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# A Partership Model with Match-specific Externalities

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**Abstract**

This paper introduces matching externalities into a partnership model and examines effects on individual's search behavior, where matching externalities are influences that an existing ill-assorted match can bring down enviousness to single individuals. Given that, I examine how matching externalities (in addition to standard search externalities) affect individual's search strategy and how equilibrium outcome change.

**Keywords:** search; matching; externalities; enviousness

**JEL classification: J12; J64; J69**

*All remaing errors are, of course, my own.

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# 1 Introduction

In a market with search frictions, it costs time and/or money to find a partner. For instance, in a labor market, if an unemployed worker fortunately finds a vacant job, or if a firm with a vacant job fortunately finds an unemployed worker, it can start on production. In a marriage market, if one fortunately finds a partner, one would be married the partner. In such circumstances when a (potential) partner is homogeneous, whether one can find a partner or not depends solely on luck. In consequence, one with no partner would resign to being unemployed (or single), and then one would not have interest in other's partnership. However, when individuals are heterogeneous, one with no partner may be interested in other's partnership. In the concrete, assume that individuals are in different character (for example, appearance, personality, educational level, skills, wealth, ancestry, and so on) and that character is called either "attractive" or "less attractive". In such circumstances matching with an attractive partner requires double good fortune; that is, to find a partner and the partner is attractive. At this time, what does an unemployed (or a single) feel when observes other partnerships? Of course, there are people those who are not intersted in other partnerships, however, some people may be interested in other partnerships. For example, consider an average college student who is seeking employment. If one of his friends who is a high achiever is informally accepted for employment from a major enterprise, he would not hurt his feelings because he would think it is suitable. If his another friend who is also an average student is accepted for employment from a major enterprise, however, he may hurt his feelings because he may think it is ill-assorted (and he is still seeking employment). As another example, consider an ordinary man who wants to have a girlfriend. If he observes a couple of a attractive man and woman, he would think it is well-assorted. If he observes another couple of an ordinary man and an attractive woman, he may feel envy at the ill-assorted partnership.

Earlier literatures have been construct a model which descrives match formation. For example, Burdett and Coles (1999) studies a theory of long-term partnership formation, which is mainly originated from a theory of marriage by Becker (1973, 1974). Partnership formation is called positive assortative matching when character of individuals is positively correlated, whereas it is called negative assortative matching when character of individuals is negatively correlated. Becker (1973) suggests that partnership formation should be positive assortative if character are complements in marriage, while it should be negative assortative if character are substitutes in marriage. Burdett and Coles (1999), which is close to this paper, analyzes the case of non-transferable utility which is gain from matching can not be negotiated, and the case of transferable utility which is gain from matching can be negotiated. Individuals are assumed to be heterogeneous as "goods" and "bads", so there is the ranking of character; all of individuals are willing to match with a good rather than a bad. Mukoyama and Şahin (2009) also constructs a matching model with heterogenous individuals, however, there is no ranking of character; there are two types of individuals, $X$-type and $Y$-type, and the flow output is larger when the pair consists of different types than when the pair consists of same types. That is, it is more beneficial for an $X$-type to match with a $Y$-type whereas it is more beneficial for a $Y$-type to match with an $X$-type.

Many earlier researches as listed off above, there exist standard externalities so-called *search externalities* or *congestion externalities*. In consequence, it costs time and/or money to form a new match. Once the match is formed, individuals those who search a partner do not have contact with a matched individual and the match does not affect single individuals at all [1]. Almost all model takes account of search externalities, however, it does not take accout of a possibility of externalities by existing matches. As mentioned above, existence of enviousness due to other's ill-assorted partnership is an example. I call these effects *matching externalities* in contrast with search externalities or congestion externalities.

It has not taken account of matching externalities in search theoretic models, however, there are similar concepts in other economic fields. In microeconomic studies, for example, Varian (1974,

[1] Of course, it is possible to contact with a matched individual if there is on-the-job search, however, I assume there is no on-the-job search thoughout paper.

1976) considers an allocation problem with envy; "If, in a given allocation, agent $i$ prefers the bundle of agent $j$ to his own, we will say $i$ *envies* $j$" [2]. In macroeconomic studies, for example, Galí (1994) studies the implications for optimal portfolio decisions of the hypotheses that individuals care about not only their own consumption level but other individuals' consumption level. Hence, it would be natural to introduce a possibility that an individual's search strategy may be affected by other's matching into a standard search model. When there is search friction, it is enough to difficult to find a partner. At this time, if a single individual observes an ill-assorted match between a less attractive individual and an attractive individual, the single individual may be displeased because the less attractive individual gets double good fortune; that is, the less attractive individual find a partner and the partner is attractive.

In this paper, I assume that an individual's attraction is observable but whether an individual is envious or not envious is not observable. Given this assumption, I examine how matching externalities (in addition to standard search externalities) affect individual's search strategy and how equilibrium outcome change. Following Burdett and Coles (1999), I consider the cases both non-transferable utility and transferable utility. The former is the gain from matching is entirely exogeneous whereas the latter can be negotiated between matched individuals. In addition, I particularly focus on the *non-selective equilibrium* and *selective equilibrium* [3]. The former is the equilibrium in which all of the individuals is willing to match with any type partner, whereas the latter is the equilibrium in which individuals are willing to match with a particular type partner.

Main results of this paper are following. When utility is not transferable, there is no multiple equilibria whereas there can exist multiple equilibria when utility is transferable. If individuals feel constantly enviousness over time (whether they are single or pair), their search behavior is the same as individuals do not feel enviousness at all. In addition, if individual's enviousness

[2]In contrast to this paper, enviousness does not directly affect individual's utility in the models of Varian (1974, 1976).

[3]These concepts come from Mukoyama and Şahin (2009).

vanishes when pairwise, the search strategy is changed; that is, change in enviousness over time affects search behavior, rather than the degree of enviousness itself affects search behavior. Finally, whether utility is non-transferable or transferable, a higher finding rate and a lower dissolution rate make the non-selective equilibrium more desirable in the sense that total utility in the steady-state equilibrium becomes higher.

