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Non-Zero-Sum Games and Nash Equilibriums: Applications to Generation Gap Problems

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Non-Zero-Sum Games and Nash Equilibriums:

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Abstract This chapter critically discusses non-zero-sum games with special reference to Nash equilibrium, which is a successor of Cournot equilibrium in the theory of duopoly and oligopoly. We first focus on the "residence game" as a typical non-zero-sum game. We pick up the old and the young couples, who have to decide whether they live together or separately, and whether they live in the country or in the city. We shed new light on " generation gaps problems," with many other applications. We next turn to the "battle of the sexes," or more plainly the "dilemma of lovers". The male and the female have to find good dating spots. Whereas the male prefers to see boxing rather than ballet, the female's preference is just the opposite. We propose that contrary to the conventional way of discussions, the introduction of a third option such as going to see movies gives each couple a second best solution. Finally, comparison between "Econs" and "Humans" is carefully discussed.

Keywords Nash as a successor of Cournot, the residence game as a non-zero-sum game, generation gaps problem, dilemma of lovers, Econs versus Humans

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I. From Antoine Cournot to John F. Nash

1.1 Appreciation of Antoine Cournot by Joseph A. Schumpeter

The main purpose of this chapter is to critically evaluate The Theory of Non-Zero-Sum Games as opposed to Zero-Sum Games and Applications, which have been extensively discussed in the last two chapters. We will start our discussion with the unique transition of equilibrium concepts from Antoine A. Cournot to John F. Nash Jr.

Joseph A. Schumpeter (1883-1950). a "giant of the 20th century," is known as the economist who very highly rated the achievement of Antoine A. Cournot (1801-1877) in the history of economic analysis. In what follows, let us introduce the reader a couple of episodes to clearly show how highly Schumpeter evaluated Cournot.

When Schumpeter taught economic science at Harvard University, he enthusiastically told his students about the history of economic giants. Among those attentive students were Paul A. Samuelson (1915-2001), then a young ambitious graduate student and later a Nobel Economic Science winner. Schumpeter's talk proceeded to concretely name "the Four Greatest Economists in History" and bravely gave great shocks to all the students by saying that among those four, three economists are prideful French scholars. According to Schumpeter, the following four were selected by him. They are François Quesnay (1694-1774), Antoine A. Cournot (1801-1877), Léon Walras (1834-1910), and the fourth economist unnamed. [1)]

Quesnay was a French physician and economist, being well-known for publishing his work *Tableau économique* (Economic Table) in 1758. This was presumably the first work to describe the working of the national economy in an analytical fashion. Cournot was a noted French mathematician, also greatly contributing to the development of oligopoly theory as an important field of economic theory. In his 1838 book *Researches on the Mathematical Principles of the Theory of Wealth*, Cournot used differential and integral calculus, apparently a highly advanced mathematical tool at that time, causing harsh criticism in economics profession. Those criticisms against Cournot, however, promptly vanished 100 years later when Game Theory was established by the collaboration of mathematician John von Neumann and economist Oskar Morgenstern in their 1944 book *Game Theory and Economic Behavior*. In retrospect, the concept of Cournot equilibrium was regarded as a important predecessor of Nash equilibrium, a very key concept in mathematical Game Theory.

Quesnay, Cournot, and Walras — they were all noted French economists of world fame. There should be no objection against such a selection. The question which would naturally occur on our mind after listening to Schumpeter's remark on the Greatest Economists in History was like this. Namely, who was the Fourth Economist Schumpeter wanted to nominate in his class at Harvard. Another Frenchman should be ruled out for sure. Someone would say that the right candidate would be Adam Smith, David Ricardo, John M. Keynes, or Karl Marx. A cynical person dare to say that Schumpeter himself would be among the Four Greatest Economist Club.

We are ready to show the reader another episode related to Cournot and Schumpeter. This is also an old story which was created when Mr. Ichiro Nakayama, then an instructor at Tokyo Institute of Commerce, did research in economics at University of Bonn, under the direction of Joseph Schumpeter. One day, old Schumpeter asked young Nakayama, " How have you been studying economics at a Japanese university ? " Then, Nakayama honestly replied to Schumpeter, " Yes, Sir, I myself was a humble assistant under the direction of Professor Tokuzo Fukuda at Tokyo, who strongly recommended me to read Antoine Cournot, Heinlich Gossen, and Leon Walras." This was evidently a nice surprise to Schumpeter, who had a hard time to swallow the fact that the honorable names of Cournot, Gossen, and Walras were already known to Japan, a rather small island nation in the Pacific. [2)]

In short, Schumpeter's appreciation of Cournot as a pioneering theorist was very high. Schumpeter's preference to equilibrium concepts in economic science was so strong that he selected Quesnay, Cournot, and Walras and the fourth person unnamed as the Four Greatest Economist in History. Although Cournot's concept of equilibrium, characterized as "Cournot equilibrium," had been was under constant attack in so many years, such a groundless attack disappeared when the concept of "Nash equilibrium" as a successor of "Cournot Equilibrium" began to sweep over Game Theory and Applications.

