). However, the welfare impact on consumers and the whole society was not discussed in her otherwise excellent work. A more complete welfare analysis which allows for complementary goods and also for positively or negatively correlated costs, was independently made by Sakai & Yamato (1989).

8) See Sakai & Yamato (1990).

9) Such a nice symmetric case was investigated by Dixit & Stern (1982) and Friedman (1983) for oligopoly models in the absence of any risks.

10) The problem of information sharing in the Cournot oligopoly has been much concern in the modern theory of oligopoly and industrial organization. Gal-Or (1985), Li (1985), and Shapiro (1986) studied the problem for a very simple case of homogeneous products (namely, $\theta = 1$) whereas Sakai (1988) worked with a more general case of product differentiation ($-1 \leqq \theta \leqq 1$) . There exist another group of papers such as Ponssard (1979), Clarke (1983), and Nalebuff & Zeckhauser (1986) which limited attention on the presence of only one risk, still maintaining the assumption of homogeneous products ( i.e., $\theta = 1$). It is noted that if all private risks are perfectly and positively correlated ( namely, $\rho = 1$), then the case of private risks may boil down to the one of a common risk.

11) For the detailed derivation of these formulas, see Sakai (1988).

12) Such a distinction between "a few" and "many" may be compared with the famous result of Selten (1973) who claims that four are "few" and six are "many". In fact, using his own cartel-making model, Selten has shown that if there are at least as many as six firms in an industry then there emerges the completely new situation: every firm intends to stay out of the cartel and act as an outsider rather than remaining an insider. It seems that for any kind of economic model, there should exist a dividing line between "a few" and "many".

13) Because any tongue-like area in Fig. 4 is not exactly a symmetric figure, the point $M_n$ is near yet not equal to the minimum point of the curve $H_nM_nK_n$ ($n$ = 10, 20, 50). As Eq. (19) above may show, this curve is not a parabola but takes a more complicate shape.

14) When there are a sufficiently large number of firms, our oligopoly framework taken here is presumably close to the monopolistic competition situation of Chamberlin (1933). In his nice work (1987. 88a, 88c), Vives studied incentives to share information and welfare in such large market.

15) For a detailed welfare analysis of the Bertrand oligopoly with private cost risks, see Sakai & Yamato (1988). Vives (1987) discussed a similar problem within the framework of monopolistic competition.

16) For the trade association laws and antitrust laws in the U.S., see Lambo & Shield (1971) and Areeda (1981). Besides, in this connection, Scherer (1980) , Vives (1992, 1999, 2008) are useful papers.

17) For the evaluation of industrial policies in the post-war Japan, see Komiya

(1975) and Suzumura & Okuno (1987).

18) For the effect of risk aversion on the information sharing in oligopoly, see Sakai & Yoshizumi (1991).

19) For the information sharing and welfare in a Stackelberg-type leader-follower model, see Gal-Or (1988), Sakai (1987), Okamura-Shinkai (1987), and others.