This paper is organized as follows. The next section states a basic setup of the model. In section 3, to illustrate essentials, I begin with a simple case in which utility is assumed to be non-transferable, and consider two types of equilibria, the non-selective equilibrium and the selective equilibrium, which is based on Mukoyama and Şahin (2009). In section 4, I extend the model to the case of transferable utility. Finally Section 5 concludes.

# 2 Setup

Consider a search-frictional market in which individuals randomly meet. Only single individuals search a partner; that is, there is no on-the-job search. Time is continuous. Population is normalized to unity and they are different in attraction; high attractive and low attractive (for example, height, intelligence, age, education, family background, and so on). Attraction is assumed to be observable. The measure of high and low attractive individuals are $x_H$ and $x_L(= 1-x_H)$, respectively. The fraction of high and low attractive individuals in the singles are $\pi_H$ and $\pi_L(= 1-\pi_H)$, respectively. In addition, low attractive individuals are whether envious or non-envious; if an envious individual observes an ill-assorted match (a low- and high-attractive pair), they are discontented with the match. I call this influence *matching externality* in contrast with the concept of *search externality* or *congestion externality.* The fraction of envious individuals is $\zeta$ and the observation rate is $\gamma$. Single individuals find another single at the Poisson rate $\alpha$ [4]. Throughout this paper I focus on the steady-state equilibrium.

[4]Most literature concerning labor search models assume a matching technology which endogenously determines the finding rate. In this model, however, it is exogenous because the population in the market is constant over time.

# 3 Non-transferable Utility

## 3.1 Non-selective Equilibrium

In this subsection I consider the steady-state equilibrium in which individuals are non-selective; that is, whenever a single individual finds another single, they always match regardless of pertner's attraction. I first describe the model in which individuals play the non-serective strategy, and then I examine the condition such that the non-selective strategy is optimal.

When single, all of the individuals enjoy an exogenous value $b > 0$ and seek a partner. As mentioned above, if envious individuals (they are low-attractive by assumption) find an ill-assorted match, they are discontented with the match. Letting $e > 0$ be the degree of dissatisfaction, disutility from envy is given by $\gamma e$. If a single individual finds another single, they form a partnership. Let $q_{ij}$ be the gain from matching when type $i(= H, L)$ is in partnership with type $j(= H, L)$ [5]. I assume that $q_{iH} > q_{iL} > 0$ for $i = H, L$ and $q_{LL} > b$. To allow a possibility of change in envy, I assume that disutility from envy when pairwise is given by $\phi\gamma e$ where $\phi \geq 0$. If $\phi = 1$, the degree of envy does not change over time. If $\phi < (>)1$, envy is mitigated (intensified) when one has a partner. Partnership is, however, collapsed at the exogenous rate $\delta$. After the separation, they again search a partner as a single.

Let $V_H(V_{Hj})$ be the asset value of high-attractive individual when single (when partner is type $j$). Given above, the Bellman equations for high-attractive individuals are

$$rV_H = b + \alpha[\pi_H(V_{HH} - V_H) + \pi_L(V_{HL} - V_H)],$$

$$rV_{Hj} = q_{Hj} + \delta(V_H - V_{Hj}), \quad \text{for } j = H, L,$$

where $r$ is the discount rate. Note that, since disutility from envy does not affect partner's utility, it is indifferent whether a low-attractive partner is envious or non-envious. Similarly,

[5] In this subsection I assume that this gain is exogenous (i.e., utility is non-transferable). Note that, in this case, whether a partner is envious or non-envious does not matter since gain from match is not affected by partner's attraction (see the following Bellman equations). If gain is negotiated, it needs to distinguish envious partner from non-envious partner, which is treated in the next subsection.

let $V_L^k(V_{Lj}^k)$ be the asset value of low-attractive individual when single and the envy's type is $k = e$(envious), $n$(non-envious) (when pairwise with type $j$ partner). The recursive formulae are thus

$$rV_L^k = b + \alpha[\pi_H(V_{LH} - V_L) + \pi_L(V_{LL} - V_L)] - \gamma e,$$
$$rV_{Lj}^k = q_{Lj} + \delta(V_L^k - V_{Lj}^k) - \phi\gamma e, \quad \text{for } j = H, L \text{ and } k = e, n,$$

where $e = 0$ for $k = n$ and I suppose $b > \gamma e$. Define $\Delta q_H \equiv q_{HH} - q_{HL} > 0$ and $\Delta q_L \equiv q_{LH} - q_{LL} > 0$. From the above equations and $\pi_L = 1 - \pi_H$, I obtain

$$(r+\delta)(V_{Hj} - V_H) = \frac{(r+\delta)(q_{Hj} - b) - \alpha\pi_H \Delta q_H}{r+\delta+\alpha},$$
$$(r+\delta)(V_{Lj}^k - V_L^k) = \frac{(r+\delta)[q_{Lj} - b + (1-\phi)\gamma e] - \alpha\pi_H \Delta q_L}{r+\delta+\alpha}.$$

Note that, since $q_{iH} > q_{iL}$ implies $V_{iH} > V_{iL}$ for $i = H, L$, $V_{iH} > V_i$ necessarily holds whenever $V_{iL} > V_i$ holds.

Next I derive the fraction of high-attractive individuals in singles, $\pi_H$, which is given by the flow conditions. In the steady state, the outflow from single to pairwise equals to the inflow from pairwise to single. Let $u$ be the number of singles. Since all individuals always match with any type individuals, in the non-selective equilibrium, the outflow of high-attractive individuals from single is then $\alpha u \pi_H$ where $\alpha$ is the finding rate. On the other hand, since the number of high-attractive individuals those who are pairwise is $x_H - u\pi_H$, the inflow of high-attractive individuals from pairwise is $\delta(x_H - u\pi_H)$ where $\delta$ is the dissolution rate. Similarly, from $\pi_L = 1 - \pi_H$ and $x_L = 1 - x_H$, the flow condition for low-attractive individuals is $\alpha u(1-\pi_H) = \delta[(1-\pi_H) - u(1-\pi_H)]$. By using these conditions, I obtain $\pi_H = x_H$. As a consequence, the fraction of high attractive in singles coincides with the fraction of high attractive in the population since matching is entirely random in the non-selective equilibrium. From the flow conditions, $u$ is also derived. The number of high-attractive single is $u\pi_H = \frac{\delta x_H}{\alpha+\delta}$ and the number of low-attractive single is $u(1-\pi_H) = \frac{\delta(1-x_H)}{\alpha+\delta}$, respectively. Hence, $u = \frac{\delta}{\alpha+\delta}$.