In passing, it should be noticed that John Hicks, one of most influential economic theorists after Cournot, Walras, and perhaps Schumpeter, also appreciated the accomplishment of Cournot very much. For instance, in his 1935 paper on the theory of monopoly, Hicks (1935) remarked as follows:

> The generally increased interest in mathematical economics during the last few years has naturally turned attention back to the work of Cournot, the great founder of the subject, and still one of best teachers. It was Cournot's creation of elementary monopoly theory which was the first great triumph of mathematical economics; yet Cournot had left much undone. It is not surprising that the endeavor to complete his work have been an attractive occupation for his successors." (Hicks, 1935, p. 1)

We agree with Hicks that Cournot should be thought of as the great founder of the theory of monopoly, duopoly and oligopoly. Remarkably, Cournot had left much undone to his successors until the present time. For instance, following his pioneering work, there have so many works on the Theory of Oligopoly and Information in the 1980s, the 1990s. and even today. As the saying goes, life may be short but art (and science) should be quite long. [3)]

1.2 John F. Nash as a Successor of Cournot

John F. Nash Jr. (1928-2015) was an eccentric American mathematician with strong economic orientation. He made outstanding contributions to game theory, differential geometry, partial differential equations and many other related fields. He shared the 1994 Nobel Memorial Prize in Economic Sciences with game theorists Reinhard Selten and John Harsanyi. He was born as a mathematical kid, and received his Ph. D. at the age of only 22 years, with a merely 28-page dissertation on non-zero-sum games.

As the saying goes, there should be no clear lines between genius and madness. Already in 1959, Nash's mental illness began to come out in the form of paranoia, followed by more serious schizophrenia. While he was admired as a man with "beautiful mind" by his beautiful wife, he was also known as "The Phantom of Fine Hall" (Princeton University's mathematical center). In this chapter, whether Nash was a genius or a madman, we would like to just limit our attention to his outstanding contributions on non-zero-sum games and applications. [4)]

We will briefly show that Cournot equilibrium is excellently succeeded to Nash equilibrium. To this end, let us first assume that there are two firms which produce a homogeneous product. Let x_i be the output of firm i $(i = 1, 2)$, and let p be its common market price $(i = 1, 2)$. Besides, let the inverse demand function be defined by $p = G(x_1 + x_2)$. If we define constant cost by c, then we can respectively define firm 1 ' s profit and firm 2's profit as $\Pi_1 = p x_1 - c x_1$ and $\Pi_2 = p x_2 - c x_2$. Then, we say that the output pair (x_1^*, x_2^*) is a Cournot equlibrium pair if and only if the following conditions are both satisfied:

$$\text{(C 1)} \qquad \Pi_1(x_1^*, x_2^*) \geqq \Pi_1(x_1, x_2^*) \quad \text{for any } x_1 ,$$

$$\text{(C 2)} \qquad \Pi_2(x_1^*, x_2^*) \geqq \Pi_2(x_1^*, x_2) \quad \text{for any } x_2 .$$

In plain English, the output pair $(x_1{}^*, x_2{}^*)$ represents a Cournot equlibrium if and only if $x_1{}^*$ maximizes firm 1' s profit $\Pi_1(x_1, x_2{}^*)$ for any given $x_2{}^*$, and $x_2{}^*$ maximizes firm 2 ' s profit $\Pi_2(x_1{}^*, x_2)$ for any given $x_1{}^*$. Once the Cournot equilibrium is reached, it would be not a good policy at all for each firm to unilaterally change its output strategy. As the catchword goes, please stay where you are !

Next, let us turn to a modern concept of Nash equilibrium in Game Theory. For simplicity, let us suppose that there are only two players, player 1 and player 2. Let S_i be the set of all possible strategies for player i ($i = 1, 2$). Let P_i be player i's payoff as a function of strategies, s_i. For the case of two players, we obtain $P_i = P_i(s_1, s_2)$ ($i = 1, 2$). Then, we say that the strategy pair $(s_1{}^*, s_2{}^*)$ is a Nash equilibrium pair if and only if the following conditions are both satisfied:

(N 1) $\quad P_1(s_1{}^*, s_2{}^*) \geqq P_1(s_1, s_2{}^*)$ for any s_1,

(N 2) $\quad P_2(s_1{}^*, s_2{}^*) \geqq P_2(s_1{}^*, s_2)$ for any s_2.