Now I show the conditions which guarantee the non-selective search strategy is optimal. Note that $V_{HH} > V_{HL}$ and $V_{LH}^k > V_{LL}^k$ for $k = e, n$ by assumption $q_{iH} > q_{iL} > 0$ for $i = H, L$. Consequently, if $V_{HH} > V_H$ and $V_{LL}^k > V_L^k$ for $k = e, n$ hold, the non-selective search strategy is optimal. These conditions are rewritten as

$$\Delta q_H < \frac{(r+\delta)(q_{HL}-b)}{\alpha x_H}, \tag{1}$$

$$\Delta q_L < \frac{(r+\delta)(q_{LL}-b)}{\alpha x_H}, \quad \text{for } k = n \tag{2}$$

$$\Delta q_L < \frac{(r+\delta)[q_{LL}-b+(1-\phi)\gamma e]}{\alpha x_H}, \quad \text{for } k = e \tag{3}$$

respectively. The term $(1-\phi)\gamma e$ in (3) represents envy's effects on search strategy. It is confirmed that, from (2) and (3), if the degree of envy does not vary over time, $\phi = 1$, the search strategies of envious individuals and non-envious individuals are the same. If the degree of envy mitigates when pairwise, $\phi < 1$, envious individuals become more non-selective than non-envious individuals (the inequality becomes less restrictive). In this case, there exists parameter range that (3) holds but (2) does not hold; envious individuals want to match with non-envious lindividuals, but not vice versa. Note that, from (3), even if envy is vanished when pairwise, $\phi = 0$, envy affects on search strategy as long as one is envious when single $e > 0$. In summary, this result suggests that change in envy over time affects search behavior rather than envy itself affects search behavior. If the degree of envy intensifies when pairwise, $\phi > 1$, envious individuals become less non-selective than non-envious individuals. In this case, there exists parameter range that (2) holds but (3) does not hold in turn; non-envious individuals want to match with envious individuals, but not vice versa. In addition, a higher $\alpha$ and a higher $x_H$ make these conditions more restrictive; if one can find a partner frequently or there are many high-attractive individuals, a possibility that one matches with an attractive partner increases. Consequently, one does not want to match with an unattractive partner, which means one is not non-selective.

Figure 1 shows the parameter range that the non-selective search strategy is optimal. As denoted above, it depends on the degree of envy when pairwise, $\phi$. This suggests that the non-selective strategy is optimal when $\Delta q_H$ and $\Delta q_L$ are small. This is because the merit of matching with a high-attractive individual is small, which makes being selective less beneficial. Consequently, the non-selective strategy is optimal.

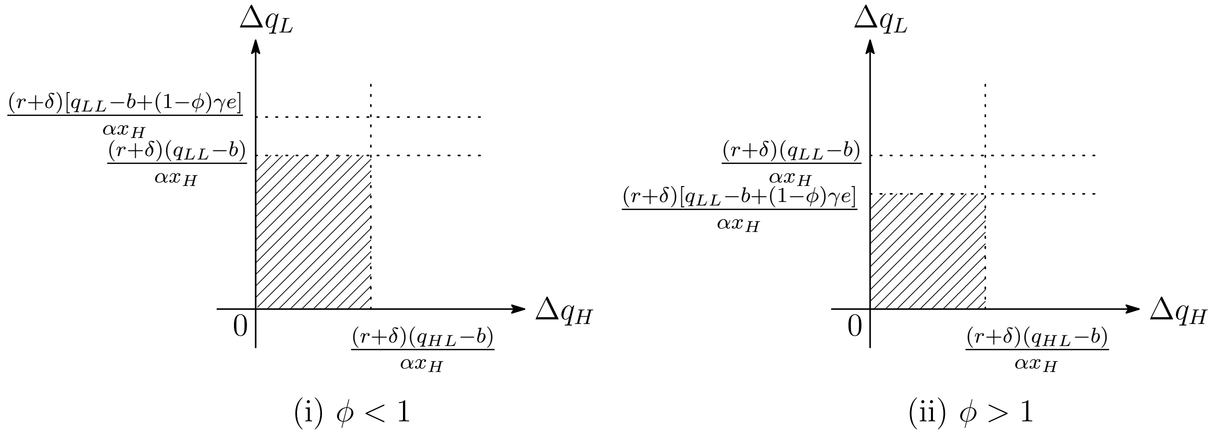


Figure 1: non-selective equilibrium

Given above, I derive total utility in the non-selective equilibrium. Recall that the number of singles is $\frac{\delta}{\alpha+\delta}$. Since all of the the singles enjoy $b$ and envious individuals are displeased at ill-assorted match $\gamma e$, total utility of single is thus $\frac{\delta}{\alpha+\delta}b - \frac{\delta\zeta(1-x_H)}{\alpha+\delta}\gamma e$. On the other hand, paired high-attractive individuals gain $q_{HH}$ ($q_{HL}$) if the partner is high (low) attractive. In the non-selective equilibrium, since matching is entirely random, the number of high attractive-high attractive matches and high attractive-low attractive matches are proportional to the population of high-attractive individuals, $x_H$. Total utility of pairwise is thus $\frac{\alpha x_H}{\alpha+\delta}(q_{HH}+q_{HL}) + \frac{\alpha(1-x_H)}{\alpha+\delta}(q_{LH}+q_{LL}) - \frac{\alpha\zeta(1-x_H)}{\alpha+\delta}\phi\gamma e$ (on average). Consequently, total utility in the non-selective equilibrium $U_n$ is