Thus, the strategy pair $(s_1{}^*, s_2{}^*)$ represents a Nash equlibrium if and only if $s_1{}^*$ maximizes player 1' s payoff $P_1(s_1, s_2{}^*)$ for any given $s_2{}^*$, and $s_2{}^*$ maximizes player 2 ' s payoff $P_2(s_1{}^*, s_2)$ for any given $s_1{}^*$. Once the Nash equilibrium is reached, it would be not a good policy at all for each player to unilaterally change its payoff strategy. Here again, please stay where you are !

If we compare a pair of Cournot equilibrium conditions, (C1) and (C2), and a pair of Nash equilibrium conditions, (N 1) and (N2), then it is unquestionably clear that those two pairs of equilibrium conditions are perfectly analogous. We can naturally declare that the idea of Cournot has safely been succeeded to that of Nash. It is really one of "The Seven Wonders of the Academic World," however, that such succession from Cournot to Nash has been neglected for so many years. Here, we confirm again the following maxim. " Life is so short but Art (or Science) is so long.

Ⅱ. The Residence Game as a Non-Zero Sum Game: the Old Couple versus the Young Couple

In what follows, we will carefully discuss the "residence game" as a typical example of a non-zero-sum game. Needless to say, living at an appropriate place and with good human relations is one of the most important factors for our life. Besides, what we may call the "generation gaps problem" is closely related to such residence game.

Let us suppose there are two couples — the Old Couple and the Young Couple — who are eager to find a new residence in a couple of months. It can naturally expected to see that these two couple have similar or different opinions as to where to live and how they live. For instance, the Old Couple want to live in the Country with beautiful natural environments, and hopefully in the neighborhood of the New Couple. While the New Couple may instead prefer to live in the City, in which housing facilities are modern and transportation means are well-developed, they wish to live separately from and independently of their parents. As Russian writer Ivan S. Turgenev (1818 - 1883) splendidly described in his famous novel *Fathers and Sons* (1862), the Old Generation may easily clash with the Young Generation over an even trifling problem, possibly causing a matter of life and death.

Let us suppose that we send a questionnaire on residence preference to the two couples. The questionnaire asks a respondent about his/her choice of living in the country or living in the city, and of living together (or living closely in the neighborhood) or living separately (or living far away) from relatives. On the one hand, we are informed that the Old Couple's first choice is to live in the country and its second choice is to live together with sons or daughters. On the other hand, however, we are informed that the Young Couple's first choice is to live in the city and their second choice is to live separately from old parents. For simplicity, as is seen in Table 3.1, let us assume four different scores: 10 points, 5 points, 3 points, and 1 point.

As is clear in Table 1, a generations-gap problem is quite severe between the Old Couple and the Young Couple, so that finding a solution seems to be quite difficult. Between the two couples, both the first and second choices are entirely different. Under such circumstances, we will show that relying on the Nash equilibrium is so helpful for clear understanding. As the saying goes, seeing is believing !

We are now in a position to set up a sort of "Residence Game" as is shown in Fig. 1. There are two players in the game. They are "the Old Couple" and "the Young Couple." Each player, the Old Couple or the Young Couple, has the two possible strategies: namely, "living in the Country" and "living in the City." It is noted that each box in the matrix has a pair of numbers, where the first and second numbers respectively denote "the payoff of the Old Couple" and "the payoff of the New Couple." For instance, the pair (10, 1) in the upper left box indicates that when the both couples agree to live together (or at least to live in the neighborhood), the Old Couple gains 10 units of payoff and the Young Couple one unit of payoff. The pair (5, 10) in upper right box shows that when the Old Couple live in the Country and the Young Couple in the City, the payoffs gained by each couple are respectively 5 units and 10 units. Similar interpretations can be done for the pair (1,3) in the lower left box, and the pair (3, 5) in the lower right box.

Table 1 Rating Results of the Questionnaire I

Payoffs	Old Couple	Young Couple
10 points	Living in the country and together	Living in the city and separately
5 points	Living in the country And separately	Living in the city and together
3 points	Living in the city And together	Living in the country and separately
1 point	Living in the city And separately	Living in the country and together

Young Couple

		Country	City
Old Couple	Country	(living together) 10, 1	(living separately) $\boxed{5,\ 10}$
	City	(living separately) 1, 3	(living together) 3, 5

Fig. 1 Residence Game I as the starting point :
The pair (5,10) represents only one solution.

If we take a careful look at Fig. 1, then we can learn that the pair (5, 10) in the upper right box stands for only one Nash Solution. (This pair is enclosed by a small rectangle.) Let us explain why this is so. Now, suppose that the Old Couple live in the country and the New Couple in the city. Then, we can show that neither the Old Couple nor the New Couple have incentives to do separate activities. On the one hand, if the Old Couple unilaterally move from the country to the city, then their payoff must decrease from 5 points to 3 points. On the other hand, if the Young Couple dares to move from the city to the country, then their payoff has to dramatically decline from 10 points to 1 point. Therefore, each player should not change his/her strategy unless his/her opponent changes residence strategy. In short, once Nash equilibrium is achieved, the best strategy each player can take is to just obey the following conservative maxim: "Stay where you are ! " .

Moreover, we can show that the remaining three pairs cannot be Nash equilibriums. For instance, let us pick up the pair (10. 1) in the north-east box. It is true that if the Old Couple move from the country to the city, then their payoffs will decrease from 10 units to one unit. So, the Old Couple has no intention whatever to change their residence. The New Couple, however, would have a different opinion, for their change of residence from the country to the city would definitely increase their payoff from 1 unit to 10 units. For similar reasons, the pairs (1,3) and (3,5) would not be qualified for solutions.

In short, in the present Residence Game Ⅰ, the pair (5, 10) would be a "tentative solution, " if not a "permanent solution." As far as we believe in usefulness of Nash equilibrium, there would exist no other solutions whatever. We bear in mind, however, that the Nash solution would represent only one possible solution, not a " perfect solution for guaranteeing happiness of all members." In fact, although the pair (5, 10) would make the Young Couple very happy with 10 units, the Old Couple's happiness would not be so high with mere 5 units. In other words, the present generation-gap solution is barely supported by suppression of old parents.

Next, we will pay attention to some other types of residence games. Let us recall that in Residence Game Ⅰ above, the Old Couple's first choice is to live in the country and their second choice is to live together with children whereas the Young Couple's first choice is to live in the city and their second choice is to live separately from parents. If we interchange the first choice and the second choice, we are expected to get into the new situation. Now, in the Residence Game Ⅱ, whereas the Old Couple's first choice is to live together with children, and their second choice is to live in the country, the New Couple's first choice is to live separately from parents, and their second choice is to live in the city. More specifically, we obtain the *new* questionnaire results as seen in Table 2, from which we can derive the *new* payoff matrix of this Residence Game Ⅱ in Fig. 2 .

Table 2 Rating Results of the Questionnaire Ⅱ

Payoffs	**Old Couple**	**Young Couple**
10 points	Living together and in the country	Living separately and in the city
5 points	Living together and in the city	Living separately and in the country
3 points	Living separately and in the country	Living together and in the city
1 point	Living separately and in the city	Living together and in the country

		Young Couple Country	**Young Couple** City
Old Couple	**Country**	(living together) 10, 1	(living separately) 3 , 10
Old Couple	**City**	(living separately) 1 , 5	(living together) 5 , 3

Fig. 2 Residence Game Ⅱ as the starting point :

There are no solutions at all..

Let us compare this payoff matrix of Residence Game Ⅱ with the last payoff matrix of Residence Game Ⅰ. Then, we will see that the two pairs of payoffs (10, 5) and (3, 5) *on* the diagonal are the same in the two matrices. The two pairs of payoffs *off* the diagonal, however, are a bit different: exactly, the locations of the two numbers, 5 and 3, are just interchanged. Remarkably, such an apparently small difference will cause a substantial difference on the existence of a solution.

To our surprise, in this new residence game, there are no Nash solutions at all ! Whichever box we would start with in Residence Game Ⅱ, either Parents or Children would have good incentives to change their strategies. For instance, even if the pair (10, 1) in the upper-left box is initially chosen, the strategy change of Children from the country to the city would guarantee a big rise in their payoff from 1 to 10. Similarly, the pair (1, 5) cannot represent a Nash solution because the strategy change of Parents from the City to the Country would surely increase their payoff from 1 to 10. Similar interpretation would be possible if the remaining pairs are chosen as starting points. .

Summing up, if Parents and Children playing Residence Game Ⅱ, the generations gap between them would be too huge to be reconcilable. Parents are not happy, nor are Children. So, this would be the worst situations we could think of.

Now, we are in a position to adequately combine Residence Games Ⅰ and Ⅱ, thus attempting to take a "middle position. " While the Old Couple's choice is firstly to live together with Children and secondly to live in the country, the Young Couple's choice is firstly to live in the city and secondly to live separately from Parents. Presumably, such "crossing of preferences between Parents and Children" seems to frequently occur in today's Japan.