$$U_n = \frac{\delta b}{\alpha+\delta} + \frac{\alpha[x_H(q_{HH}+q_{HL}) + (1-x_H)(q_{LH}+q_{LL})]}{\alpha+\delta} - \frac{(\alpha\phi+\delta)(1-x_H)\zeta\gamma e}{\alpha+\delta}, \tag{4}$$

where the first term represents singles' utility, the second term stands for pairs' utility, and the third term reflects disutility from envy. Singles' utility is increasing in the separation rate $\delta$ whereas decreasing in the finding rate $\alpha$. This is because a higher $\delta$ ($\alpha$) increases (decreases) the number of singles. Pairs' utility is increasing in the finding rate $\alpha$ whereas increasing in the separation rate $\delta$ in turn since a higher $\alpha$ ($\delta$) increases (decreases) the number of pairs. Regarding disutility from envy, the effects of the finding rate and the separation rate depend on the degree of envy when pairwise, $\phi$. It can be shown that if $\phi = 1$, both $\alpha$ and $\delta$ have no impact on disutility from envy because all of the envious individuals have envy whether they are single or pairwise. If $\phi < 1$, a higher $\alpha$ decreases disutility since the number of pairs increases, (total) disutility from envy decreases. On the other hand, if $\phi < 1$, a higher $\delta$ increases disutility because the number of singles increases, (total) disutility from envy increases. For simplicity, I assume that $\phi = 0$ hereafter; envy vanishes when pairwise (it would be more practival rather $\phi < 1$ than $\phi > 1$).

## 3.2 Selective Equilibrium

As in the previous subsection, I first assume that all of the individuals play the selective strategy, and then I derive the conditions under which the selective strategy is indeed optimal. Note that, under full segmentation, envy does not arise since ill-assorted matches are not formed. So there is no distinction between envious individuals and non-envious individuals. Since one does not match with a different type partner, the Bellman equations are

$$rV_i = b + \alpha\pi_i(V_{ij} - V_i), \quad \text{for} \quad i = j = H, L,$$
$$rV_{ij} = q_{ij} + \delta(V_i - V_{ij}), \quad \text{for} \quad i = j = H, L.$$

Following these equations, I obtain

$$(r+\delta)(V_{HL} - V_H) = \frac{(r+\delta)(q_{HL} - b) - \alpha\pi_H\Delta q_H}{r+\delta+\alpha\pi_H},$$
$$(r+\delta)(V_{LH} - V_L) = \frac{(r+\delta)(q_{LH} - b) - \alpha(1-\pi_H)\Delta q_L}{r+\delta+\alpha(1-\pi_H)}.$$

If the selective strategy is optimal, it must hold that $V_i > V_{ij}$ for $i \neq j = H, L$ and that $V_i < V_{ij}$ for $i, j = H, L$. It can be easily shown that if low-attractive individuals are willing to match with a low-attractive one, they are also willing to match with a high-attractive one (if $V_L < V_{LL}$

holds, $V_L < V_{LH}$ also holds), which is guaranteed by the assumption $q_{LL} > b$. Consequently, a low-attractive individual is willing to match with any type of partner. On the other hand, for high-attractive individuals, there is a parameter set such that $V_H < V_{HH}$ holds but $V_H < V_{HL}$ does not holds. The conditions that the selective strategy is optimal is summarized by

$$\Delta q_H > \frac{(r+\delta)(q_{HL}-b)}{\alpha x_H}. \tag{5}$$

It is clear that eq.(5) is the inverse range of eq.(1); this condition states "the non-selective strategy is not optimal" so that there is no multiple equilibria when utility is non-transferable. Figure 2 illustrates the parameter set that the selective strategy is optimal. Although a low-attractive individual is willing to match with a high-attractive one, since the advantage of matching with a same type individual is large for a high-attractive individual (or the disadvantage of matching with a low-attractive individual is small), a high-attractive individual becomes selective. Since this range becomes wider as $b$ increases, a higher utility when single makes a high-attractive individual more selective. The result of uniqueness of the equilibrium can change when utility is transferable, which will be discussed in section 3.3.

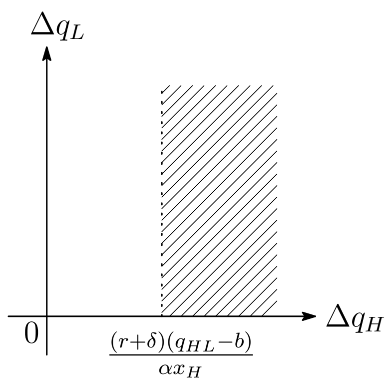


Figure 2: selective equilibrium

### 3.3 Welfare Analysis

Recall that, in the selective equilibrium, one is in partnership with a same type individual. The flow condition for the high attractive is thus $\alpha u \pi_H^2 = \delta(x_H - u\pi_H)$: the outflow of unemployment is the number of the high attractive singles $u\pi_H$ times the effective finding rate $\alpha\pi_H$, whereas the inflow of unemployment is the number of the high attractive pairwise $x_H - u\pi_H$ times the separation rate $\delta$. Similarly, the flow condition for the low attractive is $\alpha u(1-\pi_H)^2 = \delta[(1-x_H) - u(1-\pi_H)]$. To obtain an analytical solution, I assume that $x_H = \frac{1}{2}$ (as in Mukoyama and Şahin, 2009). Some algebra reveals that $\pi_H = \frac{1}{2}$ and the number of singles is $u = \frac{2\delta}{\alpha+2\delta}$. In the selective equilibrium, the singles $\frac{2\delta}{\alpha+2\delta}$ enjoy $b$ whereas the paired individuals $\frac{\alpha}{2(\alpha+2\delta)}$ gain $q_{HH}$ or $q_{LL}$. Consequently, total utility in the selective equilibrium is

$$U_s = \frac{2\delta b}{\alpha + 2\delta} + \frac{\alpha(q_{HH} + q_{LL})}{2(\alpha + 2\delta)}. \tag{6}$$

When $x_H = \frac{1}{2}$, eq.(4) is rewritten as

$$U_n = \frac{\delta b}{\alpha + \delta} + \frac{\alpha(q_{HH} + q_{HL} + q_{LH} + q_{LL})}{2(\alpha + \delta)} - \frac{\delta\zeta\gamma e}{2(\alpha + \delta)}, \tag{4'}$$

From these expressions, the condition $U_s \geq U_n$ is equivalent to

$$2b + \frac{(\alpha + 2\delta)\zeta\gamma e}{\alpha} \geq q_{HH} + q_{LL} + \frac{(\alpha + 2\delta)(q_{HL} + q_{LH})}{\delta}. \tag{7}$$

If this inequality holds, total utility in the selective equilibrium is higher than in the non-selective equilibrium. Comparison of two equilibria is summarized in the following proposition.