In this Residence Game Ⅲ, the questionnaire results are summarized in Table 3 , with the corresponding payoff matrix being shown in Fig. 3. As can easily be seen in Fig. 3, there should exist only one possible solution, which is represented by the pair (5,5) in the lower-right box. In fact, once Parents and Children are " cornered " into this box, they should have no incentives to unilaterally change their strategies. Since this pair guarantees only a "second best solution for Parents and Children, they have to feel "moderately happy" rather than " perfectly happy." In fact, under such middle situations," only each couple's first choice is realized but their second choice is neglected. At any rate, although the generations-gap problem may happen at any time and in any country, it would really be a very tough problem to perfectly solve.

Table 3 Rating results of the questionnaire Ⅲ

Payoffs	Old Couple	Young Couple
10 points	Living together and in the country	Living in the city and separately
5 points	Living together and in the city	Living in the city and together
3 points	Living separately and in the country	Living in the country and separately
1 point	Living separately and in the city	Living in the country and together

Young Couple

		Country	City
Old Couple	Country	(living together) 10, 1	(living separately) 3 , 10
	City	(living separately) 1 , 3	(living together) $\boxed{5 , 5}$

Fig. 3 Residence Game Ⅲ : The only one possible solution (5, 5) makes Parents and Children "moderately happy," but " not perfectly happy."

Now, let us turn our attention to the fourth possibility of multiple equilibrium. We are now concerned with the more friendly situation under which Parents and Children share the same residence option of living together. In spite of similarity of the first options. however. their second option is entirely different. Whereas Parents want to live in the peaceful country, Children wish to live in the convenient city. Then, we can obtain the *new* questionnaire results as seen in Table 4, from which we immediately derive the *new* payoff matrix of this Residence Game Ⅳ in Fig. 4.

Table 4 Rating results of the questionnaire Ⅳ

Payoffs	Old Couple	Young Couple
10 points	Living together and in the country	Living together and in the city
5 points	Living together and in the city	Living together and in the country
3 points	Living separately and in the country	Living separately and in the city
1 point	Living separately and in the city	Living separately and in the country

		Young Couple	
		Country	City
Old Couple	Country	(living together) 10, 5	(living separately) 3 , 1
	City	(living separately) 1 , 3	(living together) $\boxed{5 ,\ 10}$

Fig. 4 Residence Game Ⅳ : There are two Nash equilibriums, i.e. (5, 1) and (1, 5)

A "third compromise plan" would be on the agenda.

As is quite clear in Fig. 4, there are TWO solutions. They are the pair (10, 5) in the upper left box, and the pair (5, 10) in the lower right box. If Parents and Children live together, Parents want to live in the lovely country, but Children wish to live in the exciting city. Unquestionably, it should be mission impossible" to realize Parents' wish and Children's wish simultaneously.

When there are TWO solutions. we are caught in a dilemma. If Parents' wish comes first, Children' wish is unduly ignored. And vice versa. Given two solutions. we cannot say which one is better than another. If such a dilemma is to be dissolved, there should be no way out other than *rolling the dice.* Between the two players, we would imagine the friendly contract by which " an odd number " (1 or 3 or 5) gives Parents' preference a priority, but " an even number " (2 or 4 or 6) gives Children's preference a priority. Perhaps, such agreement would remind the reader of the *mixed strategies* introduced by von Neumann and Morgenstern for the solution of zero-sum games.

In order to get out of the dilemma, rolling the dice between Parents and Children would look like a good idea. A deeper reflection, however, would tell us that this is not a practical idea whatever, for interchanging their homes periodically would be very troublesome, thus giving much inconveniences to each player.

According to the Classical Theory of Yasuma Takata (1959), it is the relative strength of the POWERS of the two players that finally determines where to live. If Parents have a stronger power than Children, then they would have to agree to live in the country. If Children succeed in persuading Parents, however, their choice of living in the country would prevail. Alternatively, the " invisible *kuuki* or social atmosphere " a la Shichihei Yamamoto (1997) would possibly prevail in the whole society, thus deeply influencing individual decisions. We believe that such sociological and historical considerations are very important but have been underestimated so far.

Ⅲ. Some Other Non-Zero-Sum Games Reexamined

The modern theory of games has been closely related to the development of modern theory of economics. In this connection, there are a number of games which have drawn much attention. In what follows, we will critically reexamine several popular games and their applications.

3.1 "The Battle of the Sexes" , or plainly "the Lover's Dilemma for a Date"

The first popular game to discuss below has a very exciting name "The battle of the sexes," which literally means that the male sex and the female engage in a serious battle. We do not know, however, how serious the battle is. Since such naming sounds a bit exaggeration to us, we propose that it should be changed to a milder name " The lover's dilemma."

Let us suppose that there are two players, Jack and Betty. They are the lovers who have different hobbies but are looking for a nice dating place. While Jack is very fond of seeing boxing, Betty likes to see ballet. Although each player wishes to see sport games with his/her partner, it is not an easy job for both players to reach an agreement. The worst scenario would be that they do not meet together. More specifically, let us suppose that the payoff matrix of such a dating game can be expressed by Fig. 5.

As Fig. 5 tells us, if Jack and Betty decide to see Boxing, then Jack is very happy (5 points) but Betty feels so-so (1 point). In contrast to this, if they determine to see ballet, Betty is so happy (5 points) but Jack feels so-so (1 point). As a third possibility, we could imagine the situation under which Jack goes to see Boxing but Betty chooses

		BETTY	
		Boxing	**Ballet**
JACK	**Boxing**	$\boxed{5,\ 1}$	−1 , −1
	Ballet	−1, −1	$\boxed{1,\ 5}$

Fig. 5: The lovers' dilemma for a date

Ballet, or the situation Betty goes to see Ballet but Jack chooses Boxing. Then, both lovers would feel lonely and miserable (minus 1 point).

As far as the present date game is concerned, there exist two Nash equilibriums, that is the pair (Boxing, Boxing) and the pair (Ballet, Ballet). At the first equilibrium, Jack is very happy but Betty is not so. At the second equilibrium, Betty feels so happy but Jack does not so. Such non-symmetric situation may exaggeratedly be called "The Battle of the Sexes," but it should reasonably be named " The Lover's Dilemma." The question which would immediately occurs in our mind is in which way we can get out of this terrible dilemma.

One possible way out would be the adoption of "mixed strategies" by the two players, i.e. Jack and Betty. For instance, they could play the well-known game of Stone-Paper-Scissors to determine whose preference, Jacks' or Betty's , should be given the first priority. Careful observation of lovers would tell us, however, that this would be an unworkable idea. There would be a more reasonable idea which both Jack and Betty agree to adopt as a " second best solution." Without a certain compromise on both sides, a love relationship would sooner or later come to an end.

In order to get out of the dilemma, let us suppose that Jack and Betty cleverly agree to adopt a "*third way* option of going to see Movie" instead of seeing Boxing or Ballet. This alternative proposal would not make Jack and Betty very happy, but only acceptable.

Now, as is seen in Fig. 6, Jack and Betty will play the "Expanded Date Game," in which the *third strategy* " Movie " is newly added. Presumably, the strategy " Movie " will give each player only 3 units of payoffs. Then, the pair (Movie, Movie) = (2,2) will be the third Nash solution. Understandably, this new solution would give less happiness to each player than the old solutions (Boxing, Boxing) and (Ballet, Ballet). They would nevertheless be satisfied with being together since they are true lovers.

The Game of Dating Lovers' Dilemma has many applications to economic and related problems. Let us apply it to the Bargaining between of the Labor Union and the Manager Association. Whereas the Labor Union aims to improve the working conditions of union members such as increases in wages and pensions, the Manager Association wishes to increase the business conditions of the Company. by means of personnel reduction. Each player wants to have his/her demand approved by the opponent. Even if the opponent's reply is not perfectly satisfactory, it is the the common knowledge among all players that the total breakdown should be be avoided at any cost. In the light of these considerations, we can form the

BETTY

	Boxing	* Movie	Ballet
Boxing	5 , 1	−1 , −1	−1 , −1
JACK * Movie	−1 , −1	* 3 , 3	−1 , −1
Ballet	−1 , −1	−1 , −1	1 , 5

Fig. 6 Introduction of a " third compromise option " of going to a movie

payoff matrix of the labor bargaining game as in Fig. 7.

As is seen in Fig. 7, there are *two* Nash solutions. The *first* solution (2, −1) shows the case of the worker's victory, but the *second* solution (−2, 3) means that of the manager's victory. In the real world, however, coexistence of two solutions may hardly be thinkable. In fact, the worker's position is generally weaker than the manager's position. As a result, a sort of "third compromise plan with a slight manager's bias" is likely to be carried out. It is in such a situation that the famous Theory of Power a la Yasuma Takata (1959) could be highly effective.