**Proposition 1.** (i) *A higher $\alpha$ makes the non-selective equilibrium more desirable whereas a higher $\delta$ makes the selective equilibrium more desirable.* (ii) *When the parameters appertaing to enviousness ($\zeta$, $\gamma$, and $e$) is high, the selective equilibrium is more desirable.*

*Proof.* It is straightforward from eq.(7), so is omitted. ■

Intuition of Proposition 1 is as follows. Since the LHS is decreasing and the RHS is increasing in $\alpha$, a higher $\alpha$ makes holding of this condition more difficult; if individuals find a partner frequently, the non-selective strategy brings the equilibrium to higher total utility even though envy arises. This is because envy vanishes when one has a partner by assumption ($\phi = 0$). As a result, although the non-selective strategy generates ill-assorted matches, matching removes envy from single individuals. A higher finding rate makes the second effect dominates the first effect. On the other hand, since the LHS is increasing and the RHS is decreasing in $\delta$, a higher $\delta$ makes holding of this condition easier; if pairs dissolve frequently, the selective strategy brings the equilibrium to higher total utility even though the number of singles is large. This is the inverse effect of a higher finding rate; although the selective strategy does not generates envy, it brings a larger singles. Note that, from the assumption $q_{LL} > b$, this inequality does not hold if there is no envy ($e$, $\gamma$, or $\zeta = 0$). In that case, all of the individuals should not be selective. In other words, existence of envy supports rationality of being selective so that envy affects search behavior. In addition, this circumstance would be interpreted as a kind of a prisoner's dilemma; envious individuals (those who are low attractive) want to match with a high attractive rather than a low attractive, which is, of course, rational. This strategy, however, gives rise to ill-assorted matches which brings forth envy. As a result, on average, total utility can decrease.

# 4 Transferable Utility

Up to here gains from matching ($q_{ij}$) is exogenous. In this section I suppose that matching surplus is equally divided by bilateral Nash bargaining. Let $p(i,j)$ be the total gain from matching and $p_{ij}$ be the gain for a type-$i$ individual whose partner is a type-$j$ for $i, j = H, L, L^e$ [6]. Note that, since matching surplus is equally divided, $p_{HH} = \frac{p(H,H)}{2}$, $p_{LL} = \frac{p(L,L)}{2}$, and $p_{L^eL^e} = \frac{p(L^e,L^e)}{2}$. To exclude a trivial steady state, I suppose $p_{L^eL^e} > b$.

[6] In a labor market model, assuming that $i$ is a worker and $j$ is a firm, $p(i,j)$ corresponds to output, $p_{ij}$ is a flow wage for the worker, and $p_{ji} = p(i,j) - p_{ij}$ is a flow profit for the firm.

## 4.1 Non-selective Equilibrium

In the non-selective equilibrium, all of the individuals is willing to match whenever a single individual finds another single one, regardless of the pertner's attraction. As in the previous section, I first describe the model in which all of the individuals plays the non-serective strategy, and then I examine the condition such that the non-selective strategy is optimal indeed. Note that, unlike the previous section, it needs to distinguish whether a low-attractive partner is envious or non-envious since gain from match is endogenously determined by negotiation (see below). The Bellman equations are

$$rV_H = b + \frac{\alpha}{2}[(V_{HH} - V_H) + \zeta(V_{HL^e} - V_H) + (1-\zeta)(V_{HL} - V_H)],$$
$$rV_{Hj} = p_{Hj} + \delta(V_H - V_{Hj}), \quad \text{for} \quad j = H, L \text{ (non-envious), and } L^e \text{ (envious)},$$

where $\zeta$ is the fraction of envious individuals in low-attractive individuals. Since gain from match is negotiated, $p_{HL}$ and $p_{HL^e}$ might be different. For a non-envious individual, the Bellman equations are

$$rV_L = b + \frac{\alpha}{2}[(V_{LH} - V_L) + \zeta(V_{LL^e} - V_L) + (1-\zeta)(V_{LL} - V_L)],$$
$$rV_{Lj} = p_{Lj} + \delta(V_L - V_{Lj}), \quad \text{for} \quad j = H, L, \text{and } L^e,$$

and for an envious individuals,

$$rV_L^e = b + \frac{\alpha}{2}[(V_{L^eH} - V_L^e) + \zeta(V_{L^eL^e} - V_L^e) + (1-\zeta)(V_{L^eL} - V_L^e)] - \gamma e,$$
$$rV_{L^ej} = p_{L^ej} + \delta(V_L^e - V_{L^ej}), \quad \text{for} \quad j = H, L, \text{and } L^e.$$

Note that there is enviousness since an ill-assorted match can exist in the non-selective equilibrium. From these equations, I obtain

$$(r+\delta)(V_{Hj} - V_H) = p_{Hj} - \frac{2(r+\delta)(b-\gamma e) - \alpha[p_{HH} + \zeta p_{HL^e} + (1-\zeta)p_{HL}]}{2(r+\delta+\alpha)}$$

for $j = L, L^e$ and $e = 0$ when $j = L$, and

$$(r+\delta)(V_{Lj} - V_L) = p_{Lj} - \frac{2(r+\delta)(b-\gamma e) - \alpha[p_{LH} + \zeta p_{LL^e} + (1-\zeta)p_{LL}]}{2(r+\delta+\alpha)}$$

for $j = H, L^e$ and $e = 0$ when $j = H$. The Bellman equations for an envious individual are

$$(r+\delta)(V_{L^e j} - V_L^e) = p_{L^e j} - \frac{2(r+\delta)(b-\gamma e) - \alpha[p_{L^e H} + \zeta p_{L^e L^e} + (1-\zeta) p_{L^e L}]}{2(r+\delta+\alpha)}$$

for $j = H, L$. To examine that the non-selective strategy is optimal, it needs to check that $V_{ij} > V_i$ for $i, j = H, L, L^e$. Recall that the gain from matching $p_{ij}$ (for $i \neq j$) is determined by Nash bargaining. For example, the gain of a high-attractive individual with a non-envious individual, $p_{HL}$, is given so as to satisfy $V_{HL} - V_H = V_{LH} - V_L$ (and the non-envious individual receives $p_{LH} = p(H, L) - p_{HL}$). Since I focus on enviousness, the key parameters are $e$ and $\zeta$. Regarding the rest of parameters I set $r = 0.05$, $\delta = 0.02$, $\alpha = 0.2$, $\gamma = 0.1$, $b = 0.1$. The value from matching, $p(i, j)$, is assumed that $p(H, H) = 1.0$, $p(H, L) = p(H, L^e) = 0.8$, $p(L, L) = p(L, L^e) = 0.5$, and $p(L^e, L^e) = 0.2$.

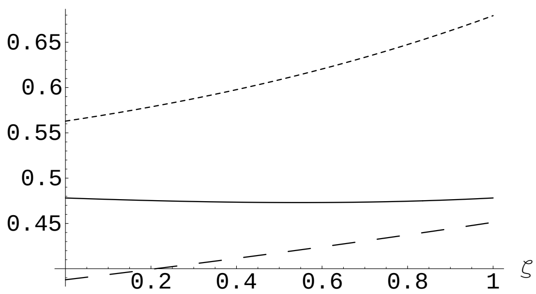


Figure 3: gain ($e = 0.1$)

Figure 3 illustrates the gains $p_{HL}$ (the solid line), $p_{HL^e}$ (the dotted line), and $p_{LL^e}$ (the broken line) with various $\zeta$, given $e = 0.1$. It shows that $p_{HL^e}$ and $p_{LL^e}$ are increasing in $\zeta$. When a high-attractive or non-envious individual matches with an envious one, they can receive transfer comes from enviousness. If $\zeta$ is high, such a situation is more likely to arise, and then the reservation value increases. As a result, $p_{HL^e}$ and $p_{LL^e}$ are increasing in $\zeta$. In addition since a high-attractive individual has a larger potential gain than a non-envious individual, $p_{HL^e}$ is always higher than $p_{LL^e}$. Of course, $p_{HL}$ is affected by the fraction of envious individuals through bargaining, however, its effect is indirect so that $p_{HL}$ almost does not change in $\zeta$.

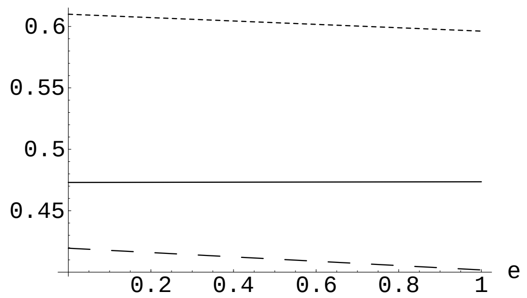


Figure 4: gain ($\zeta = 0.5$)

Figure 4 illustrates the gains with various $e$ given $\zeta = 0.5$. It shows that, in contrast to Figure 3, $p_{HL^e}$ and $p_{LL^e}$ almost do not change but are decreasing in the degree of enviousness, $e$. This result is counterintuitive because a higher enviousness brings a higher matching surplus given $\zeta$.

In Figure 6, it can be confirmed that the matching surplus for a high-attractive and a non-envious individual are increasing in $e$ indeed. As mentioned above, $p_{HL}$ is affected by $e$ through bargaining, however, it almost does not change because its effect is indirect.

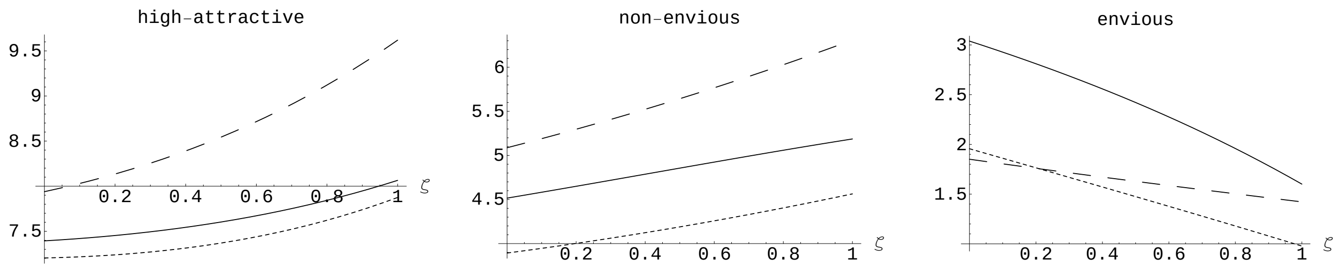


Figure 5: matching surplus ($e = 0.1$)

Given the gains above, when the condition $V_{ij} - V_i > 0$ for $i, j = H, L, L^e$ holds, the non-selective strategy is optimal. Figure 5 shows the matching surplus with various $\zeta$. In each graph the solid line is when the partner is a high attractive, the dotted line is when the partner is a non-envious, and the broken line is when the partner is an envious individual. Figure 3 shows that all of the matching surplus, $V_{ij} - V_i$ for $i, j = H, L, L^e$, is positive; that is, the non-selective strategy is optimal under parameters I set. It illustrates that, for a high-attractive and a non-envious individual, a higher $\zeta$ increases the matching surplus. This is because when $\zeta$ is high, the gain from envious individual increases through bargaining (Figure 3) and such a matching is more likely to occur. In addition the matching surplus when the partner is a high-attractive (the solid line) is larger than when the partner is a non-envious one (the dotted line). This result mainly comes from the assumption $p(H,H) > p(H,L) > p(L,L)$; that is, a high-attractive individual has a higher potential gain. On the other hand, for an envious individual, a higher $\zeta$ decreases the matching surplus. This is because when $\zeta$ is high, the transfer to the partner increases through bargaining and such a matching is more likely to occur. Note that, for an envious individual, the surplus becomes maximum when the partner is high attractive, even though it is required a higher transfer. This result comes from the assumption $p(H,L^e) > p(L,L^e) > p(L^e,L^e)$; since the total gain is the largest when the partner is a high-attractive, the gain for an envious individual is large [7]. In addition, the surplus when the partner is the same type is larger than when the partner is a non-envious individual if $\zeta$ is high. Since an envious individual does not have to transfer when the partner is also envious (and such a matching is more likely to occur when $\zeta$ is high), it is less beneficial to match with a non-envious individual.