		MANAGER	
		Wage Increase	Personnel Reduction
WORKER	Wage Increase	(settlement) $\boxed{2,\ -1}$	(breakdown) −5 , −3
	Personnel Reduction	(breakdown) −5, −3	(settlement) $\boxed{-2\ ,\ 3}$

Fig. 7 The Worker-Manager Bargaining Game

3.2 The Prisoner's Dilemma

We will turn to the next problem of the prisoner's dilemma. After this problem was skillfully formalized by game theorist Albert W. Tucker, its popularity has constantly increased until today, with many economic and related applications.

Let us suppose that one day, after a bank robbery took place in the city, two persons, Adolf and Otto, were arrested as suspects. While they are now in the jail, each prisoner is in solitary confinement with no communications between them. Although, the prosecutors do not have sufficient evidence to convict the pair on the serious charge of bank robbery, they obtain some evidence of convicting both on the minor charge of possession of illegal articles. So, the prosecutors has decided to offer each suspect a plea bargaining. Then, each suspect is given the chance either to unilaterally confess the pair's crime or to still keep silent. To sum up, as is seen in Fig. 6, we will have the have the following payoffs:

(1) If both Adolf and Otto keep silent, then they will serve two years in prison.
(2) If both Adolf and Otto confess the pair's crime, then they will serve 6 years in prison.
(3) If Adolf (or Otto) keeps silent but Otto (Adorf) confesses the crime, Adolf (or Otto) will have to serve 9 long years in prison but Otto (or Adorf) will be set free.

In this prisoner's dilemma game, there exists only one solution (Confess, Confess) = (−6, −6) . This surely represents a Nash equilibrium, for a unilateral different action by one person would damage his position badly. The golden rule here is to "just stay where you are."

To tell the truth, the solution (−6, −6) is not only a Nash solution, but also a "stronger solution" or a "dominant strategy solution." In order to understand this, let us first assume that opponent Otto keeps silent. Then, Adolf 's payoff is (−2) if he also keeps silent, and 0 if he makes a confession, which shows that when Otto keeps silent, Adolf 's best strategy is to confess a crime. Next, let us assume that Otto makes a confession. Then, Adolf 's payoff is (− 1) if he keeps silent, and (−6) if he makes a confession, which shows that when Otto makes a confession, Adolf 's best strategy is also to confess a crime. Therefore, whether Otto keeps silent or makes a confession, Adolf 's best strategy is to confess a crime. In other words, to make a confession is Adolf 's " dominant strategy. " Similarly, to confess is also Otto's "dominant strategy." In short, the pair (Confess, Confess) represents a "dominant strategy equilibrium."

		OTTO	
		Silent	**Confess**
ADOLF	**Silent**	(lesser charge for both) −2, −2	(9 yrs prison. set free) −9 , −1
	Confess	(set free, 9 yrs prison) −1, −9	(6 yrs in prison for both) $\boxed{-6\ ,\ -6}$

Fig. 8 The prisoner's dilemma

Now, we are ready to compare the following two pairs; (Silent, Silent) = (−2, −2) and (Confess, Confess) = (−6, −6). Clearly, for the two persons, the strategy " Silent " is better than the strategy " Confess ." If a communication between both were allowable, then each suspect would make the following promise between both ; " I will keep silent whenever you keep silent. Please keep the promise." Such exchange of private opinions, however, is not possible because each prisoner is in solitary confinement with no communications allowable. .

To summarize, when we are engaged in the prisoner's dilemma, we can recognize a serious gap between " individual rationality " and " whole rationality." In the prisoner's dilemma aforementioned, whereas the pair (Confess, Confess) seems to represent a sort of rationality, it does not produce the maximum level of the whole group . Each member's interest at the " micro level" does not necessarily correspond to the group interest at the " macro level. " Micro is Micro, and Macro is Macro. Those two do not always go in the same direction.

The present prisoner's dilemma has many politico-economic applications. One of good examples is concerned with the " arms race. " Let us recall the famous 1940 Charlie Chaplin Movie " The Great Dictator," in which Hynkel as the dictator of Tomainia, and Napoloni as the dictator of Bacteria are fiercely involved in arms race negotiations for the world hegemony. As is seen in Fig. 9, the two rival countries have the two strategies: arms reduction and arms expansion.

There are several possibilities. If Hynkel and Napoloni maintain friendly relations and succeed in arms reduction negotiations, then they can utilize a part of military budget to the economy and welfare (plus 1 unit for payoff). If both countries are in hostile relations and decide arms expansion, then they will have to use more money for military budget (minus 1 unit). If Hynkel (or Napoloni) unilaterally pushes arms reduction, then the power of Tomania (or that of Bacteria) will decrease (minus 3 units) and the power of Bacteria (or that of Tomania) will increase (plus 3 units).