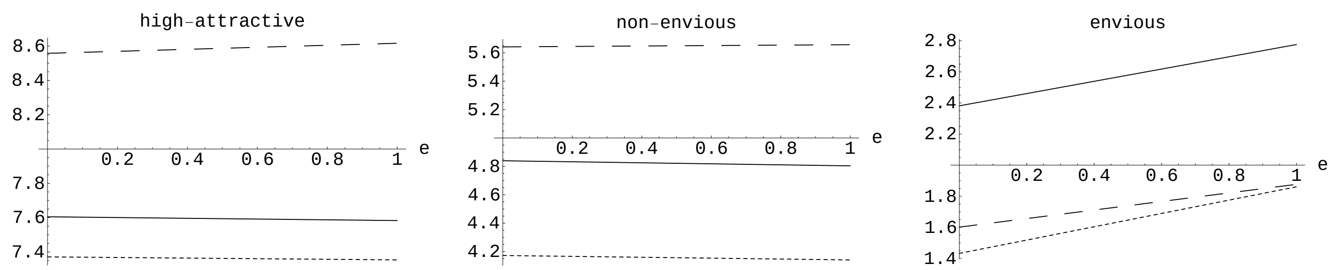


Figure 6: matching surplus ($\zeta = 0.5$)

The matching surplus with various $e$ ($\zeta = 0.5$ is fixed) is in Figure 6. It illustrates that, as in Figure 5, the surplus becomes maximum when the partner is an envious individual for a high-attractive and a non-envious individual. It comes from the same reason above. As shown in Figure 4, the gain almost does not change in $e$ so that the matching surplus also almost does not change in $e$.

[7]It can be checked from Figure 3 by a comparison of $p_{L^eH} = p(H,L^e) - p_{HL^e}$ with $p_{L^eL} = p(L,L^e) - p_{LL^e}$.

Comparing Figure 5 with Figure 6 suggest that the matching surplus is affected by the fraction of envious individuals, $\zeta$, rather than the degree of enviousness, $e$. For an envious individual, the surplus is, however, increasing in $e$. The reason is straightforward. This is because matching vanishes enviousness, which makes the surplus higher.

In the non-selective equilibrium, since derivation of the flow condition is similar to the case of non-transferable utility, it is omitted. Letting $u$ be the number of singles, $u = \frac{\delta}{\alpha+\delta}$ in the steady state. All of the single individuals, $u$, enjoy $b$ and envious singles, $\frac{\delta\zeta}{2(\alpha+\delta)}$, have displeasure $\gamma e$. In aggregate, total utility in the non-selective equilibrium is

$$\begin{aligned} U_n = & \frac{\delta b}{\alpha+\delta} - \frac{\delta\zeta\gamma e}{2(\alpha+\delta)} + \frac{\alpha[p_{HH} + \zeta p_{HL^e} + (1-\zeta)p_{HL}]}{2(\alpha+\delta)} \\ & + \frac{\alpha\zeta[p_{L^eH} + \zeta p_{L^eL^e} + (1-\zeta)p_{L^eL}]}{2(\alpha+\delta)} + \frac{\alpha(1-\zeta)[p_{LH} + \zeta p_{LL^e} + (1-\zeta)p_{LL}]}{2(\alpha+\delta)}, \end{aligned} \tag{8}$$

where the third term of RHS stands for utility of high attractive individuals in pairs, the fourth term reflects utility of non-envious individuals in pairs, and the fifth term represents utlity for envious individuals in pairs (on average).

## 4.2 Selective Equilibrium

In the selective equilibrium since one matches only with a same type partner, an ill-assorted match is not formed and thus enviousness does not arise. Hence, as in the previous section, it does not need to distinguish an envious individual from a non-envious individual. As in the previous section, the Bellman equations are

$$rV_i = b + \frac{\alpha}{2}(V_{ij} - V_i),$$

$$rV_{ij} = p_{ij} + \delta(V_i - V_{ij}),$$

for $i = j = H, L$. It can be easily shown that, from these equations, $(2r + 2\delta + \alpha)(V_{ij} - V_i) = 2(p_{ij} - b) > 0$ for $i = j = H, L$. All of the individuals is thus willing to match with a same type partner. The selective strategy is optimal if a deviation (matching with a different type partner) is not beneficial. That is, $V_{ij} < V_i$ for $i \neq j = H, L, L^e$. As shown in the previous subsection, however, this condition almost does not hold in my parameterization so that the selective strategy is not optimal (see Figure 5 and Figure 6) [8]. In contrast to the case of non-transferable utility, there exist multiple equilibria in which both the non-selective strategy and the selective strategy are played.

Since the flow condition is quite similar to the case of non-transferable utility, it is omitted. In the selective equilibrium, total utility is given by

$$U_s = \frac{2\delta b}{\alpha + 2\delta} + \frac{\alpha(p_{HH} + p_{LL})}{2(\alpha + 2\delta)}, \tag{9}$$

where the first term denotes singles' utility and the second term represents pairs' utility. Note that since there is no difference between an envious and a non-envious individual, it does not depend on $\zeta$ and $e$.

## 4.3 Welfare Analysis

As in the case of non-transferable utility, the number of singles is higher in the selective equilibrium than in the non-selective equilibrium, while, enviousness reduces aggregate utility in the non-selective equilibrium.

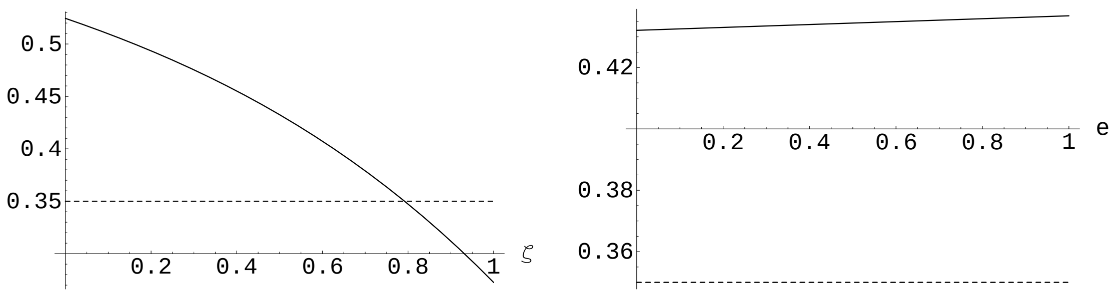


Figure 7: aggregate utility

[8]It does not mean that the selective strategy is not beneficial and is not played. Here I state "the selective strategy is not optimal" in the sense that a deviation (playing the non-selective strategy) is also beneficial.

Figure 7 draws aggregate utility in the steady-state equilibrium. The solid line is the non-selective equilibrium (8) and the dotted line is the selective equilibrium (9). As mentioned above, aggregate utility in the selective equilibrium does not depend on $\zeta$ and $e$. In the non-selective equilibrium aggregate utility is decreasing in $\zeta$. As shown in Figure 3, $\zeta$ has impacts on the gains; $p_{HL^e}$ and $p_{LL^e}$ are increasing, $p_{HL}$ is almost constant, and $p_{L^eH}$ and $p_{L^eL}$ are decreasing in $\zeta$. In addition, a higher $\zeta$ directly decreases aggregate utility due to enviousness (the second term of eq.(8), $-\frac{\delta\zeta\gamma e}{2(\alpha+\delta)}$). In all, the selective equilibrium is more desirable than the non-selective equilibrium when $\zeta$ is high, even though both strategies are played. With regard to the degree of enviousness, $e$, total utility in the non-selective equilibrium almost does not change (but is slightly increasing). This result comes from Figure 4; the gains have little relation to the degree of enviousness $e$ (but $p_{L^eH}$ and $p_{LL^e}$ are slightly increasing in $e$). In summary, the outcome is affected by the fraction of envious people $\zeta$ rather than the degree of enviousness $e$ under transferable utility.

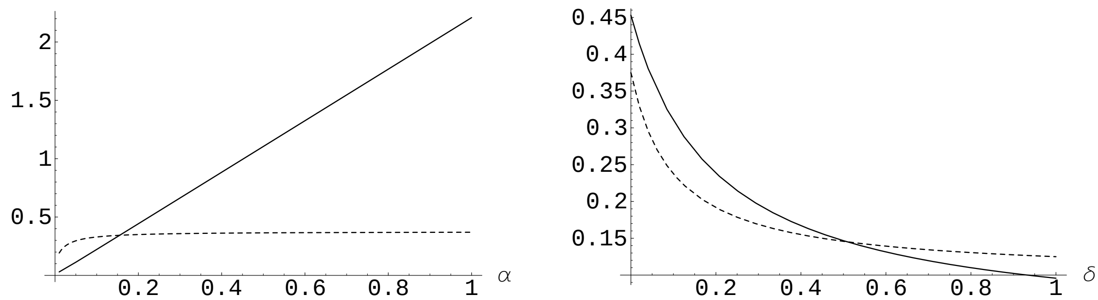


Figure 8: aggregate utility ($\zeta = 0.5$ and $e = 0.1$)

Figure 8 illustrates total utility with various meeting rate $\alpha$ and dissolution rate $\delta$. Since a higher $\alpha$ increases the number of pairs, total utility is increasing in $\alpha$ in both equilibria. On the other hand, since a higher $\delta$ increases the number of singles, total utility is decreasing in $\delta$ in both equilibria. Note that the non-selective equilibrium is more desirable when $\alpha$ is high whereas the selective equilibrium is more desirable when $\delta$ is high. This result concides with the case of non-transferable utility (see Proposition 1). When one cannot find a partner easily (a lower $\alpha$), the number of singles becomes large. In such a circumstance, if one plays the non-selective strategy, single envious individuals suffer from enviousness and if one plays the selective strategy, it does not arise enviousness. Consequently, total utility in the selective equilibrium is higher than in the non-selective equilibrium when $\alpha$ is low. On the other hand, if one can find a partner easily (a higher $\alpha$), the number of singles decreases and enviousness vanishes for envious individuals. Accordingly, one should not be selective even though enviousness arises in the steady state. When partnership is firm (a lower $\delta$), the number of singles gets fewer. In such a circumstance, even if one plays the selective strategy, the advantage that enviousness does not arise is small. If one plays the non-selective strategy, a lot of pairs are in partnership for a long time. Consequently, it is beneficial to play the non-selective strategy. On the other hand, if partnership is frequently dissolved (a higher $\delta$), the number of singles increases, and thus enviousness reduces total utility when one plays the non-selective strategy. Hence, one should be selective even though the number of singles is large.

# 5 Conclusion

This paper have investigated a search model with enviousness by an ill-assorted matching. First, in Section 3, I have shown that there is no multiple equilibria when utility is nontransferable. In addition, given finding rate and dissolution rate, the parameters appertaing to enviousness are high, the selective equilibrium is more desirable. Second, in Section 4, I have shown that there can exist multiple equilibria when utility is transferable. In addition, it is numerically illustrated that aggregate utility is affected by the fraction of envious individuals rather than the degree of enviousness. Finally, I have shown that a higher finding rate and a lower dissolution rate make the non-selective equilibrium more desirable whether utility is transferable or not.

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