Clearly, a sort of the prisoner's dilemma takes place in the arms race. For Tomainia and Bacteria, arms expansion is a dominant strategy, so that the pair (Expansion, Expansion) = (−1, −1) represents a dominant strategy equilibrium. This equilibrium point is not only inferior to the pair (Reduction, Reduction) = (1, 1), but also undesirable from global welfare. As far as Tomainia and Bacteria do not trust each other, however, they will have to go on the way of arms expansion. As history tells us, human history is no more than a history of expansion, explosion and destruction.

Another example of the prisoner's dilemma can be given by the "advertisement competition," which seems very excessive in reality. The basic framework of the

		NAPALONI	
		Reduction	Expansion
HYNKEL	Reduction	(friendly) 1 , 1	(unilateral) −3 , 2
	Expansion	(unilateral) 2, −3	(hostile) $\boxed{-1 , -1}$

Fig. 9 The arms race: friendly, hostile, or unilateral

advertisement competition has exactly the same as that of the arms race. We should now carry out a skillful reinterpretation task. Specifically, the two players Hynkel and Napoloni should be replaced by Tsukuba Company and Fuji Company respectively. Moreover, the two strategies "Reduction" and "Expansion" should be reinterpreted as " Advertisement Reduction " and " Advertisement Expansion ," but not as " Arms Reduction" and " Arms Expansion." Then, following the same analogy as before, we can easily show that the pair (Ad Expansion, Ad Expansion) is a dominant strategy equilibrium, thus the resulting competition between Tsukuba and Fuji Companies will be fierce and even excessive. Needless to say, excessive competition will yield the waist of financial and human resources. As the saying goes, doing too much is like doing too little.

Ⅳ Game Theory and its Possible Defects : " Econs " versus " Humans "

Soseki Natsume (1867-1916) is one of the most popular writers in Modern Japan.. In the first page of his well-known book *Kusa-makura, or Lying on the Grass*, Natsume (1906) once remarked:

> Going up a mountain path, I have been thinking of the present world. If you approach

> everything too rationally, you will make a lot of troubles. If you use a pole in the stream of big emotions, you will be swept away by the strong current. If you faithfully rely on strong passions, you will be unduly confined. In short, the present world is not a very good place to live in. (Natsume 1906, p. 1)

As Natsume tells us, we live in the world where so many people with so many personalities live together. Roughly speaking, there are THREE factors which control human nature. They are Rationality, Emotion, and Passion. If we firmly believe in the power of rationality on the basis of cost and benefit, we may often offend the feeling of other people. If we are influenced by the strong power of emotion, we may sometimes lose the sense of balance, thus being swept away from important common sense. If we are strong passion lovers, our behavior will tend to go from one extreme to another. In short, any person should have a balanced mixture of Rationality, Emotion, and Passion.

Let us remind the reader of the exact title of the great book by John von Neumann and Oskar Morgenstern , *Theory of Games and Economic Behavior*. Here, "economic behavior," NOT "human behavior" is connected with traditional game theory. Needless to say, "economic behavior" should be closely related to Rationality, thus underestimating the influence of Emotion and Passion. It is in this sense that traditional game theory has a defect in a system, which must be mended as soon as possible.

According to Richard H. Thaler (2015), once a Rochester classmate and later a Nobel Prize winner, many traditional models tend to employ a fictional creature called "Econs." Econs are a kind of "rational fools" a la Amartia Sen (1987) in the sense that, as is seen in traditional game theory, they rationally choose strategies by optimizing their payoffs. In other words, among the Three Natures of Human Beings, only Rationality is unduly picked up, with negligence of Emotion and Passion.

In sharp contrast to the above argument, "Humans" are no more than human beings, namely homo sapience, Humans do a lot of misbehaving, meaning that traditional economic models lead to a lot of wrong predictions. Humans are supposed to possess many non-rational feelings such as envy, hatred, optimism, pessimism, sympathy, antagonism, and the like.

In the real world, not in the fictional one, we are Humans rather than Econs ! It is high time for us to bravely go beyond traditional game theory with the aim of establishing a brand new theory of human behavior. We need the second von Neumann and the second Morgenstern !

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Footnotes

1)　For this point, see Yasui (1979).　Yasui was one of the pioneers who tried his best to build a bridge between the old traditional economics and the modern economic science for his long years.

2)　See Yasui (1979).

3)　For details, see Yasui (1979)

4)　For details, see Sakai (1990a), Sakai (1990b), Sakai (1991), and Sakai & Sasaki (2021).　.

5)　For details, see Nasar (1998)